A Theory of Gravity in the Framework of the Lorentz Covariant and Second Quantized Formalism

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On the basis of the remark that the gravitational interaction really acts with the binding energy, the emphasis is laid on the necessity of treating the gravitational interaction by the method of particle physics irrespective of the general relativity.

Accordingly, the interaction of a massless particle with spin 2 (graviton) is investigated by using the method which has been applied to the reformulation of quantum electrodynamics by the present author. As a result, we can formulate the theory of gravitational interaction in an unambiguous manner, in which the interaction of gravitational field with the binding potential is successfully derived.

The rotation of the perihelion is calculated by taking into account higher order corrections to the gravitational interaction, and a value which is about 8% smaller than the result of general relativity is obtained. This is exactly the value expected by Dicke and Goldenberg from their measurement of the solar oblateness.

As regards the behaviour of the photon in the gravitational field, our theory reveals some features different from general relativity. This difference, which concerns the universality of the coupling constant in our theory, will be tested by more precise experiments about the gravitational deflection of light or by experiments about the change of light velocity in the gravitational field.

§1. Introduction and summary

Einstein's theory of general relativity is one of the most elegant theories in physics. Its beautiful mathematical formulation has attracted many physicists, and the famous three tests of general relativity have generally been considered as tokens of the correctness of the theory.

The difficulty encountered in these tests, however, still casts some questions about the truth of the general relativity. Recently, Dicke and Goldenberg\(^1\) have observed the solar oblateness which is responsible to the discrepancy of 8% of the Einstein value for the perihelion motion of Mercury.

Apart from such experimental tests, the theory of general relativity reveals its difficult feature when viewed from the quantum theoretical viewpoint. Many attempts to accommodate Einstein's theory with quantum theory have never been successful. The quantization of Einstein's equation has been possible only with its expansion in powers of the gravitation constant.\(^2\)

If we consider the prominent successes achieved by quantum electrodynamics and meson theory, this difficulty may be taken as a crucial defect of general relativity, and this fact seems to imply the need for a new formulation of theory.
of gravitational interaction. At this point, one may argue that only a theory of gravity in the macroscopic world is enough for us, or, even if the need for the quantum theoretical treatment is admitted, the conclusion such as above may be thought too premature.

The following simple remark, however, seems to strongly support the necessity of the new formulation. Namely, the mass defect in nuclear weight means undoubtedly that the gravity really acts not only with the sum of the rest mass but also with the binding energy. What is the mechanism which makes the gravity interact with the binding energy? No such theory of gravitational interaction as is incapable of answering this question should be taken as a final one. The answer to this question may be possible only when the gravitational interaction itself is treated as a fundamental interaction in the microscopic world. Even if a theory such as Einstein's one possesses a correct aspect in the macroscopic world, it should be derived from the fundamental theory after some approximate manipulations.

The above remark about the binding energy leads us at the same time to the following conclusion: Since the gravity interacts with the binding energy, it must also interact with the kinetic energy of particles. Therefore the entity concerning the gravitational interaction is not the rest mass of a particle, but must be its total energy. It is important to remark that with the equivalence of mass and energy in the theory of special relativity alone we cannot derive this conclusion. Here again, one may find the importance of our naive question about the interaction between gravity and binding energy.

From the viewpoint mentioned above, we intend in this paper to construct a theory of the gravitational interaction irrespective of general relativity. The results are fully satisfactory. It turns out that the main feature of gravity, including its interaction with the binding potential, are well explained in the framework of the Lorentz covariant and second quantized theory. Hiida and Yamaguchi have also tried to treat the gravitational interaction in the form of the conventional field theory, but they confine themselves only to treating the interaction of particles with the unquantized external gravitational field.

The prominent character of our theory is to treat the gravity completely by using the second quantized formalism. We have tried to investigate the formulation of particle physics by incorporating Foldy's covariant particle equation with the Foch-Cook-Friedrichs method of second quantization. For the present purpose, such a method turns out to be very useful. It enables us to treat easily the interaction of a massless particle with spin 2 (graviton) in quite an analogous manner to the treatment of the quantum electrodynamics. The gravitational interaction is introduced completely in parallel with that of Coulomb interaction in case of quantum electrodynamics.

Furthermore, our method facilitates to handle the higher order corrections to the gravitational interaction. As a consequence of the effect of the higher order
correction, we obtain a value 8% smaller than the result of the general relativity for the perihelion rotation. This value, together with the observation of the solar oblateness by Dicke and Goldenberg, precisely explains the well-known experimental value for Mercury.

