

## Identifying parameter sensitivity in a water quality model of a reservoir

Yongtai Huang and Don Pierson

### ABSTRACT

Sensitivity analysis can provide useful insights into how a model responds to the variations in its parameter values (i.e. coefficients). The results can be very helpful for model calibration, refinement and application. A one-dimensional model has been set up to simulate the hydrothermal and water quality conditions of Cannonsville Reservoir, which provides water supply for New York City. This paper aims at identifying the most influential parameters in the model through sensitivity analysis. Firstly, the Morris method (a screening method) is used to identify influential parameters. It is found that 18 parameters are important in simulations of variables that include temperature, dissolved oxygen (DO), total phosphorus (TP) and chlorophyll *a* (Chl<sub>a</sub>). Secondly, the method is enhanced to investigate the global sensitivity of the parameters. It highlights 20 parameters that are sensitive in the simulations of the above-mentioned variables. The 18 parameters identified by the original Morris method are among the 20 parameters and the other two parameters are not very sensitive. The results show that similar results can be obtained through the original and enhanced Morris methods, although they each have their own strengths and weaknesses.

**Key words** | coefficients, Morris method, sensitivity analysis, water quality model

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### INTRODUCTION

Water quality models are usually developed to simulate a large number of variables, such as water temperature, dissolved and particulate nutrients, phytoplankton and dissolved oxygen (DO). Such models often contain many parameters (also called coefficients) associated with the many processes and state variables simulated. Identifying the model coefficients that are of greatest importance in influencing the model output of interest can help a modeler focus attention on the most important processes being simulated and the coefficients and algorithms associated with these. This can in turn lead to informed calibration, critical examination of model algorithms, and process studies to better define coefficient values and confirm the validity of the model algorithms. Sensitivity analysis can be used to identify sensitive parameters, and support the process of model calibration and refinement.

Several methods can be used to perform sensitivity analysis. These methods can be divided into two broad

categories: local and global. Local sensitivity methods (such as differential analysis) calculate local gradients of the model output with respect to infinitesimal variations of a factor (such as a model coefficient). They are simple to implement and are not computationally demanding.

Global sensitivity analysis methods are receiving considerable attention since they can provide information about the influences of the variations of multiple factors within their feasible range of variability. These methods are excellent in performing sensitivity analysis of model parameters and some of them, such as the Fourier amplitude sensitivity test (FAST) (Deflandre *et al.* 2006), generalized sensitivity analysis (GSA) (Cox & Whitehead 2005), adjoint sensitivity method (Piasecki 2004), Monte Carlo analysis (Bobba *et al.* 1996), and global variance-based sensitivity analysis (Estrada & Diaz 2010), have been applied in surface water quality modeling. Unfortunately, these methods are computationally demanding, meaning that

they require a large number of model executions. Hence, their applications are mostly limited to the models that are not computationally expensive or have a small number of parameters. For the water quality models that are computationally costly, the sensitivity analysis methods that require a small number of model executions are preferred.

The Morris method is not computationally demanding for sensitivity analysis in comparison with other global methods (Morris 1999; Saltelli *et al.* 2004). Although it is a 'one factor at a time' (OAT) method, in which only one input factor is modified between two successive runs of the model, it may cover the whole range of input factors in successive runs, i.e. the input factors are varied over their possible ranges. In this sense, the method can be regarded as global (Saltelli *et al.* 2004). This method has been used in several fields, such as water quality modeling (Huang & Liu 2008), watershed modeling (Francos *et al.* 2003), climate change prediction (Campolongo & Braddock 1999) and laboratory ground water flow and solute transport modeling (Larsbo & Jarvis 2006).

In this paper sensitivity analysis is applied to a one-dimensional model which was set up to simulate the hydrothermal and water quality conditions of Cannonsville Reservoir, which is located in New York State and is part of the New York City water supply. The model is being applied to the evaluation of the effects of climate change and human activities on the hydrothermal structure and water quality of the reservoir. The main purpose of this study is to identify the sensitive parameters in the model. First of all, the Morris method is used to identify the sensitive parameters affecting the simulations of four output variables including temperature, DO, total phosphorus (TP) and chlorophyll *a* (Chl*a*). Secondly, the method is enhanced to find the overall sensitivity of the parameters in the simulations of the above-mentioned variables as a whole. The sensitive parameters are the ones to which most of the calibration efforts should be directed.

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## HYDROTHERMAL AND WATER QUALITY MODEL

A one-dimensional reservoir model has been developed by the Upstate Freshwater Institute (UFI) (Doerr *et al.* 1998; Owens 1998; UFI 2001) to simulate the hydrothermal and

water quality conditions in Cannonsville Reservoir. The model consists of three components: a hydrothermal sub-model, a nutrient sub-model, and a phytoplankton sub-model.

The hydrothermal sub-model simulates the vertical dynamics of reservoir thermal stratification and related transport regimes, based on changes in such critical (state) variables as meteorological, hydrological and operational conditions. The hydrothermal model can simulate the formation and break-up of ice cover, water temperature, stratification and mixing. It functions as the physical/mass transport framework for the water quality models.

The nutrient sub-model describes the transformation and fate of the nutrient loads in the reservoir (Doerr *et al.* 1998). Nutrient transformations occur within the model, which affect the form and bioavailability of the nutrient.

The PROTBASE (PROtech Based Algal Simulations) sub-model (Markensten & Pierson 2007) is used to simulate phytoplankton biomass. In the model, phytoplankton biomass is predicted in terms of algal carbon and is a balance between growth (photosynthesis) and losses due to respiration, grazing, sedimentation and outflow. Phytoplankton growth is limited by temperature, light and nutrient availability.

