Letters to the Editor

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Lepton-Baryon Resonances and Duality in Weak Processes

Yuji NAKAWAKI, Takesi SAITO, Yosiki SAKAI* and Yasutaka TANIKAWA *

Department of Physics
Osaka University, Toyonaka, Osaka
*Department of Physics
Kobe University, Kobe

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In a previous paper, 1 we proposed the theory of duality for weak interactions and studied the process \( \nu + n \rightarrow \mu + p \) by using the Veneziano model, in which the \( \mu^+p \) resonance (T boson) recently observed by the CERN group 2 was considered to lie on a Regge trajectory \( \alpha_T(s) \). In the present letter we assume also an existence of a \( \mu^+n \) resonance, i.e., \( \mu^-p \) channel resonance (we call this the \( T' \) boson), lying on a Regge trajectory \( \alpha_{T'}(u) \), and introduce it into the previous model. Then, we can show that a new model reduces the energy dependence of quantities corresponding to the vector and axial-vector form factors to be much smaller than that in the previous model.

Let us write the scattering amplitude for the process \( \nu n \rightarrow \mu^-p \) as

\[
\sqrt{\mu_\nu p_\nu}<\mu p|T|\nu n>\sqrt{\nu_\nu n_\nu} = \left[ \bar{\nu}(p) \left( F_{\nu} \gamma_\mu - \frac{F_{\nu}}{M} \sigma_\mu q_\nu \right) u(n) \right] + \left[ \bar{\nu}(p) \left( F_{\nu} \gamma_\mu - \frac{F_{\nu}}{M} \sigma_\mu q_\nu \right) u(n) \right] \times \left[ \bar{\nu}(\mu) \gamma_\mu u(\nu) \right] + \cdots ,
\]

where \( u(\nu) = \gamma_\mu u(\nu) \) is the two-component spinor of \( \nu \), \( q_\mu \) is a four-momentum transfer between proton and neutron: \( q_\mu = p_\mu - n_\mu \), \( F_\nu \)'s are invariant amplitudes, \( M \) is the nucleon mass and \( m_\mu \) is the muon mass. The invariant amplitudes \( F_\nu \) depend on both variables \( s = (n + \nu)^2 \) and \( t = (p - n)^2 \). The \( T \) and \( T' \) bosons appear in \( F_\nu \) as pole terms in the \( s \) and \( u = (n - \mu)^2 \) planes. Their residues can be obtained by making use of Fierz identities. The result is, for spin-one-resonances, as follows:

\[
F_{\nu} = \frac{g_\mu g_\nu}{s - M_{T}^2} \left( 1 - \frac{M^2}{2M_{T}^2} \right) + \frac{g_\mu' g_\nu'}{u - M_{T'}^2} \left( 1 - \frac{M^2}{2M_{T'}^2} \right) + \cdots ,
\]

\[
F_{A} = \frac{g_\mu g_\nu}{s - M_{T}^2} \left( 1 - \frac{M^2}{2M_{T}^2} \right) + \frac{g_\mu' g_\nu'}{u - M_{T'}^2} \left( 1 - \frac{M^2}{2M_{T'}^2} \right) + \cdots ,
\]

\[
F_{S} = \frac{g_\mu g_\nu}{s - M_{T}^2} \left( M_{nT} \right) + \frac{g_\mu' g_\nu'}{u - M_{T'}^2} \left( M_{nT'} \right) + \cdots ,
\]

\[
F_{P} = \frac{g_\mu g_\nu}{s - M_{T}^2} \left( M_{nT} \right) + \frac{g_\mu' g_\nu'}{u - M_{T'}^2} \left( M_{nT'} \right) + \cdots ,
\]

\[
F_{T} = \frac{g_\mu g_\nu}{s - M_{T}^2} \left( M_{nT} \right) - \frac{g_\mu' g_\nu'}{u - M_{T'}^2} \left( M_{nT'} \right) + \cdots ,
\]

where \( g_\mu \) and \( g_\nu \) are the \( \mu^-pT \) and \( \nu nT \) - coupling constants, \( g_\mu' \) and \( g_\nu' \) are the \( \mu^-nT' \) and \( \nu pT' \) - coupling constants, and \( M_T \) and \( M_{T'} \) are the \( T \) - and \( T' \) - boson masses, respectively. No other amplitudes contain the \( T \) - and \( T' \) -boson pole terms.

Rüjula and Zia 3,1) have shown that the interaction of the \( T \) boson is superweak. In the following, we assume, for the interaction of the \( T' \) boson, that \( g_\mu g_\nu = g_\mu' g_\nu' \), i.e., the \( T' \) interaction is also superweak.
(We further set $M_T'=M_T=2\text{GeV}$.)

