The Calculation of the Electromagnetic Fields of a Sheet Current Source with Arbitrary Spatial Intensity Distribution over a Layered Half Space—II. The Computer Program and its Application

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Summary

A general computer program to solve the electromagnetic induction problem for an arbitrary sheet current source over a half space with up to three layers is given. The arbitrary source is constructed from a number of elementary sources by superposition. A description of the program is included. A sample run to illustrate the use of the program is given.

1. Introduction

Calculations to solve the problem of electromagnetic induction in the Earth have been undertaken by many workers. In most cases either uniform or relatively simple non-uniform sources have been employed. More general natural source configurations exist and various ones have been observed experimentally (Kisabeth & Rostoker 1971).

In electromagnetic induction studies for magnetotelluric work as well as for geomagnetic deep sounding natural sources are used. It is advantageous to be able to represent non-uniform source fields and to calculate their effects over layered media in order to enable the removal of source effects from the observed data as one aspect of data reduction from such studies.

In this work a general computer program is given to solve the electromagnetic induction problem for an arbitrary sheet current source over a layered half space with up to three layers.

2. Description of the program

The general two-dimensional program for a sheet current source with an arbitrary intensity distribution is composed of two programs in which the solution for a general source is constructed from a number of elementary sources by superposition. The first program calculates the field components over a two-dimensional grid with a layered subsurface due to a current source with a Gaussian spatial intensity distribution of small half width (termed an elemental Gaussian). The second program
**********ELEMENTAL GAUSSIAN**********

**CAUTION**
The parameters $z_1, d_1, d_2, p_9, z_0, h_w$ are expressed in meters.
The values in the array $z$ are expressed in kilometers.

$z_1$ --- height of upper boundary above surface (a negative value)
$d_1$ --- thickness of first layer --- if half space is desired set $d_1$
greater than 1.0E+6 meters.
$d_2$ --- thickness of second layer --- if only one layer is used set $d_2$
greater than 1.0E+6 meters --- likewise for a half half space.
$O_1$ --- maximum intensity of the Gaussian current distribution centered
at the origin.
$z_0$ --- height of current source above surface (a negative value)
$h_w$ --- half width of Gaussian current distribution
$n_1$ --- number of levels to be calculated
$n_2, n_3, n_4, n_5$ --- the meaning of these values is explained in
format statement 112.
$n_8$ --- number of kilometers between calculated field values
$n_t$ --- total number of points per level
$n_5$ --- one half the number of integration intervals. This number
must be an even number.

The MKS system of units is used throughout this program.

***************

INTEGER Z(25)
DIMENSION CON(4), Y(2001)
COMMON/AL/OI, OR, HW, EX, ZO, A, B, D1, D2, NT, N5, N8
READ(5,100) Z1, D1, D2, OI, ZO, HW, N1, N2, N3, N4, N8
READ(5,101) NT, N5

2 --- this array contains the level spacings. The first
spacing is the distance between the first level and the top boundary.
These values are in kilometers.

IF (N1.EQ.1) GO TO 103
READ(5,102) (Z(K), K=1,N1)
GO TO 104

103 READ(5,102) Z(1)

FREQ --- frequency in hertz.
A, B --- limits of integration. integrand is assumed negligible at B, A=0.

104 READ(5,105) FREQ, B

CON --- array containing the conductivities of the various layers
starting with the air layer (conductivity zero) and proceeding downward.
Four conductivities must be specified. if a half space subsurface
is desired, the three subsurface conductivities must be specified
to be the same value.

Fig. 1. ELEMENTAL GAUSSIAN main program.

combines the solutions for the field components of a number of spatially shifted
elemental Gaussian sources such that a rectangular distribution of current intensity is
closely approximated. With an appropriate change in parameters the second program
is also used to approximate the arbitrary current intensity distribution in a piecewise
continuous manner by weighting a number of spatially shifted rectangular current
distributions in accordance with the trapezoidal rule. The solution for the field
components due to the arbitrary source and the layered subsurface is thus obtained.

2.1 The program ELEMENTAL GAUSSIAN

The ELEMENTAL GAUSSIAN program consists of a main program and two sub-
programs, INTEG and CONST. The main program listing is given in Figs 1 and 2. The
subprograms INTEG and CONST are given in Figs 3, 4 and 5, 6 respectively.
The main program calculates the field component values at a specified number of points (NT) along a level at a constant height for positive y values for a Gaussian current distribution at \( z_0 = -|\hat{z}| \).