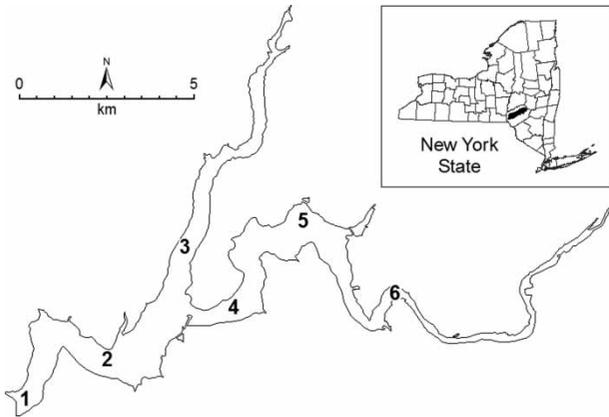
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## STUDY RESERVOIRS AND MODEL CONFIGURATION

### Cannonsville Reservoir

Located in Delaware County, New York, Cannonsville Reservoir was constructed as part of the water supply for New York City (see Figure 1). The reservoir is about 24 km long and on average 0.8 km wide. It has a surface area of 19.2 km<sup>2</sup> and a capacity of 362 million m<sup>3</sup>. Its maximum depth is about 42.7 m with an average depth of 18.6 m.

With the drainage area of 1,178 km<sup>2</sup>, the Cannonsville watershed has a relatively large percentage of agricultural land use, and there are a number of sewage treatment plants and other point sources in the watershed. These factors in the past led to high point and non-point nutrient loading to the reservoir, high average chlorophyll concentrations and frequent phytoplankton blooms. An extensive program of watershed management, as well as wastewater,



**Figure 1** | Cannonsville Reservoir and sampling stations 1 through 6.

storm water and septic system infrastructure improvement has greatly reduced these problems in recent times.

### Model configuration

To set up the one-dimensional model to simulate the hydro-thermal and water quality conditions, the reservoir is discretized into 35 vertical layers with the average thickness of 1.5 m per layer. The model inputs include daily meteorological data (air temperature, dew-point temperature, wind speed and solar radiation) and other data related to the water balance (water elevation, discharge, dam spill and aqueduct outflow). In addition, the following time series input data (which are generated by a separate watershed model) are also input for the model: (1) streamflow, (2) dissolved phosphorus and nitrogen from non-point and point sources, (3) particulate phosphorus from non-point and point sources, (4) dissolved organic carbon from non-point sources, (5) total suspended solids (TSS), (6) silicon load and (7) inflow temperature.

The model can be configured to simulate daily values for a large number of variables, such as DO, TP, Chl<sub>a</sub>, in the epilimnion and hypolimnion layers or averaged over the water column as a whole.

According to model description (UFI 2001), 116 hydro-thermal and kinetic parameters (coefficients) are used in the model. Among them, 52 can be adjusted during model calibration. The names and definitions of the sensitive parameters, as well as their lower and upper bounds, are presented in Appendix A (available online at <http://www.iwaponline.com/wqjrc/047/016.pdf>).

## METHODOLOGY

### Morris method

The Morris method was employed to identify the sensitive coefficients in the model described above. It determined a sensitivity ranking for parameters in terms of their mean effects on model outputs as well as their non-linear and interaction effects (Saltelli *et al.* 2004). This method is based on what is called an elementary effect and is described in detail as follows (refer to literature such as Saltelli *et al.* (2004) for simple examples).

Assume that the model output  $y = y(\mathbf{x})$  is a scalar function of the vector  $\mathbf{x}$  of parameters. The vector  $\mathbf{x} = (x_1, \dots, x_i, \dots, x_N)$  has  $N$  parameters. The range of each  $x_i$  is normalized to be within 0 and 1. Assume each  $x_i$  may have a number of discrete values in the set  $\{0, 1/(p-1), 2/(p-1), \dots, 1\}$ , where  $p$  is the number of levels. A perturbation factor  $\Delta$  is defined as a multiple of  $1/(p-1)$ . The elementary effect of the  $i$ th parameter is defined as:

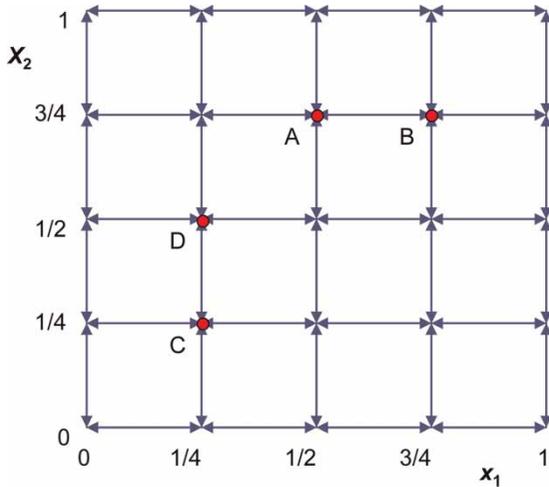
$$d(x_i) = [y(x_i, \dots, x_{i-1}, x_i + \Delta, x_{i+1}, \dots, x_N) - y(\mathbf{x})] / \Delta \quad (1)$$

where  $d(x_i)$  represents the elementary effect of parameter  $x_i$  on the model output. In the case that a water quality model outputs simulated values at several locations in a water body for a number of time points, the elementary effect of the  $i$ th component of  $\mathbf{x}$  on the output can be calculated using the following equation:

$$d(x_i) = \frac{1}{M} \sum_{k=1}^M [y_k(x_1, \dots, x_{i-1}, x_i + \Delta, x_{i+1}, \dots, x_N) - y_k(\mathbf{x})] / \Delta \quad (2)$$

where  $M$  represents the total number of simulated values at all locations and time points;  $y_k(\mathbf{x})$  is the  $k$ th output.

A five-level grid in a two-dimensional input space ( $N = 2$ ) is provided as an example to demonstrate how elementary effects are calculated (see Figure 2). To evaluate the elementary effect of  $x_1$  between A and B on the output, the outputs at points A and B need to be evaluated. In the same way, to evaluate the elementary effect of  $x_2$  between C and D on the output, the outputs at points C and D need also to be



**Figure 2** | The representation of a five-level grid ( $p = 5$ ) in the two-dimensional input space ( $N = 2$ ,  $\Delta = 1/4$ ). Each arrow identifies the couple of points needed to compute one elementary effect. The horizontal arrows identify the 20 elementary effects of  $x_1$ , while the vertical ones identify the 20 elementary effects of  $x_2$ .

calculated. It can be observed in this figure that  $5^2 = 25$  model executions are needed to evaluate 40 elementary effects of  $x_1$  and  $x_2$ . For an  $N$ -dimensional input space, the number of model executions is  $p^N$ . It is computationally unaffordable to perform such a large number of model executions for most numeric water quality models. In other words, it is impossible to evaluate all elementary effects of model parameters on the model. As a result, the Morris method tends to obtain the finite distribution of elementary effects associated with the parameters.

