possible to determine the quantities at any instant. The usefulness of the method for detailed studies of deformation characteristics in plastic deformation problems was demonstrated locally and overall in the two selected problems. The plastic zone development, load-displacement curves, geometrical changes of the free surface, and stress and strain distributions were computed and shown to predict well the actual behavior of the material. It was pointed out further that the computed solutions apply quantitatively not only to the specific material used for the analysis but also to a group of materials that produce similar flows. Evidently, the finite element method is a powerful tool for the analysis. However, questions remain with regard to the accuracy of the solutions and the efficiency of the computation. It appears that still another method may be needed for the detailed analysis of plastic deformation processes.

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DISCUSSION

Yoshiaki Yamada[2]

The discusser would like to welcome the fact that Professor S. Kobayashi, who is eminent in slip-line field theory and upper bound solution of plastic-rigid materials, has employed the finite element method in his study of large deformation processes with his colleague Mr. C. H. Lee. Particularly they should be congratulated for the finding that confirms the unloading phenomena in plane-strain side pressing of a cylindrical specimen.

In the stringent opinion of the discusser, however, papers which concern the application of the finite element method should take care to contain either detailed novel features of the solution procedures adopted or the assessment of pertinent numerical accuracies which compare with the existing solutions of similar problems. In this connection, the discusser hopes that the authors would be aware of recent data in the area of material nonlinearities which are included, for example, in *Recent Advances in Matrix Methods of Structural Analysis and Design* (University of Alabama Press, 1971).

Questions that this discusser would like to raise are as follows:

1 Why did the authors use the hypothetical stress-strain curve instead of actual data as in Figs. 4 and 14? Was the experimental curve in Fig. 5 obtained for Aluminum 1100-F or other material? The discusser believes that it is rather easy to duplicate computations for actual material.

2 Was the sticking condition realized perfectly by using the dies which were provided with machined concentric circular grooves? Could the authors give the details of the grooves' dimensions and their configurations after deformation?

Authors' Closure

The authors wish to thank Professor Yamada for his discussion. Regarding the first question, the computation for the problem was made first with an arbitrarily selected stress-strain curve in order to find the applicability of the method to the problem involving large plastic deformation. The experimental results were quoted afterwards for comparison purposes. The authors agree with the discusser in that it is easy to duplicate computations for actual material. However, the authors utilized the computed results for comparison with the materials of similar property, because it is too costly to repeat the computation. It was also noted that a set of solutions computed with a specific stress-strain curve is applicable to a group of materials with similar plastic properties.

With respect to the second question, the authors think that the complete sticking condition was not achieved in the experiment. The authors stated in the text, therefore, that the deviation between theoretical and experimental load-displacement curves was partly attributed to the fact that some differences exist between the theory and experiment with regard to the constraint conditions at the tool-workpiece interface.

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