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letters to the editor

On "Those Magnificent Men and Their Controlling Machines"[1]

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Ziegler's Forum on the early history of control published in this *Journal* is very interesting and very good. One point raises a question in my mind. George Philbrick joined Foxboro in 1936, the same year that I did, and the firm was already building and selling the complete M/10 Stabilog at that time. This model is the full two-term controller, with adjustable gain and integral time (proportional rate floating), not the briefly used (in 1929 I believe) preliminary device described. So George couldn't have designed the M/10, although he did contribute a great deal to our work on control.

The Spitzglass to whom he refers later changed his name to Sperry, being the Al Sperry who founded Panellit.

Ed Smith's early book on control[3] includes a chronological bibliography, and at the editor's request, I selected papers from it that seemed representative:

Grebe, J. J., R. H. Boundy, R. W. Cermak
The Control of Continuous Processes
Trans. Am. Inst. Chem. Eng., June 1933

Mason, C. E.
Control in Continuous Distillation
Oil and Gas Jr., March 7, 1929

Mason, C. E.
Science of Automatic Control in Refining
World Petroleum, Vol. 34, June, July, August 1933

Invanoff, A.
Theoretical Foundations of the Automatic Regulation of Temperature
Jr. Inst. Fuel, London, February 1934

Smith, E. S., Jr.
Analysis of Fluid Rate Control Systems
Instruments, pp. 54–60, March 1932

Smith, E. S., Jr.
Automatic Regulators, Their Theory and Application
Trans. ASME, Vol. 9, No. 4, pp. 291–303, May 1936

Ivanoff, A.
The Influence of the Characteristics of a Plant on the Performance of an Automatic Regulator
Chem. Eng. Group, Conf. on Auto. Regulators, Soc. Chem. Ind., London, 1936

Keppler, P. W.
Combustion Instruments and Control at Hell Gate
Combustion, pp. 29–33, April 1937

Smith, E. S., Jr. and C. O. Fairchild
Industrial Instruments, Theory and Application
Trans. ASME, Vol. 59, pp. 595–607, PRO 59-9, October 1937

Mason, C. E.
Quantitative Analysis of Process Lags
Trans. ASME, Vol. 60, No. 4, pp. 327–334, May 1938

Haigler, E. D.
Application of Temperature Controllers
Trans. ASME, November 1938, PRO 60-5, p. 633

Sptizglass (Sperry), A. F.
Quantitative Analysis of Single-Capacity Processes
Trans. ASME, November 1938, PRO 60-9, p. 665
Trans. ASME, January 1940, p. 1

Mason, C. E. and G. A. Philbrick
Automatic Control in the Presence of Process Lags
Trans. ASME, Vol. 62, No. 4, pp. 295–308, May 1940

Mason, C. E. and G. A. Philbrick
Mathematics of Surge Vessels and Automatic Averaging Control
Trans. ASME, Vol. 63, pp. 589–601, 1941

Ziegler, J. G. and N. B. Nichols
Optimum Settings for Automatic Controllers
Trans. ASME, 1942, pp. 759–768

Ziegler, J. G. and N. B. Nichols
Process Lags in Automatic Control Circuits
Trans. ASME, July 1943, pp. 433–444, Discussion and Closure

[1]By J. G. Ziegler, published in the September, 1975, issue of the Journal of Dynamic Systems, Measurement, and Control, Trans. ASME, Series G, Vol. 97, No. 3, p. 279.

[2]Concord, Mass.

[3]*Automatic Control Engineering*, Ed. Sinclair Smith, McGraw-Hill, New York and London, 1944.

A New View of the Modal Control Technique[1]

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This correspondence is in regard to the paper "A New View of the Modal Control Technique" by H. Ju, M. J. Rabins, and C. D. Han which appeared in the Sept., 1975 issue of your *Journal*. Having carefully read a pre-publication copy of that article and having discussed the paper at some length, we conclude that the technique of the paper does not result in a true modal control, but that it does offer the possibility (under certain limited conditions) of quadratic optimization with pole movement in a desired direction. We suggest, therefore, that the

[1]By H. Ju, M. Rabins, and C. D. Han, published in the September, 1975, issue of the Journal of Dynamic Systems, Measurement, and Control, Trans. ASME, Series G, Vol. 97, No. 3, pp. 300–308.

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present paper offers both less and more than it implies, and we give our reasons below.

In equation (11) of the paper by Ju, et al., i.e.:

$$TK_{fb}^* T^{-1} = BK_{fb}D \tag{1}$$

assume for purposes of this discussion that matrix D is an $n \times n$ identity matrix, or, in other words, that all of the n states are available. (We will not concern ourselves at all in this discussion with the observer problem.)

Further, assume that the $n \times p$ matrix B is of rank p and, as in equation (10) of the paper,

$$\Lambda - \Lambda_d = K_{fb}^* \tag{2}$$

Equation (1) is then only correct if K_{fb}^* is taken as rank p. Since K_{fb}^* is $n \times n$ diagonal this means that only p of its elements are $\neq 0$. The remaining $(n - p)$ elements must $= 0$ so, consequently, $(n - p)$ of the eigenvalues cannot be moved to desired precise new locations. Further, these elements should be ordered as:

$$K_{fb}^* = \begin{bmatrix} \lambda_1 & & & & & & 0 \\ & \lambda_2 & & & & & \\ & & \cdot & & & & \\ & & & \lambda p & & & \\ & & & & 0 & & \\ & & & & & \cdot & \\ 0 & & & & & & 0 \end{bmatrix} \tag{3}$$

where

$$|\lambda_1| > |\lambda_2| > \ldots. > |\lambda p|, \tag{4}$$

for clarity.