Three steps are involved in obtaining the finite distribution of elementary effects associated with the parameters. Firstly, a model run is performed based on the random selection of parameters from the set  $\{0, 1/(p-1), \dots, 1-\Delta\}$ . Secondly, a single parameter,  $x_i$ , is randomly selected and modified by an increment  $\Delta$ , and a second run is performed. Thirdly, the elementary effect of  $x_i$  is calculated using Equation (2). This procedure is repeated until stable output statistics, including (arithmetic) mean and standard deviation of elementary effects for each parameter, have been achieved.

However, in the above method, two model runs are needed for evaluating each elementary effect. The total computational effort required for obtaining  $R$  elementary effects for each of  $N$  parameters is  $2NR$  model runs, which is computationally demanding. Morris suggested a more efficient

sampling method. The sampling method starts by randomly selecting a 'base' vector  $\mathbf{x}^*$ . Each component  $x_i$  of  $\mathbf{x}^*$  is sampled from the set  $\{0, 1/(p-1), 2/(p-1), \dots, 1\}$ . Note that the vector  $\mathbf{x}^*$  is used to generate the other sampling points but it is not one of them. The model is never evaluated at  $\mathbf{x}^*$ . The first sampling point,  $\mathbf{x}^{(1)}$ , is obtained by increasing one component of  $\mathbf{x}^*$  by  $\Delta$ . The second sampling point is generated from  $\mathbf{x}^*$  with the property that it differs from  $\mathbf{x}^{(1)}$  in its  $i$ th component that has been either increased or decreased by  $\Delta$ . The third sampling point,  $\mathbf{x}^{(3)}$ , is generated from the base value  $\mathbf{x}^*$  with the property that  $\mathbf{x}^{(3)}$  differs from  $\mathbf{x}^{(2)}$  for only one component  $j$ , for any  $j \neq i$ . The sampling method proceeds producing a succession of  $(N+1)$  sampling points  $\mathbf{x}^{(1)}, \mathbf{x}^{(2)}, \dots, \mathbf{x}^{(N+1)}$ , with the key property that two consecutive points differ from each other in only one component. These sampling points define what is called a trajectory in the input space. In this sampling method, any component of the base vector  $\mathbf{x}^*$  has been selected at least once to be increased or decreased by  $\Delta$  in order to calculate one elementary effect for each factor.

In this sampling method, the total computational effort required for obtaining  $R$  elementary effects for each of  $N$  parameters is  $(N+1)R$  model runs. Although a characteristic of this sampling method is that the points belonging to the same trajectory are not independent, the  $R$  elementary effects for each parameter are from different trajectories and are independent. The mean of elementary effects for parameter  $x_i$  can be calculated by:

$$\mu(x_i) = \sum_{j=1}^R d_j(x_i)/R \quad (3)$$

where  $d_j(x_i)$  represents the  $j$ th elementary effect of  $x_i$  on the output  $y$ , where  $j = 1, \dots, R$ ;  $\mu(x_i)$  represents the mean of  $d_j(x_i)$  over  $R$  elementary effects and estimates the overall effect of the  $i$ th component of  $\mathbf{x}$  on the output. The standard deviation of the elementary effects for parameter  $x_i$  is given by:

$$\sigma(x_i) = \sqrt{\sum_{j=1}^R [d_j(x_i) - \mu(x_i)]^2 / R} \quad (4)$$

where  $\sigma(x_i)$  represents the standard deviation of  $d_j(x_i)$ .

The Morris measure,  $\mu(x_i)$ , has a drawback. If  $d_j(x_i)$  contains positive and negative values, their effects may cancel each other out. Thus, the measure is not reliable for ranking factors in order of sensitivity. To overcome this drawback, the absolute mean effect across  $R$  runs can be used, which is calculated by:

$$\mu^*(x_i) = \sum_{j=1}^R |d_j(x_i)|/R \quad (5)$$

where  $\mu^*(x_i)$  represents the absolute mean of  $d_j(x_i)$ .

The statistic  $\mu^*(x_i)$  reveals the influence of parameter  $x_i$  on the model output. The larger the value of mean  $\mu^*(x_i)$ , the more sensitive the parameter is.

The statistic  $\sigma(x_i)$  discloses important information about the model. A large measure of spread, i.e. a high value of the standard deviation,  $\sigma(x_i)$ , indicates that the parameter has a non-linear effect on the output, or is involved in interaction with other parameters, or both. That is to say, the elementary effects of this parameter are significantly different from each other and are strongly dependent on the location in input space at which they are computed. On the contrary, a low  $\sigma$  value indicates very similar values of the elementary effects, implying that the effect of  $x_i$  is almost independent of its value or the values taken by other factors.

The following example is used to illustrate the Morris method. Considering a vessel representing a volume segment of surface water, the concentration of DO in it can be calculated using the following equation (Thomann & Mueller 1987):

$$c = c_s - \frac{k_d L_0}{k_r} [1 - \exp(-k_r t)] \quad (6)$$

where  $c$  is the concentration of DO in the vessel (mg/L);  $c_s$  is the saturated concentration of DO in the vessel (mg/L);  $k_d$  is the effective deoxygenation rate ( $T^{-1}$ );  $L_0$  is the initial dissolved and particulate carbonaceous biochemical oxygen demand (CBOD) concentrations (mg/L);  $k_r$  is the overall loss rate of CBOD from the water column due to both settling and oxidation of soluble BOD; and  $t$  is time ( $T$ ).