As for the behaviour of photon in the gravitational field, our result shows a somewhat interesting aspect. Assuming the universality of the gravitational coupling constant to the case of photon, we obtain for the gravitational deflection of light the value three fourths of the result of general relativity. This is almost near the allowable lower bound of the existing experimental values, or seems to be a little smaller when compared to the recent observation of radio source. Since it is possible to change our result by modifying the strength of coupling with photon, we think it important that the universality is tested by more precise experiments about the gravitational deflection of light. Since this phenomenon is closely connected with the change of light velocity in gravitational field, it is also desirable that the experiment proposed by Shapiro on the delay time of the radar pulse will be performed.

Finally, the problem of interaction between gravity and binding energy, which is a motivation of the present work, is attacked for the case of the scalar-meson-exchange second-order potential. The result obtained is exactly what was expected. Although the general discussion of this problem is not yet possible, the fact that we can obtain the satisfactory explanation at least by a simple model may be thought as a remarkable progress in our understanding of the nature of the gravitational interaction.

The successful evaluation of higher order corrections to gravitational interaction and the solution for the naive question concerning the interaction with binding energy could be considered to show the correctness of our idea of formulating the gravitational interaction in the framework of special relativity by using the method of second quantization from the outset.

§ 2. Treatment of massless particle with spin 2

In this paper we adopt Foldy’s form of representation of the inhomogeneous Lorentz group for one-particle system. Thus the ten generators are given by

\[ P = p, \quad H = \omega = \sqrt{m^2 + p^2}, \quad J = r \times p + s, \quad K = \frac{1}{2} (r \omega + \omega r) - \frac{s \times p}{m + \omega} - t p. \]

Here, \( K \) are the generators of Lorentz transformations along the three coordinate axes. \( m \) denotes the mass of the relevant particle and \( s \) are spin operators of
an irreducible representation of the three-dimensional rotation group. (We use units $\hbar = c = 1$.)

In the case of a particle of vanishing rest mass, each generator comes to commute with $sp/p = sn$. Therefore the representation given by (2.1) $\sim$ (2.4) does not immediately give an irreducible representation for a massless particle. This, however, is not serious. Obviously, the irreducible representation is given by (2.1) $\sim$ (2.4) with $m = 0$ only if we select proper components of wave function by using a suitable projection operator.

The treatment of a particle with spin 2 is simplified according to the following tensor representation. The wave function is given by

$$\varphi = \Theta (\varphi_{12})$$

where $\Theta$ is a traceless symmetric matrix of three dimensions. The spin operators are defined by

$$s_k \varphi = [S^{(i)}_k, \varphi]$$

where $S^{(i)}_k$ are those of the irreducible 3-dimensional representation. (2.6) is only correct with the imaginary antisymmetric representation for $S^{(i)}_k$, and we use in this paper the following representation:

$$S^{(i)}_1 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & -i \\ 0 & i & 0 \end{pmatrix}, \quad S^{(i)}_2 = \begin{pmatrix} 0 & 0 & i \\ 0 & 0 & 0 \\ i & 0 & 0 \end{pmatrix}, \quad S^{(i)}_3 = \begin{pmatrix} 0 & -i & 0 \\ -i & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}.$$ (2.7)

The components of $\varphi$ belonging to the eigenvalues $sn = \pm 2$ are given by

$$\varphi^i = \frac{1}{2i} (sn)^2 ((sn)^2 - 1) \varphi$$

$$= \frac{1}{2} ((S^{(i)}_n)^2 \Theta (S^{(i)}_n)^3 - (S^{(i)}_n) \Theta (S^{(i)}_n))$$

$$= (\varphi^i_0).$$ (2.8)

Denoting the complete orthonormal system of $\varphi^i$ by $\{\varphi^i_0\}$ we get from (2.8)

$$A_k \varphi^i_{l\alpha} A_l \cdot B_m \varphi^\ast_{\lambda\mu} B_n = \frac{1}{2} \{ (A_k (S^{(i)}_n)_{\alpha\mu} B_l)^3 + (A_k (S^{(i)}_n)_{\lambda\nu} B_l)^3 \},$$ (2.9)

where $\varphi^\ast$ denotes the complex conjugate of $\varphi$. This is an important relation for the calculations in the next section.

Among the ten generators of the inhomogeneous Lorentz group, the most complicated are $K$, but in the case of a particle with $m = 0$ and $s = 2$ the following property simplifies their treatment:

$$\frac{1}{\sqrt{p}} K_k \varphi^i = r_k p \frac{1}{\sqrt{p}} \varphi^i + i \frac{1}{\sqrt{p}} (p_{k\alpha} \varphi^\ast_\alpha + (\varphi^i_k p_l)).$$ (2.10)

In what follows we shall call a particle with $m = 0$ and $s = 2$ as tensor graviton.
§ 3. Fundamental gravitational interaction

We begin our theory by considering the interaction of the tensor graviton. The calculations are performed first for the interaction with a scalar particle.