If an attempt is made to move all n eigenvalues (i.e., K_{fb}^* of rank n) when B is of rank p, then equation (1) cannot be correct since the left and right-hand sides of that equation will be of different rank. The result will be that the closed loop poles will not be conditioned as desired.

Even beyond this limitation on K_{fb}^*, there is a problem with this method. If K_{fb}^* is of rank p, ordered as above, we may then write, from (1) premultiplied by B':

$$B'TK_{fb}^* T^{-1} = (B'B)K_{fb} \tag{5}$$

where B' is the transpose of B, and $(B'B)$ is a $p \times p$ square and invertable matrix. We may solve (5) for:

$$K_{fb} = (B'B)^{-1}B'TK_{fb}^* T^{-1} \tag{6}$$

which will always be correct since $(B'B)^{-1} B'$ is always of rank p.

Ignoring disturbance imput $\mathbf{U}(t)$ in equation (1) of the paper, and taking reference imput $\mathbf{S}$ as $\mathbf{0}$ (to simplify this discussion), then equation (3) of the paper (with $D = I_n$) becomes:

$$\mathring{\mathbf{X}} = [A - B(B'B)^{-1} B'TK_{fb}^* T^{-1}]\mathbf{X}, \tag{7}$$

or,

$$\mathring{\mathbf{X}} = G\mathbf{X}, \text{ where } G = [A - B(B'B)^{-1} B'TK_{fb}^* T^{-1}]. \tag{8}$$

The nature of the closed-loop eigenvalues of G may be better studied if we replace matrix A in G by its natural basis,

$$A = T\Lambda T^{-1} \tag{9}$$

in which Λ is the diagonal matrix of open-loop eigenvalues. Using equations (1), (8), and (9), we get:

$$G = T[\Lambda - T^{-1}B(B'B)^{-1} B'BK_{fb}]T^{-1}$$

or,

$$G = T[\Lambda - T^{-1}BIpK_{fb}]T^{-1}$$

or,

$$G = T[\Lambda - PK_{fb}]T^{-1} \tag{10}$$

where

$$P = T^{-1}B \tag{11}$$

Now, the eigenvalues of G are those of $(\Lambda - PK_{fb})$ and unless $PK_{fb} = \Lambda - \Lambda_d$, then the solution is not the one sought in the first place. But from equations (1) and (11) (with $D = I_n$):

$$TK_{fb}^* T^{-1} = BK_{fb} = TPK_{fb}$$

or,

$$K_{fb}^* T^{-1} = PK_{fb} \neq \Lambda - \Lambda_d \tag{12}$$

since $K_{fb}^* = \Lambda - \Lambda_d$ by the assumption of equation (2) and $K_{fb}^* T^{-1}$ cannot be diagonal in any case. The solution of the paper is therefore not generally correct from the modal control view point unless certain limitations are observed (which are currently being studied), or certain approximations in the desired pole placements are accepted. Of course, when conditions on the matrix D are considered as well, additional constraints on the applicability of this computational method may emerge.

However, we believe that it may be demonstrated that this "pseudo-modal control" may lead under certain conditions to a very simple and perfectly correct quadratic optimal control computation (nonrecursive) that may be well-adapted to industrial applications, and which simultaneously allows conditioning of some of the eigenvalues. The feedback portion of the system is, from equation (8),

$$B(B'B)^{-1}B'TK_{fb}^* T^{-1} = -G + A \tag{13}$$

in which $(B'B)$ is $p \times p$ symmetric and positive definite by construction. We introduce the notation:

$$(B'B) = R \text{ and } (B'B)^{-1} = R^{-1} \tag{14}$$

so that (13) becomes:

$$G = A - BR^{-1}B'TK_{fb}^* T^{-1} \tag{15}$$

if

$$TK_{fb}^* T^{-1} \text{ is symmetric and positive definite} \tag{16}$$

and if

$$A'TK_{fb}^* T^{-1} + TK_{fb}^* T^{-1}A - TK_{fb}^* T^{-1}(B'B)TK_{fb}^* T^{-1} = -Q \tag{17}$$

where Q is symmetric and semi-positive definite, and the $p \times p$ matrix $(B'B)$ is augmented to dimension $n \times n$ by adding $(n - p)$ rows and columns of zeros then, $TK_{fb}^* T^{-1}$ = solution of a Riccati Equation and the resulting system gives a stable optimal control. Equations (16) and (17) above are generally true when:

$$K_{fb}^* = \propto \begin{bmatrix} 1 & & & 0 \\ & 1 & & \\ & & \cdot & \\ 0 & & & 1 \end{bmatrix}; \ \propto = \text{const.}$$

which is exactly the case chosen in several of the numerical examples of the paper.

To conclude, we believe that the computational method proposed in this paper offers the possibility of optimal control *and* some simultaneous pole assignments under certain limited conditions. The investigation of these conditions is the subject of continued study by the correspondents.