Equation (6) can be taken as a model with two parameters and one output:  $c = f(k_d, k_r)$ . Assume  $c_s = 16$  mg/L,  $L_0 = 10$  mg/L,  $t = 5$  d, the range of  $k_d$ : 0.1–0.4/d, and the range of  $k_r$ : 0.2–0.5/d.

Suppose the ranges of  $k_d$  and  $k_r$  are discretized into four levels (i.e.  $p = 4$ ). The discrete values are shown in Figure 3. To calculate the absolute mean of elementary effects and the standard deviation of elementary effects of  $k_d$  and  $k_r$ , the Morris method was performed for five runs. (It is worthy of mention that more runs are necessary in practice uses.) The first run of the Morris method started from ( $k_d = 0.1$  and  $k_r = 0.2$ /d). As shown in Figure 3, the concentrations of DO with three pairs of different parameter values were calculated using Equation (6) as  $f(0.1, 0.2) = 12.8$  mg/L,  $f(0.2, 0.2) = 9.7$  mg/L,  $f(0.2, 0.3) = 10.8$  mg/L. The element effects of  $k_d$  and  $k_r$  were calculated as  $d(k_d) = [f(0.2, 0.2) - f(0.1, 0.2)]/\Delta = -9.5$  mg/L, and  $d(k_r) = [f(0.3, 0.2) - f(0.2, 0.2)]/\Delta = 3.4$  mg/L (in the example  $\Delta = 1/3$ ). In the same way, four more runs of the Morris method were performed. The calculated concentrations of DO and the element effects of  $k_d$  and  $k_r$  are presented in Figure 3.

The absolute mean of elementary effects and the standard deviation of elementary effects of  $k_d$  and  $k_r$  were calculated using Equations (5) and (4), respectively. The results are:  $\mu^*(k_d) = (9.5 + 6.5 + 5.5 + 7.8 + 5.5)/5 = 7.0$  mg/L,  $\sigma(k_d) = 1.69$  mg/L and  $\mu^*(k_r) = (3.4 + 5.1 + 1.3 + 2.0 + 3.9)/5 = 3.1$  mg/L and  $\sigma(k_r) = 1.54$  mg/L. The results show that  $k_d$  are more sensitive than  $k_r$ . In addition, the elementary effects of both  $k_d$  and  $k_r$  are different from each other and are dependent on the parameter values at which they are computed.

The Morris method is easy to implement and computationally economic in the sense that the number of model evaluations is linearly related to the number of parameters. Although the method relies on a sensitivity measure,

		$k_d$			
		0.1	0.2	0.3	0.4
$k_r$	0.2	12.8	-9.5*	9.7	6.5
	0.3	13.4	3.4**	10.8	5.1**
	0.4	13.8	-6.5*	11.7	-7.8*
	0.5	14.2	-5.5*	12.3	2.0**
	0.6	14.6	-4.5*	12.9	7.4

Figure 3 | Illustration of the calculation of the elementary effects of  $k_d$  and  $k_r$  on the concentrations of DO. \*Indicates the elementary effect of  $k_d$  and \*\*indicates the elementary effect of  $k_r$ .

called the elementary effect, which uses incremental ratios and is apparently a local measure, the final measure,  $\mu^*$ , is obtained by averaging several elementary effects at different points of the input space, so that it does not depend on the specific points at which the elementary effects are computed. In this sense, the method can be regarded as global in nature (Saltelli *et al.* 2004).

The Morris method described above can be used to identify the sensitive parameters affecting one output variable. After the sensitive parameters for each variable are found, these parameters can be listed together to visualize the important parameters for multiple output variables. The advantage of this method is that parameters can be added to or removed from the list easily according to the output variables of interest. The disadvantage is that if a large number of output variables are considered, examining the results may be a tedious task. In addition, the overall sensitivity of model output to the combined influence of parameters is unknown. The disadvantages can be overcome by the enhanced Morris method.

### Enhanced Morris method

The Morris method is enhanced to evaluate the overall sensitivity of model output to the combined affects of multiple parameters as follows:

By adding a subscript to indicate the variable, the elementary effect in Equation (2) can be rewritten as the following equation:

$$d_s(x_i) = \frac{1}{M} \sum_{k=1}^M [y_{sk}(x_1, \dots, x_{i-1}, x_i + \Delta, x_{i+1}, \dots, x_N) - y_{sk}(\mathbf{x})] / \Delta \quad (7)$$

where  $d_s(x_i)$  represents the elementary effect of the  $i$ th component of  $\mathbf{x}$  on the model output of the  $s$ th variable;  $y_{sk}(\mathbf{x})$  is the  $k$ th output of the  $s$ th variable, where  $s = 1, \dots, V$  and  $V$  is the total number of variables on which the overall sensitivity of the model output to the parameter are to be investigated.

The elementary effect,  $d_s(x_i)$ , in Equation (7) has the same unit as the model output of the  $s$ th variable. For different variables, it may have different units. For instance, the unit for the elementary effect for DO may be mg/L and the unit for temperature may be °C. To make the elementary

effects independent of the units of the model output, normalized elementary effects can be applied, which can be obtained by dividing the elementary effect by the average absolute simulated values and expressed mathematically as:

$$e_s(x_i) = M \times d_s(x_i) / \sum_{k=1}^M |y_{sk}(\mathbf{x})| \quad (8)$$

where  $e_s(x_i)$  is the normalized elementary effect of parameter  $x_i$  on the model output of the  $s$ th variable.

The overall elementary effect can then be calculated by:

$$e(x_i) = \sum_{s=1}^V w_s e_s(x_i) \quad (9)$$

where  $w_s$  is the weight of the  $s$ th elementary effect on the overall elementary effect and the sum of weights is equal to 1. Assigning different values to the weights allows overall elementary effects (sensitivity) of the variables to reflect the relative importance of the different model output variables to the particular model application. For example, when evaluating a model's ability to predict reservoir trophic status, a greater weighting would be given to the simulated output of chlorophyll and TP.

The mean of  $R$  overall elementary effects of parameter  $x_i$  can be calculated by:

$$\mu^a(x_i) = \sum_{j=1}^R e_j(x_i) / R \quad (10)$$

where  $\mu^a(x_i)$  represents the mean of  $e_j(x_i)$ .