The second quantization for the free system is easily done by application of the Fock-Cook-Friedrichs method.¹¹ The interaction is treated by adding suitable terms to $K$ so that the requirement of Lorentz covariance is satisfied. Among the commutation relations required for the generators, the most important relation is

$$[K_t, K_r] = -i\varepsilon_{tir}\mathbf{J}_k. \quad (3·1)$$

Separating $K$ into free and interacting parts as $K = K_0 + K$, we obtain as a restriction for $K_1$

$$[K_{0t}, K_{1r}] + [K_{1t}, K_{1r}] + [K_{1t}, K_{1r}] = 0. \quad (3·2)$$

Once this condition is satisfied, the Hamiltonian is given by $[P_t, K_r] = -i\delta_{t}^{r}H$.⁵

As for the interaction of the tensor graviton with a scalar particle, the simplest of the interaction term is given by

$$\sum b_{\alpha\beta}^{(s)}A_{\alpha}^{+}A_{\beta}B_{\gamma} + \text{h.c.}, \quad (3·3)$$

$$b_{\alpha\beta}^{(s)} = g \int \left( \varphi_{\alpha}(p), \varphi_{\beta}(q) \right) (p + q)\kappa(p + q)\partial_{p_{i}}\varphi_{\gamma}^{(s)}(p - q) dpdq - ig \int \frac{\omega_{p} + \omega_{q}}{|p - q|} \left( \varphi_{\alpha}(p), \varphi_{\beta}(q) \right)$$

$$\times \left\{ (p + q)\varphi_{\gamma}^{(s)}(p - q) + (p + q)\kappa\varphi_{\gamma}^{(s)}(p - q) \right\} dpdq. \quad (3·4)$$

Here $A_{\alpha}^{+}$ are the creation-annihilation operators of the scalar meson, and $B_{\gamma}$ those of the tensor graviton. $\varphi$ is the wave function of the scalar meson multiplied by $1/\sqrt{\omega}$, and $\varphi^{(s)}$ means $(1/\sqrt{p})\varphi$. Throughout this paper any wave function should similarly be understood to involve a factor $1/\sqrt{\omega}$. The Greek suffix is used to denote a state of a complete orthonormal system for each particle.

This interaction term is expressible as

$$i[K_{0t}, T], \quad (3·5)$$

with

$$T = \sum d_{\alpha\beta}A_{\alpha}^{+}A_{\beta}B_{\gamma} + \text{h.c.}, \quad (3·6)$$

$$d_{\alpha\beta} = -ig \int \frac{1}{\omega_{p} - \omega_{q} - |p - q|} \left( \varphi_{\alpha}(p), \varphi_{\beta}(q) \right) (p + q)\kappa(p + q)\varphi_{\gamma}^{(s)}(p - q) dpdq. \quad (3·7)$$

Therefore the covariance is obviously satisfied to first order in $g$.⁶

⁵) For the detail of the method, we refer to the papers in Refs. 4)~7).
The next step is to examine the second order covariance. Since our present purpose is to investigate the gravitational interaction between \( A \)-particles, it is enough to consider, at present, the term of the type \( A^+ A^+ A^- A^- \). We see at once that the requirement of covariance is not satisfied with the above-introduced interaction term alone.

Now it is important to remember the situation in the quantum electrodynamics. There the transverse photon takes the place of our tensor graviton, and its interaction of the type of \((3\cdot3)\) also fails to satisfy the covariance. The role of Coulomb interaction is nothing but to recover the covariance, \(^6\) and it is formulated by introducing the negative energy scalar photon. \(^7\)

This fact teaches us that the present situation may also be rescued if we introduce, besides the tensor graviton, two sorts of gravitons, namely, massless particles with \( sn = \pm 1 \) and \( sn = 0 \) respectively. We shall call the former one "vector graviton" and the latter "scalar graviton". The vector graviton can be treated in a way similar to that of the transverse photon. After some lengthy but straightforward calculations, it is found that the vector graviton must be a negative energy particle. This is the character corresponding to that of the scalar photon in quantum electrodynamics and causes no trouble for our physical understanding. The additional interaction necessary to recover the covariance for the term of \( A^+ A^+ A^- A^- \)-type is determined as follows:

\[
\sum b^{(3)}_{\alpha \beta \gamma \delta} A^+ \alpha \beta A^- \gamma \delta B^- \gamma \delta + \sum b^{(0)}_{\alpha \beta \gamma \delta} A^+ \alpha \beta A^- \gamma \delta B^- \gamma \delta + \text{h.c.}, \tag{3\cdot8}
\]

\[
b^{(3)}_{\alpha \beta \gamma \delta} = 2\sqrt{2} \int (\omega_p + \omega_q) (\omega_p - \omega_q + \|p - q\|) (\varphi_\alpha(p), \varphi_\beta(q)) \times (p + q)i^\prime \frac{\partial}{\partial p_i} \frac{1}{|p - q|} \varphi^{(1)}_{\gamma \delta} (p - q)dpdq
\]