Now, we take the Veneziano formula for $F_v$ in the following form:

$$
F_v= \frac{G}{\sqrt{2}} \sum_{i=1}^{n} \left[ V_i^t \Gamma(1-\alpha_T(s)) \frac{\Gamma(i-\alpha_T(t))}{\Gamma(1+i-\alpha_T(s)-\alpha_T(t))} 
+ V_i^u \frac{\Gamma(1-\alpha_T(u)) \Gamma(i-\alpha_T(t))}{\Gamma(1+i-\alpha_T(u)-\alpha_T(t))} 
+ (s, u) \text{ term} \right],
$$

(3)

where $\alpha_T(s)=(1/30)(s-M_T^2)+1$ is the $T$ boson trajectory. Here we have considered only the $\rho$ trajectory $\alpha_\rho(t)$ as the Regge trajectory appearing in the $t$-channel. A necessity of satellite terms is due to the following reasons:

i) At $s=(M+m_T^2)=1\text{GeV}^2$ and $t=0$, $F_v$ must give the Fermi coupling constant $G$.

ii) The residue of the pole $\alpha_T(s)=1$ (or $\alpha_T(u)=1$) has the magnitude of the order $10^{-3}G$, if the $T$-boson interaction is superweak.

These conditions cannot be satisfied simultaneously by $F_v$ without satellite terms. In Eq. (3) we have chosen a special form for the satellite terms. This is one of possible combinations to give $F_v$ of the dipole type $(G/\sqrt{2})(1-t/0.71)^{-2}$, which is experimentally favoured. In the following we shall neglect the $(s, u)$ term in Eq. (3), because it violates the dipole behavior of $F_v$.

Comparing the pole terms at $s=M_T^2$ and $u=M_T^2$ in Eq. (2) with those in Eq. (3), we get

$$
\frac{G}{\sqrt{2}} \sum_{i=1}^{n} V_i^t = \frac{G}{\sqrt{2}} \sum_{i=1}^{n} V_i^u
= g_\rho g_T \left( 1 + \frac{M_T^2}{2M_T^2} \right) a_T,
$$

(4)

where $a_T=\frac{1}{30}$ is the slope of $\alpha_T(s)$. We now take $n=3$, and set $V_1^t=-V_2^t/2 = V_3^t$ and $V_1^u=V_2^u$, when $s=M_T^2$ and $u=M_T^2$. Then, in this case the $F_v$ becomes $(s-u)$ even form, and one can see that for fixed $t$, as $s$ increases the $(s, t)$ term increases while the $(u, t)$ term decreases, so that $F_v$ remains almost constant.

Near the poles we may take, in order to satisfy the conditions ii) and (4),

$$
V_1^t = -2V_2^t + \delta_1^t, \quad V_1^u = -2V_2^u + \delta_1^u,
$$

$$
V_3^t = V_2^t - \delta_2^t, \quad V_3^u = V_2^u - \delta_2^u
$$

(5)

where $\delta_1^t+\delta_1^u=\delta_2^t+\delta_2^u$, $\delta_1^{(u)} = 10^{-3}$.

The $\delta_1^{(u)}$ can be neglected in the region far from the poles, because $V_t^{(u)}$ is of order 1. The curve of $F_v$ against $t$ is given in Fig. 1.

In the same way the Veneziano formula for $F_A$ is given by

$$
F_A = \frac{G}{\sqrt{2}} \sum_{i=1}^{n} A_4^t \Gamma(1-\alpha_T(s)) \frac{\Gamma(i-\alpha_A_1(t))}{\Gamma(1+i-\alpha_T(s)-\alpha_A_1(t))} 
+ A_4^u \frac{\Gamma(1-\alpha_T(u)) \Gamma(i-\alpha_A_1(t))}{\Gamma(1+i-\alpha_T(u)-\alpha_A_1(t))},
$$

(6)

where $\alpha_A_1(t)=t-0.02$ is the $A_1$ trajectory degenerating with the $\pi$ trajectory. The curves of $F_A$ against $t$ are given in Fig. 2 for $n=3$ and Fig. 3 for $n=4$, respectively.

The other amplitudes $F_\theta$, $F_r$ and $F_T$ may

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\*\*\* The present experimental situation seems not to be conclusive. Here, we tentatively assume the coupling constants and the masses of $T$ and $T'$ bosons as orders of those values estimated in Ref. 3). A possible existence of weak baroleptons $T$ and $T'$ was predicted by Tanikawa and Watanabe.\(4\)

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be written in the same manner. In general the larger \( n \) gives the less energy dependent form factors.

4) Y. Tanikawa and S. Watanabe, Phys. Rev. 113 (1963), 1344.