The mean of  $R$  absolute overall elementary effects of parameter  $x_i$  can be calculated by:

$$\mu^{a*}(x_i) = \sum_{j=1}^R |e_j(x_i)| / R \quad (11)$$

where  $\mu^{a*}$  represents the absolute mean of  $e_j(x_i)$ .

The standard deviation of the overall elementary effects for parameter  $x_i$  is given by:

$$\sigma^a(x_i) = \sqrt{\sum_{j=1}^R [e_j(x_i) - \mu^a(x_i)]^2 / R} \quad (12)$$

where  $\sigma^a(x_i)$  represents the standard deviation of  $e_j(x_i)$ .

The statistic  $\mu^{a*}$  reveals the influence of parameter  $x_i$  on the overall model output. The larger the value of  $\mu^{a*}$ , the more sensitive the parameter is.

The statistic  $\sigma^a(x_i)$  indicates the relationship between parameter  $x_i$  with overall model output and other parameters. A high value of the standard deviation,  $\sigma^a(x_i)$ , indicates that the parameter has a non-linear effect on the overall model output, or a parameter that is involved in interaction with other parameters, or both.

## RESULTS AND DISCUSSION

### Sensitivity of parameters by Morris method

In the Morris method, the choice of the number of levels,  $p$ , is an open question. The value of  $p$  is linked to the sampling step  $\Delta$ . A convenient choice for the parameters  $p$  and  $\Delta$  is to assign an even value to  $p$  and let  $\Delta$  equal to  $p/[2(p-1)]$  (Saltelli *et al.* 2004). In addition, the choice of the value of  $p$  is related to the choice of the sample size,  $R$ . When  $R$  is small, it is likely that not all the possible factor levels are explored, while a high value of  $R$  implies more model runs. Morris (1991) used a sample size of  $R=4$ , which is taken as the minimum value to place confidence in the results. Previous experiments (Campolongo & Saltelli 1997; Campolongo *et al.* 1999; Saltelli *et al.* 2000) demonstrated that the choice of  $p=4$  and  $R=10$  produced valuable results. In this study,  $p=4$ ,  $\Delta=2/3$ , and  $R=30$  are used.

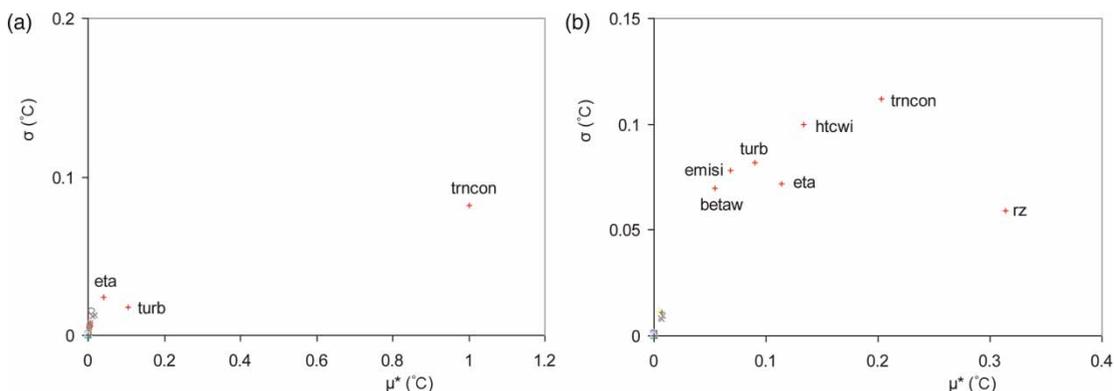
The sensitivity of the 52 parameters in the simulations of daily temperature, DO, TP and Chla in the epilimnion and

hypolimnion layers in Cannonsville Reservoir over the time periods of 1984–2004 is investigated. Stable results are achieved (the order of parameters did not change with the increase of the sample size  $R$ ) within 1,590 model executions [ $=R(N+1)=30 \times (52+1)=1,590$ ]. The results are analyzed and discussed for temperature, DO, TP and chlorophyll.

### Temperature

The Morris sensitivity measures  $\mu^*$  (the averaged absolute elementary effect) and  $\sigma$  (the standard deviation of elementary effects) are plotted in Figure 4 for the 52 model parameters in the simulation of temperature. It can be observed in Figure 4(a) that *trncon* (evaporation multiplier, see Appendix A – available online at <http://www.iwaponline.com/wqrj/047/016.pdf> for the definitions of all parameters) is the most sensitive parameter in the simulation of temperature in the epilimnion layer. Parameter *trncon* influences the evaporation of water and thus it affects the heat exchange between water and air. The second sensitive parameter is *turb* (atmospheric turbidity), which is used in the calculation of the vertical attenuation of solar radiation in the model. It affects the heat absorption of the epilimnion layer, but its effect is small. The third sensitive parameter is *eta* (wind mixing). It also affects the heat absorption of the epilimnion layer and its influence is small too. The other parameters have low means and standard deviations of elementary effects, meaning that the variations of their values have small influences on the simulated temperature.

As shown in Figure 4(b), the parameter, *rz* (diffusion exponent), has a high  $\mu^*$  and a low  $\sigma$  value. That is to say,



**Figure 4** | The Morris sensitivity measures  $\mu^*$  and  $\sigma$  for the 52 parameters in the simulation of temperature. Only the sensitive parameters are labeled. (a) Epilimnion, (b) Hypolimnion.

the variation of its value has a large influence on the simulated temperature in the hypolimnion layer no matter what value it takes or what values other parameters take. The additional six parameters including *trncon* (evaporation multiplier), *htcwi* (ice transfer), *eta* (wind mixing), *turb* (atmospheric turbidity), *emisi* (ice emissivity) and *betaw* (surface adsorption fraction) have high  $\mu^*$  values (although lower than that *rz* (diffusion exponent)) and their  $\sigma$  values are also high. That means changing the value of each parameter may have a great influence on the simulated temperature in the hypolimnion layer, but the influence depends on what value the parameter has and what values the other parameters have. The above seven coefficients are sensitive in the simulation of temperature in the hypolimnion. The remaining 45 parameters have low means and standard deviations of elementary effects, suggesting that the variations of their values result in only slight influences on the simulated temperature in the hypolimnion.