\[
+ 2\sqrt{2}i \int \frac{(p - q)_t}{|p - q|^2} (\omega_p + \omega_q) (\varphi_\alpha(p), \varphi_\beta(q)) (p + q) \varphi^{(1)}_{\gamma \delta} (p - q)dpdq
\]

\[
+ \sqrt{2} \int (p + q)_t (\varphi_\alpha(p), \varphi_\beta(q)) (p + q) \varphi^{(1)}_{\gamma \delta} (p - q)dpdq
\]

\[
+ \sqrt{2}i \int \frac{(\omega_p + \omega_q)^2}{|p - q|^2} (\omega_p - \omega_q + |p - q|) (\varphi_\alpha(p), \varphi_\beta(q)) \varphi^{(1)}_{\gamma \delta} (p - q)dpdq, \tag{3\cdot9}
\]

\[
b^{(0)}_{\alpha \beta \gamma \delta} = \sqrt{-\frac{3}{2}} g \int (\omega_p + \omega_q)^2 (|p - q|^2 - (\omega_p - \omega_q)^2) (\varphi_\alpha(p), \varphi_\beta(q)) \times \frac{\partial}{\partial p_i} \frac{1}{|p - q|} \varphi^{(0)}_{\gamma \delta} (p - q)dpdq
\]

\[
- \sqrt{-\frac{3}{2}} 2g i \int \frac{(p + q)_t}{|p - q|^2} (\omega_p + \omega_q) (\omega_p - \omega_q + |p - q|)
\]
\[ S. \text{ Sato} \]

\[ \times (\varphi_{a}(p), \varphi_{\beta}(q)) \varphi_{i}^{(0)}(p-q) \, dp \, dq \]

\[ + \sqrt{\frac{3}{2}} 2g \i \left( \frac{p-q}{|p-q|^2} \right) (\omega_{p} + \omega_{q})^{2} (\varphi_{a}(p), \varphi_{\beta}(q)) \varphi_{i}^{(0)}(p-q) \, dp \, dq \]

\[ + \sqrt{\frac{3}{2}} 2g \i \left( \frac{p-q}{|p-q|^2} \right) (\omega_{p} + \omega_{q})^{2} (\varphi_{a}(p), \varphi_{\beta}(q)) \varphi_{i}^{(0)}(p-q) \, dp \, dq \]

\[ - \sqrt{\frac{3}{2}} \frac{1}{3} \i \left( (\omega_{p} + \omega_{q})^{2} - |p+q|^{2} \right) (\varphi_{a}(p), \varphi_{\beta}(q)) i \frac{\partial}{\partial p} \varphi_{i}^{(0)}(p-q) \, dp \, dq . \]

(3·10)

Here, \( B_{1}^{\pm} \) and \( B_{0}^{\pm} \) are the creation and annihilation operators of vector and scalar gravitons, respectively. \( \varphi^{(0)} \) and \( \varphi^{(0)} \) are their wave functions.

Among these interaction terms described above, the one for the scalar graviton corresponds to the desired gravitational interaction. If we treat \( \varphi^{(0)} \) as an external field, we are led from (3·10) to the following interaction Hamiltonian for the classical motion of \( A \)-particle:

\[ \frac{\kappa M \omega_{p}}{c^{2}r} - \frac{\kappa M p^{2}}{2 \omega_{p} r} + \frac{3 \kappa M (r \times p)^{2}}{4 \omega_{p} r^{2}} , \]

(3·11)

where \( M \) is the mass of the source of the external field and \( \kappa \) is given by

\[ \kappa = \frac{1}{16} (8\pi g)^{2} . \]

(3·12)

Thus we can obtain the gravitational interaction in which the main term involves the total energy of the particle concerned. The contributions of the second and third terms in (3·11) cancels each other if we take an average with respect to the direction of the momentum assuming a symmetric distribution.

Similar calculations have been performed in the case that \( A \)-particle is a spinor and the photon, and we have also been led to the same result (3·11).

For a system composed of many particles, the contributions of the second and third terms in (3·11) cancel each other if we take an average with respect to the direction of the momentum assuming a symmetric distribution.

Without this cancellation, the kinetic energy for the internal motion comes to have an additional effect to the one contained in the first term in (3·11), and to it follows the breakdown of the equivalence principle. This feature relating to the equivalence principle is one of the prominent character of our theory.

A detailed comparison between our formulation and other theories will be given in § 7.

\[ \S \, 4. \text{ Behaviour of the photon in the gravitational field} \]

The Hamiltonian which governs the behaviour of the photon in the gravitational field is given by
Now let us first consider the gravitational red shift. This problem is treated on the basis of the energy conservation. Consider that an object in a state with energy $E_1$ emits a photon changing its state to the one with energy $E_2$, then according to the conservation of energy including gravitational energy we get

$$\left(1 - \frac{\phi}{c^2}\right)(E_1 - E_2) = \left(1 - \frac{3\phi}{2c^2}\right)\hbar \nu.$$

Here we have assumed that the momentum of the emitted photon is parallel with its coordinate vector. Thus owing to the factor $3/2$ in (4·1) the frequency of the emitted photon in the gravitational field is different from that in the absence of the gravitational field, namely, $\nu_0 = (E_1 - E_2)/\hbar$.