### Dissolved oxygen (DO)

The Morris sensitivity measures  $\mu^*$  and  $\sigma$  of the 52 model parameters in the simulation of DO are plotted in Figure 5. For the epilimnion, it can be observed in Figure 5(a) that *trncon* (evaporation multiplier) is the most sensitive parameter. It has a high value of  $\mu^*$  and a low value of  $\sigma$ , which means that it has a great effect on the simulated DO in the epilimnion layer and its effect does not depend on the values of other parameters. Additionally, seven other parameters including *sod* (sediment oxygen demand), *eta* (wind mixing), *kldoc* (oxidation of labile

DOC), *kc* (chlorophyll multiplier), *turb* (atmospheric turbidity), *rz* (diffusion exponent) and *betaw* (surface adsorption fraction) also have effects on the simulation of DO in the epilimnion. The remaining 44 parameters have low means and standard deviations of elementary effects, indicating that the variations of their values result in small influences on the simulation of DO in the epilimnion.

As shown in Figure 5(b), *sod* (sediment oxygen demand) is the most sensitive parameter in the simulation of DO in the hypolimnion. It has a high value of  $\mu^*$  but a low value of  $\sigma$ , which means that it has a great effect on the simulated DO in the hypolimnion layer and its effect does not depend on its value or the values of other parameters. Seven other parameters including *betaw* (surface adsorption fraction), *eta* (wind mixing), *rz* (diffusion exponent), *trncon* (evaporation multiplier), *htcwi* (ice transfer), *turb* (atmospheric turbidity) and *kc* (chlorophyll multiplier) are also found to influence the simulation of DO in the hypolimnion layer. The remaining 44 parameters have low means and standard deviations of elementary effects, implying that the variations of their values result in only slight influences on the simulation of DO in the hypolimnion layer.

### Total phosphorus (TP)

Figure 6 presents the sensitivity measures  $\mu^*$  and  $\sigma$  for the 52 parameters in the simulation of TP. It can be observed in the epilimnion layer (Figure 6(a)) that the parameters can be divided into four groups. The first group consists

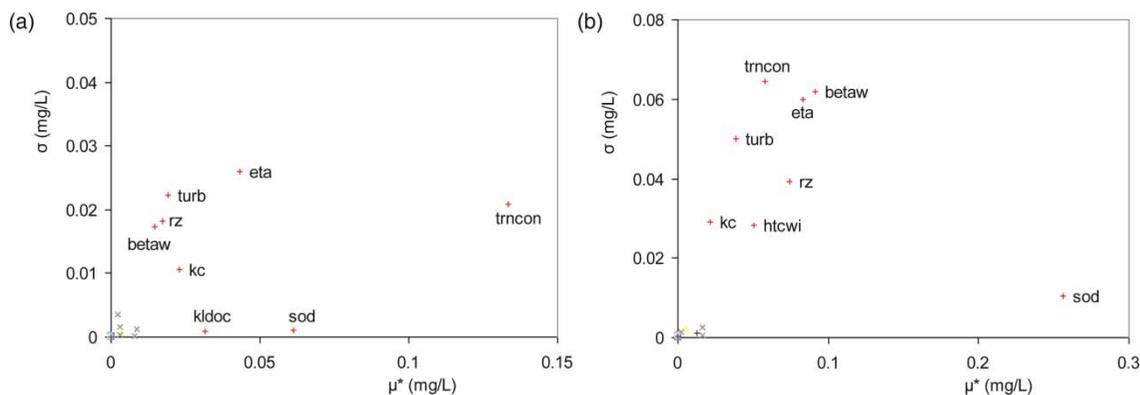
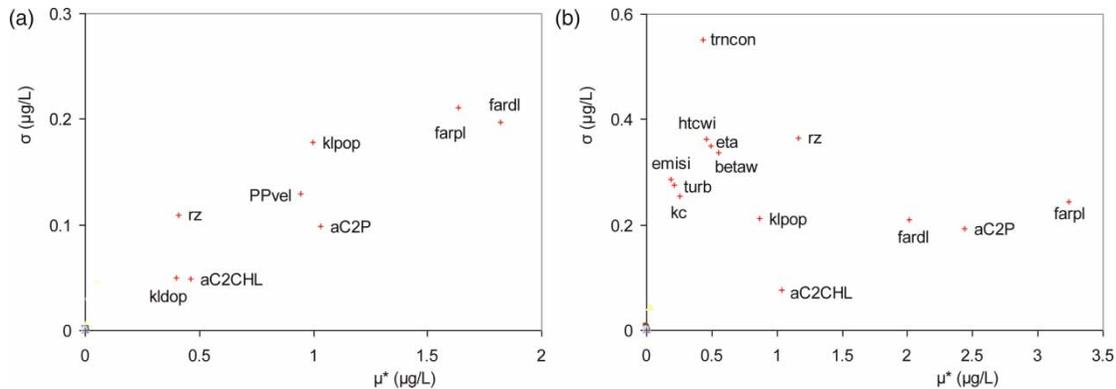


Figure 5 | The Morris sensitivity measures  $\mu^*$  and  $\sigma$  for the 52 parameters in the simulation of DO. (a) Epilimnion, (b) Hypolimnion.



**Figure 6** | The Morris sensitivity measures  $\mu^*$  and  $\sigma$  for the 52 parameters in the simulation of TP in Cannonsville Reservoir. (a) Epilimnion, (b) Hypolimnion.

of two parameters including *fardl* (fraction of algal respiration as dissolved labile) and *farpl* (fraction of algal respiration as particle labile). They have high means and standard deviations of elementary effects, which means that they are the most sensitive parameter in the simulation of TP in the epilimnion layer and have high nonlinear relationships with the simulated TP. The second group is composed of three parameters including *aC2P* (ratio carbon to phosphorus), *klpop* (hydrolysis of labile particulate organic phosphorus (POP)) and *PPvel* (settling velocity of organic particle phosphorus). They also have high means of elementary effects. That is to say, the variations of their values affect the simulated TP in the epilimnion layer. The third group consists of three parameters including *aC2CHL* (ratio carbon to chlorophyll), *rz* (diffusion exponent) and *kldop* (decay of labile dissolved organic phosphorus). Changing their values also influences the simulated TP, but their influence is relatively weak. The fourth group consists of the remaining 44 parameters. Their means and standard deviations of elementary effects are low, implying that the variations of their values result in small influences on the simulated TP in the epilimnion layer.