It is important to remark here that the violation of the equivalence principle for this phenomenon is caused by the absence of the cancellation mentioned at the end of the previous section. Even for the phenomena relevant to photons the equivalence principle still holds with such a cancellation.

When the emitted photon goes out of the gravitational field, its frequency changes from $\nu$ to $\nu'$ given by

$$\nu' = \left(1 - \frac{3\phi}{2c^2}\right)\nu.$$

This is, however, not the relation to be compared with experiment. The existing experiment on this phenomenon is nothing but to compare $\nu'$ to $\nu_0$, that is, to the frequency of the photon which is absorbed by the inverse transition in the absence of the gravitational field. For this, we get from (4·2) and (4·3)

$$\nu' = \left(1 - \frac{\phi}{c^2}\right)\nu_0.$$

This is exactly the same result as that of general relativity. It is important to notice that this relation still holds if we replace $3/2$ in (4·1) by an arbitrary number.

Next, the motion of a photon in the gravitational field is calculated using (4·1), and we get for the gravitational deflection of light a value $3/4$ times smaller than that of general relativity. This value is almost near the allowable lower bound of the existing experimental values, or seems to be slightly smaller than the recent observation of radio source. Since (4·1) is used with the tacit assumption of the universality of the coupling constant, our result can be changed if we disregard the universality. To determine whether the universality holds or not, we must await more precise experiments for the gravitational deflection.

* In this paper we are concerned only with the transverse photons, and this violation of the universality does not cause any direct trouble to the interaction of gravity with Coulomb energy.
of light. Otherwise it may also be tested by direct measurement of the change of light velocity in the gravitational field using the delay time of the radar pulse, as was proposed by Shapiro.\(^9\)

§ 5. The perihelion rotation

According to general relativity the angle of the perihelion rotation is given by

\[ \delta \varphi = 6\pi \frac{\kappa^2 m^3 M^3}{c^2 L^2}, \]  

where \( L \) is the magnitude of the angular momentum of the planet. On the other hand we get from the Hamiltonian given by (3·11)

\[ \delta \varphi = 3.5\pi \frac{\kappa^2 m^3 M^3}{c^2 L^2}. \]  

The difference of these values does not mean the inadequacy of our theory. Since the perihelion rotation is affected by a potential which is of the second order in \( \kappa \), we must consider higher order corrections to (3·11). For this purpose, it is necessary to examine the second order covariance for the terms of \( \Lambda \Lambda B B \)-type. The calculations have been performed using the \( 1/m \) expansion, and we have found it necessary to add new interaction terms of \( \Lambda \Lambda B B \)-type to recover the covariance.

Here again, we can find an analogous situation in the quantum electrodynamics. In the case of the interaction of the photon with a scalar particle, the similar type of interaction is needed from the requirement of covariance, by which the Thomson scattering is explained. In addition, such a situation has conveniently been used for the treatment of the low-energy meson-nucleon interaction.\(^\text{13}\)

At first, from the computation of the covariance in the order of \( m \) the following interaction terms are derived:

\[ (\sum c^{(1)\alpha}_{\imath \alpha \beta \gamma} A^\alpha B^\beta \rho B_\rho + \text{h.c.}) + \sum c^{(2)\alpha}_{\imath \alpha \beta \gamma} A^\alpha A^\beta B_\rho B_\rho, \]  

\[ c^{(1)\alpha}_{\imath \alpha \beta \gamma} = -16g^2 m^2 \int (\varphi_\alpha(p), \varphi_\beta(q)) \varphi_\gamma(t) \]  

\[ \times \frac{(p - q - t)_\xi}{|p - q - t|} \left( -i \frac{\partial}{\partial t} \right) \varphi_\gamma(t) \varphi_\gamma(p - q - t) d\rho dq dt + O(m^6), \]  

\[ c^{(2)\alpha}_{\imath \alpha \beta \gamma} = -32g^2 m^2 \int (\varphi_\alpha(p), \varphi_\beta(q)) \varphi_\gamma(t) \]  

\[ \times \frac{(p - q + t)_\xi}{|p - q + t|} \left( -i \frac{\partial}{\partial t} \right) \varphi_\gamma(p - q + t) d\rho dq dt + O(m^6). \]  

Since \( \varphi \) involves the factor \( 1/\sqrt{\omega} \), the first terms in (5·4) and (5·5) are of the
order of \(m\), and \(O(m^0)\) denotes a term of the order of \(m^0\) which has no contribution to the Hamiltonian.