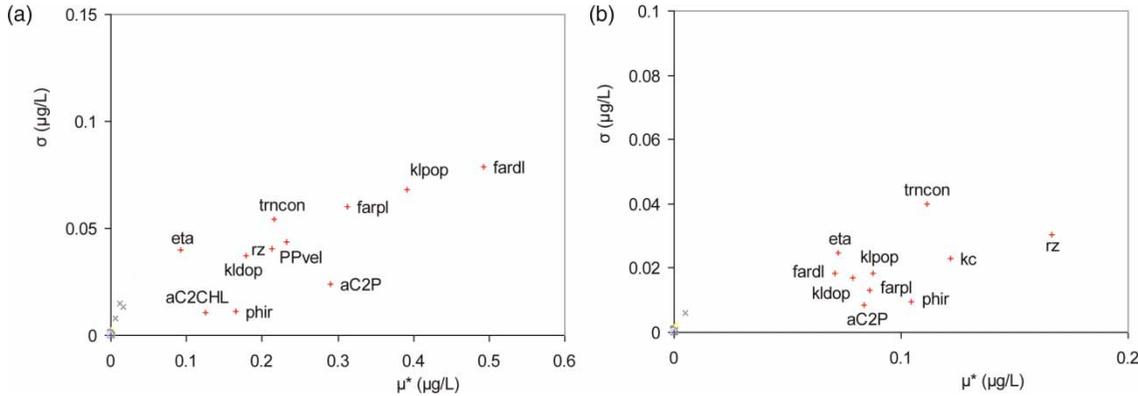
It can be observed in [Figure 6\(b\)](#) that 13 parameters apparently have significant effects on the simulated TP in the hypolimnion layer. Among them, three parameters including *farpl* (fraction of algal respiration as particle labile), *aC2P* (ratio carbon to phosphorus) and *fardl* (fraction of algal respiration as dissolved labile) are the most sensitive parameters because of their high absolute means of elementary effects. The other 10 parameters such

as *rz* (diffusion exponent), *klpop* (hydrolysis of labile POP) and *aC2CHL* (ratio carbon to chlorophyll) are also sensitive. The remaining parameters have low means and standard deviations of elementary effects, implying that the variations of their values result in little influence on the simulated concentration of TP in the hypolimnion layer.

### Chlorophyll (*Chl $\alpha$* )

The sensitivity measures  $\mu^*$  and  $\sigma$  for the 52 parameters in the simulation of *Chl $\alpha$*  are presented in [Figure 7](#). It can be observed in [Figure 7\(a\)](#) that 11 parameters including *fardl* (fraction of algal respiration as dissolved labile), *klpop* (hydrolysis of labile POP), *farpl* (fraction of algal respiration as particle labile), *aC2P* (ratio carbon to phosphorus), *PPvel* (settling velocity of organic particle phosphorus), *trncon* (evaporation multiplier), *rz* (diffusion exponent), *kldop* (decay of labile dissolved organic phosphorus), *phir* (respiration multiplier – growth), *aC2CHL* (ratio carbon to chlorophyll) and *eta* (wind mixing) are well separated from the others. They are sensitive parameters in the simulation of *Chl $\alpha$*  in the epilimnion layer. The remaining 41 parameters have low means and standard deviations of elementary effects, meaning that the variations of their values result in only slight influences on the simulated concentration of *Chl $\alpha$*  in the epilimnion layer.

In [Figure 7\(b\)](#), 10 parameters including *rz* (diffusion exponent), *kc* (chlorophyll multiplier), *trncon* (evaporation multiplier), *farpl* (fraction of algal respiration as particle labile), *phir* (respiration multiplier – growth), *klpop* (hydrolysis of labile POP), *aC2P* (ratio carbon to phosphorus), *kldop* (decay of labile dissolved organic



**Figure 7** | The Morris sensitivity measures  $\mu^*$  and  $\sigma$  for the 52 parameters in the simulation of Chl $a$ . (a) Epilimnion, (b) Hypolimnion.

phosphorus), eta (wind mixing) and fardl (fraction of algal respiration as dissolved labile) are the sensitive parameters affecting the simulation of Chl $a$  in the hypolimnion layer. The remaining 42 parameters have low means and standard deviations of elementary effects, indicating that the variations of their values result in small influences on the simulated concentration of Chl $a$  in the hypolimnion layer.

To sum up, the simulation of each of the variables including temperature, DO, TP and Chl $a$  is sensitive to a different set of parameters. Considering both mean absolute elementary effect ( $\mu^*$ ) and standard deviation of elementary effects ( $\sigma$ ), sensitive parameters are presented in Table 1, where, in total, 18 parameters are found to be sensitive in the simulations of the above variables. They therefore become the focus in model calibration.

It is worth mentioning that the Morris method is able to provide the mean absolute elementary effect and the standard deviation of elementary effects for model parameters, but there are no strict criteria for selecting the sensitive parameters. Their selection is somewhat subjective. In addition, the appropriate ranges (i.e. lower and upper bounds) of parameters are important to the results. A narrow range of a parameter may result in a small effect, while a wide range may enlarge its effect. Therefore, the lower and upper bounds of parameters must be determined carefully. The literature, in particular the documents describing the model, may be helpful.