We can see, however, that these terms do not contribute to the potential for the classical motion of \(A\)-particle, and we must proceed to examination of the covariance in the order of \(m^0\). The over-all calculation of \([K_{Ri}, K_{Ri}]\) in the order of \(m^0\) is very cumbersome, but it is enough to treat those terms which are directly connected with the interaction terms of the order of \(m\). The necessary terms turn out to be given by the following coefficients:

\[
A_\alpha A_\beta B_{\alpha}^* B_{\beta}^* : \frac{2}{3} \cdot 8g^2 \int 2m(p + q)_t (\varphi_\alpha(p), \varphi_\beta(q)) \varphi_t^{(0)}(t)
\]

\[
\times i \frac{\partial}{\partial p_j} \varphi_3^{(0)}(p - q - t) d\rho dq dt \quad (i \neq j),
\]

(5.6)

\[
A_\alpha A_\beta B_{\alpha}^* B_{\beta}^* : \frac{2}{3} \cdot 16g^2 \int 2m(p + q)_t (\varphi_\alpha(p), \varphi_\beta(q)) \varphi_t^{(0)}(t)
\]

\[
\times i \frac{\partial}{\partial p_j} \varphi_3^{(0)}(p - q + t) d\rho dq dt \quad (i \neq j).
\]

(5.7)

Furthermore, we can find that the factor \(2m\) in (3.6) and (3.7) is replaced by \((\omega_p + \omega_q)\) if we avoid the use of \(1/m\) expansion. Consequently we are led to the additional interaction given by the following coefficients:

\[
A_\alpha A_\beta B_{\alpha}^* B_{\beta}^* : - \frac{2}{3} \cdot 4g^2 \int (\omega_p + \omega_q)_t (\varphi_\alpha(p), \varphi_\beta(q)) \varphi_t^{(0)}(t)
\]

\[
\times i \frac{\partial}{\partial p_j} \varphi_3^{(0)}(p - q - t) d\rho dq dt ,
\]

(5.8)

\[
A_\alpha A_\beta B_{\alpha}^* B_{\beta}^* : - \frac{2}{3} \cdot 8g^2 \int (\omega_p + \omega_q)_t (\varphi_\alpha(q), \varphi_\beta(q)) \varphi_t^{(0)}(t)
\]

\[
\times i \frac{\partial}{\partial p_j} \varphi_3^{(0)}(p - q + t) d\rho dq dt.
\]

(5.9)

Treating \(\varphi^{(0)}\) as an external field, we get from (5.8) and (5.9)

\[
- \frac{\kappa^2 m M^2}{c^2 \rho^2}
\]

(5.10)

as a correction to (3.11). This is a potential expressing the quadratic effect of the external field. With the contribution of this potential, the rotation angle in our theory is given by

\[
\delta \varphi = (3.5 + 2) \pi \pi \frac{\kappa^2 m M^2}{c^2 L^2} = 5.5 \pi \frac{\kappa^2 m^3 M^2}{c^2 L^3}.
\]

(5.11)

This value is about 8.3% smaller than that given by (5.1). Recently, Dicke
and Goldenberg have observed the solar oblateness and have found a result which is responsible, up to about 8%, for the perihelion rotation of Mercury. Taking this fact into account, we find that our theory seems to be more suitable than general relativity for the explanation of perihelion rotation.

Unfortunately, the above results are by no means unique. For example, the factor \((\omega_0 + \omega_0)^2\) in (5.8) and (5.9) can be replaced by \((p + q)^2\), and also some ambiguity is involved in the determination of (5.4) and (5.5). For justification of the present choice of interactions, we need a further inquiry about the covariance without using the 1/m expansion. It is interesting, however, that the simplest choice of the Hamiltonian leads us to the fairy good explanation of the experiment. This implies the correctness of our method.

§ 6. Interaction with binding potential

By the results of preceding sections, we have shown the possibility of explaining the famous three tests of general relativity in our theory. Our final task is to attack the problem of the interaction between gravity and binding potential, which was the motivation of the present work. At present it is not yet possible to treat the problem in general, and here we confine ourselves to treating only a scalar meson exchange second order potential. We shall see below that the question is answered in the complete affirmative.

To mediate the interaction between A-particles, we consider the following Yukawa-type interaction with C-meson:

\[
\sum f_{\alpha \beta r}A_\alpha^+A_\beta^-C_r^- + \text{h.c.},
\]

\[
f_{\alpha \beta r} = \int (\varphi_\alpha(p), \varphi_\beta(q)) i \frac{\partial}{\partial p_1} \phi_r(p - q) dp dq.
\]

As in the previous sections, we examine the covariance for the terms of AAAAC-type. Here A-particle is treated also by using 1/m expansion. Since C-particle must be treated in a completely covariant way, we need interactions of gravitons with a C-meson pair. Such interactions are immediately inferred from those given in § 3, and we do not give details here.