**Sensitive parameters by enhanced Morris method**

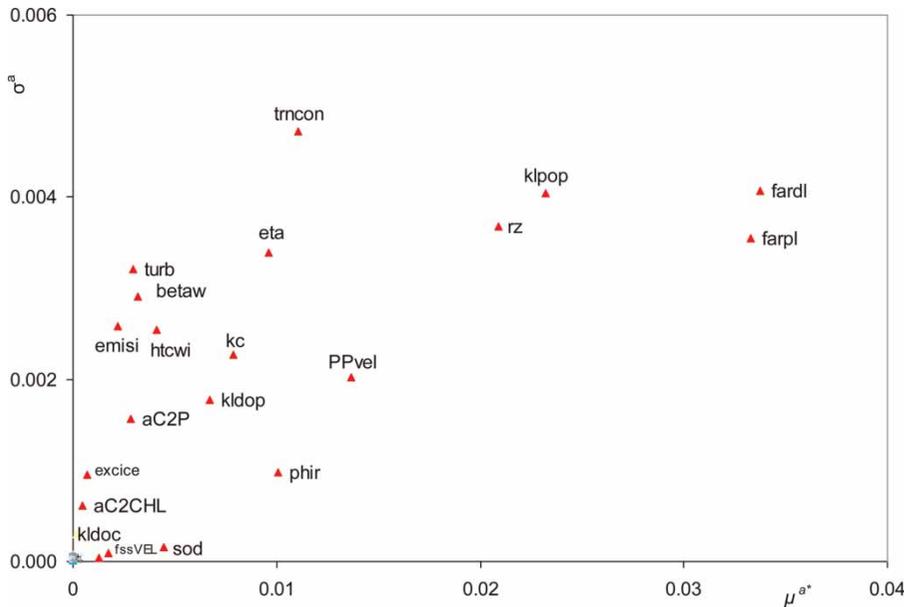
The enhanced Morris method is used to investigate the sensitivity of the 52 parameters in the simulations of daily

**Table 1** | Summary of the sensitive parameters in the simulation of temperature, DO, TP and Chl $a$  in the epilimnion and hypolimnion layers

Variables	Layer	Sensitive parameters
Temperature	Epilimnion	eta, trncon, turb <sup>a</sup>
	Hypolimnion	betaw, emisi, eta, htcwi, rz, trncon, turb
DO	Epilimnion	betaw, eta, kc, kldoc, rz, sod, trncon, turb
	Hypolimnion	betaw, eta, htcwi, kc, rz, sod, trncon, turb
TP	Epilimnion	aC2CHL, aC2P, fardl, farpl, kldop, klpop, PPvel, rz
	Hypolimnion	aC2CHL, aC2P, betaw, eta, fardl, farpl, htcwi, kc, klpop, rz, trncon
Chl $a$	Epilimnion	aC2CHL, aC2P, eta, fardl, farpl, kldop, klpop, phir, PPvel, rz, trncon
	Hypolimnion	aC2P, eta, fardl, farpl, kc, kldop, klpop, phir, rz, trncon
Overall		aC2CHL, aC2P, betaw, emisi, eta, fardl, farpl, htcwi, kc, kldoc, kldop, klpop, phir, PPvel, rz, sod, trncon, turb

<sup>a</sup>Note that parameters are ordered alphabetically in the table.

temperature, DO, TP and Chl $a$ . For illustrative purposes, uniform weight values are applied to all individual elementary effects in Equation (9). The number of variables is four, but the effects on the variables in two layers (epilimnion and hypolimnion layers) are investigated separately. So 1/8 is assigned to each weight factor. The sensitivity measures  $\mu^{a**}$  and  $\sigma^a$  are presented in Figure 8. It can be observed that 20 parameters are well separated from the others. They are



**Figure 8** | The Morris overall sensitivity measures  $\mu^{a*}$  and  $\sigma^a$  for the 52 parameters in the simulation of temp, DO, TP and Chla, obtained by the enhanced Morris method.

the most sensitive parameters in the simulation of temp, DO, TP and Chla. In addition, by comparing the parameters in Figure 8 and Table 1, it can be found that the 18 most sensitive parameters that identified sensitive with the original Morris method are among the 20 parameters. An additional two parameters (i.e. fssVel and excice) are identified sensitive with the enhanced Morris method only but their  $\mu^{a*}$  and  $\sigma^a$  values are low. That is to say, the variations of their values have low effects on the simulated results. So it can be concluded that the same results can be achieved with the original and enhanced Morris methods.

## SUMMARY AND CONCLUSION

In this study, the original and enhanced Morris methods were used to identify sensitive parameters for the simulation of four variables including temperature, DO, TP and Chla using a water quality model of a reservoir. Both methods produced similar short lists of sensitive parameters. The main conclusions are as follows:

1. The original Morris method is capable of ranking the parameters in order of sensitivity in affecting a model output variable. Sensitive parameters can then be determined according to their statistics of elementary effects (i.e. the

averaged absolute elementary effect and the standard deviation of elementary effects) on model output. A short list of sensitive parameters for multiple variables can be obtained by combining the parameter sets sensitive to each individual variable. The advantage to this method of sensitivity identification is that parameters associated with different output variables can be added to or removed from the list according to the importance of the output variables for a particular application. The disadvantage is that if a large number of model output variables are considered, the data analysis may become a tedious task and the overall sensitivity of model output to the parameter variations may be difficult to discern.

2. The enhanced Morris method is able to rank the parameters in order of their overall sensitivity in affecting all important model output variables. Therefore, sensitive parameters can then be determined according to their overall effects on all model outputs. Compared with the original Morris method, the enhanced method does not require the selection of the sensitive parameters affecting each output variable and it is possible to weigh the relative importance of the model outputs used to define overall parameter sensitivity.
3. The results of the original and enhanced Morris methods both identify similar groups of sensitive parameters. The results reveal that most of the parameters that have

high mean elementary effects on model outputs also have high standard deviations of elementary effects, suggesting that the parameters that have high effects on model outputs also have a high nonlinear relationship with the model outputs or high interactions with other parameters, or both. This suggests that it is better to calibrate the parameters in the model simultaneously.

The above conclusions are helpful in model calibration and refinement. Although parts of the conclusions are location-specific, the methods can be used for other reservoirs/lakes.

The original and enhanced Morris methods are tools for identifying a list of sensitive parameters. The knowledge about the water quality model is needed to understand and explain the results.

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