As a result, it turns out that the new interaction term of AAAAC-type is demanded only for the vector graviton to the order of 1/m, namely,

\[
\sum e_{\alpha \beta r}^{(1)} A_\alpha^+A_\beta^-C_r^-B_1^-B_3^- + \sum e_{\alpha \beta r}^{(2)} A_\alpha^+A_\beta^-C_r^-B_1^-B_3^- + \text{h.c.},
\]

\[
e_{\alpha \beta r}^{(1)} = 4\sqrt{2} g f i \int (\varphi_\alpha(p), \varphi_\beta(q)) \phi_r(t) \frac{1}{|p - q - t|} \varphi_{\text{st}}^{(1)}(p - q - t) dp dq dt,
\]

\[
e_{\alpha \beta r}^{(2)} = 4\sqrt{2} g f i \int (\varphi_\alpha(p), \varphi_\beta(q)) \phi_r(t) \frac{1}{|p - q + t|} \varphi_{\text{st}}^{(1)}(p - q + t) dp dq dt.
\]

Next we examine the covariance for the terms of AAAAC-type, which appear...
from the cross terms between interactions given by (6·3) and (3·8). According to the similar discussion used to derive (3·8) from (3·3), the following interaction terms involving the scalar graviton is required, which do not contradict the covariance for terms of $\Lambda\Lambda C\Lambda B$-type:

\[
\sum g^{(1)}_{\Lambda\Lambda C\Lambda B} A_\alpha^+ A_\beta^- C_\gamma^- B_\delta^+ + \sum g^{(2)}_{\Lambda\Lambda C\Lambda B} A_\alpha^+ A_\beta^- C_\gamma^+ B_\delta^- + \text{h.c.}, \quad (6\cdot6)
\]

\[
g^{(1)}_{\Lambda\Lambda C\Lambda B} = \frac{8}{3} \sqrt{\frac{3}{2}} \int_{\frac{\partial}{\partial p}} \varphi_\delta (p) (p - q - t) d^3 p d^3 q d^4 t, \quad (6\cdot7)
\]

\[
g^{(2)}_{\Lambda\Lambda C\Lambda B} = \frac{8}{3} \sqrt{\frac{3}{2}} \int_{\frac{\partial}{\partial p}} \varphi_\delta (p) (p - q + t) d^3 p d^3 q d^4 t. \quad (6\cdot8)
\]

From the Hamiltonian thus determined, we eliminate the first order interaction with respect to $C$-particle. The resulting Hamiltonian contains a term corresponding to the binding potential between $A$-particles. In addition, it also contains the following term:

\[
\sum a_{\alpha\beta\gamma\delta} A_\alpha^+ A_\beta^- A_\gamma^- B_\delta^+ + \text{h.c.}, \quad (6\cdot9)
\]

\[
a_{\alpha\beta\gamma\delta} = -\frac{16}{3} \sqrt{\frac{3}{2}} \int_{\frac{\partial}{\partial p}} (\psi_\alpha (p), \psi_\beta (q)) (\psi_\gamma (u), \psi_\delta (u))
\]

\[
\times \varphi_\phi (p + q - t - u) d^3 p d^3 q d^4 u + \sqrt{\frac{3}{2}} \int_{\frac{\partial}{\partial p}} (\psi_\alpha (p), \psi_\beta (q)) (\psi_\gamma (u), \psi_\delta (u)) \varphi_\phi (p + q - t - u) \frac{1}{W_{p-t} W_{q-u}}
\]

\[
\times \left[ \frac{2}{3} (W_{p-t} + W_{q-u}) + \frac{2}{3} (W_{p-t} - W_{q-u})^2
\]

\[-\frac{2}{3} \frac{1}{|p + q - t - u|^3} ((p + q - t - u) \cdot (p - q + t + u))^3
\]

\[+ \frac{2}{3} (p - q + t + u)^3 \right] d^3 p d^3 q d^4 u. \quad (6\cdot10)
\]

In (6·10) the symmetrization between $\alpha$ and $\beta$, and $\gamma$ and $\delta$ are omitted for simplicity. The long-range interaction caused by this term of the Hamiltonian exactly corresponds to the following classical potential for the two-particle system:

\[
-\frac{\kappa M}{c^2} \frac{V(|r_1 - r_2|)}{|r_1 - \frac{1}{2}(r_1 + r_2)|}, \quad (6\cdot11)
\]

where $V$ is the Yukawa potential mediated by $C$-meson. To derive (6·11) an average with respect to the orientation of the relative coordinate is taken, which is the procedure similar to an average with respect to the direction of momentum as was mentioned at the end of § 3.

Thus we have arrived at the satisfactory answer to our original question.
Even though the simplest model is used, this may offer a remarkable progress in our understanding of the nature of the gravitational interaction.

§ 7. Comparison with other theories

In the course of conventional field theory, the quantization of the weak gravitational field was accomplished by Gupta. There, eleven sorts of gravitons participated, and the indefinite metric was inevitably used to eliminate the redundant components.

When we consider the \( \varphi_{\mu\nu}T_{\mu\nu}\)-type interaction between a scalar particle and gravitons, where \( \varphi_{\mu\nu} \) are the tensor field corresponding to the gravitons and \( T_{\mu\nu} \) are the components of the energy-momentum tensor of the scalar field, one can easily find that the part of the interaction related to the gravitons with \( sp/p = \pm 2 \) corresponds to our initial interaction Hamiltonian given by \([P_0, K_f] = -i\delta_{\mu\nu}H\) and \( K_f \) characterized by (3·3) and (3·4).

In spite of this correspondence, the whole scheme of our theory is much different from that of the conventional one. The most important feature of our theory is that we can construct a consistent covariant Hamiltonian formalism using only five gravitons corresponding to five values of \( sp/p \), and without using the indefinite metric.

Because of the difference of the number of gravitons in each theory, it is not appropriate to compare the Hamiltonians of both theories directly. To clarify the difference between our theory and conventional one, it is convenient to consider the scattering of two scalar particles. We denote the four-momenta of incident and scattered particles by \( p_1, q_1 \) and \( p_2, q_2 \), respectively, and put \( p_\mu = p_{1\mu} + p_{2\mu}, q_\mu = q_{1\mu} + q_{2\mu} \) and \( t_\mu = p_{1\mu} - p_{2\mu} \). Then, the Feynman amplitude for the scattering process in the conventional theory turns out to be proportional to

\[
2(p_\nu q_\nu)^2 - \frac{1}{3}p^2 q^2 - \frac{1}{3}p q^2 - 3(p^2)^2 / t^2,
\]

where the symmetrization with respect to two particles is omitted for simplicity. On the other hand we obtain instead of (7·1)

\[
2(p_\nu q_\nu)^2 - \frac{1}{3}p^2 q^2 / t^2
\]

according to the present theory.

The amplitude (7·2) surely satisfies the requirement of invariance and gives a justification for the correct covariant nature of our Hamiltonian formalism.

Weinberg\(^4\) has also intended to treat the massless particles, avoiding the indefinite metric. He used only transverse photons in the case of the electromagnetic interaction, and \( sp/p = \pm 2 \) gravitons in the case of the gravitational interaction. To recover the non-invariant character of the one-photon-exchange (or one graviton exchange) scattering process, he introduced a direct potential interaction, and the potential interaction was chosen so that the result of the conventional theory was reproduced. As for the electromagnetic interaction, the
physical consequences of Weinberg's theory are completely equivalent to those obtained by the present author.\(^6\)

An essential difference lies between Weinberg's theory and the present one. Weinberg treated only the invariance of the S-matrix. In our theory, however, the covariance of the Hamiltonian formalism is examined as the fundamental framework of the theory.

The covariance of the Hamiltonian formalism may be more fundamental than the invariance of the S-matrix. If one persists in the result of Einstein's theory, it is even possible in our theory to add a suitable direct interaction between two particles. The interactions introduced in §3, however, are minimal first-order interactions from the viewpoint of the covariant Hamiltonian formalism, and our theory may claim its position as a possible candidate for the theory of gravitational interaction other than Einstein's one.

§8. Discussion

In this paper we have succeeded in constructing the theory of gravity according to the method which was used to reformulate the quantum electrodynamics. The method gives the same result as conventional field theory in the case of quantum electrodynamics. Nevertheless, one can find its utility in the fact that it enables us to avoid the use of the indefinite metric. It is also a merit in this paper that we need not worry about the treatment of gauge invariance. On the other hand, our method obliges us to examine the covariance in detail at each stage of construction of the interaction Hamiltonian. Although such a recipe seems cumbersome at first glance, it is in itself important to recognize clearly the role of each term of the interaction Hamiltonian from the viewpoint of covariance.

To say about comparison with general relativity, our theory must be tested by more precise experiments. However, we may claim its merit rather than general relativity when viewed from the quantum theoretical viewpoint. In particular, the result of the higher order corrections and the discussion of the interaction of gravity with the binding potential are the most important consequences, which show a quite natural character of our theory.

Besides the quadratic effect of the external field given in §5, another correction, namely, higher-order correction to the external field itself still remains. We can see that additional potentials which are proportional each to \(1/r\) and \(1/r^2\) come out as a result of such an effect. Of these potentials, the former one plays only as a small correction to the gravitation constant, and this is not important. The latter is of the same type as the one due to the quadrupole moment of the sun, but we have disregarded it since its effect is estimated to be about one order smaller than that of the solar oblateness observed by Dicke and Goldenberg.
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