The integrations for \( E_x^+, H_y^+ \) and \( H_z^+ \) are performed through the call to INTEG. The integration subroutine (INTEG) calls the subroutine CONST in which the integrands, excluding the terms \( \cos (sy) \) for \( E_x \) and \( H_y \) and \( \sin (sy) \) for \( H_z \) (see Paper I), are calculated for all values of \( s \) at the specified level. For \( E_x \) the integrand is:

\[
\{A_n \exp(-\theta_n z) + B_n \exp(\theta_n z)\} C(s) \cos (sy). \tag{1}
\]

For \( H_y \) the integrand is:

\[
(-i/\omega \mu_0) \{\theta_n [A_n \exp(-\theta_n z) - B_n \exp(\theta_n z)] C(s) \cos (sy) \} \tag{2}
\]
For $H_z$ the integrand is:

$$(i/\omega\mu_0)(s[A_n \exp (-\theta_n z) + B_n \exp (\theta_n z)] C(s)). \sin (sy).$$

$C(s)$ is as given in equation (2) of Paper I. Upon returning to INTEG the integration is performed by the use of Filon's method (Tranter 1965). The main program then writes the electric and magnetic field values for that level on a file or magnetic tape. This process is then repeated for subsequent levels.

2.2 The program RECTAZOID

This program sums a finite number of elemental Gaussian solutions to obtain an approximate solution for a rectangular current distribution. The user specifies the
number of elemental Gaussians to approximate the rectangular current distribution and assigns their source coefficients the magnitude one. The elemental Gaussian solution for positive values of \( y \) calculated by the previous program is read into both positive and negative \( y \) regions. In this, the vertical magnetic field quantities must be made negative in the negative \( y \) region since the vertical magnetic field is asymmetric about the origin. This solution is then shifted and summed to approximate the rectangular current solution. The result is then stored. This program is again used to give the solution for a piecewise continuous source composed of a summation of rectangular current sources. The general source configuration must be digitized by the use at regular intervals (equal to the width of the rectangles) and these source coefficients are then supplied to the program in the array \( SA \). The general solution is obtained by superimposing rectangular source solutions which are shifted with respect to one another and multiplied by the appropriate source coefficients in accordance with the trapezoidal rule. The final solution is thus obtained for the layered Earth and is that of a piecewise continuous source which approximates a continuous source. The listing of \( \text{RECTAZOID} \) is given in Figs 7 and 8.

3. Various parameters and their effects on the solution

In the programs many parameters can be varied and the values of these can affect the accuracy of the final solution. It is important to consider these parameters and the reasons for choosing their values.

```
203 G=G*5.5
W=1.0000-20
CALL CONST(V,22,CONSTT,CONSHY,CONSHZ,CON)
DO 205 I=1,NT,NB
ORIGHY(I)=CONSHY
ORIGHZ(I)=CONSHZ
ORIGIN(I)=CONST
C C FILOK'S INTEGRATION METHOD
C
THE(I)=S
IF (THE .LE. .1) GO TO 206
ALPHA=THE**2+THE*SINTH(2.*SINTH)**2)/THE
BETA=2.*THE*(1.+(COSTH)**2)-2.*SINTH*COSTH)/THE*3
GAMMA=4.*SINTH*THE*COSTH)/THE**3
GO TO 207
ALPHA=(2.*THE**4/4.1.)-(2.*THE**6/315.1.4.2.*THE**7/7225.)
207 HX(I)=S*(ALPHA*EVENHY(I)+BETA*ODDHY(I)+GAMMA*ODDHZ(I))
HZ(I)=S*(ALPHA*EVENZ(I)+BETA*ODDHZ(I)+GAMMA*ODDHY(I))
C C LINEAR INTERPOLATION BETWEEN CALCULATED POINTS IN A GIVEN LEVEL.
C
N9=NB+1
N10=NB-1
DO 208 I=N9,N10,N9
ELINE=(EX(I)+EX(I-N9))/FLOAT(N9)
HYLINE=(HY(I)-HY(I-N9))/FLOAT(N9)
HZLINE=(HZ(I)-HZ(I-N9))/FLOAT(N9)
DO 208 J=1,N10
EX(I-N9*J)=EX(I-N9)+ELINE*FLOAT(J)
HY(I-N9*J)=HY(I-N9)+HYLINE*FLOAT(J)
HZ(I-N9*J)=HZ(I-N9)+HZLINE*FLOAT(J)
RETURN
END
```

Fig. 4. Subroutine \( \text{INTEG} \) (continued).
SUBROUTINE CONST(S,Z2,E1,H1,H2,CON)

DIMENSION CON(U)

COMPLEX TH(3),W(3),Q,CEXP,R,E1,CSQRT,A2,A3,A4,B2,B3,Q1,Z1

DIMENSION H1,H2

COMPLEX CPM,CMPR,CMPRF

COMMON/A1/OI,OM,UM,WN,PI,ZO,A,B,D1,D2,N5,N8

THIS SUBROUTINE CALCULATES THE INTEGRAL VALUES.

THE SUBSURFACE CAN CONTAIN A MAXIMUM OF TWO LAYERS AND ONE HALF SPACE.

CPRII=(0.0,-1.0)/(OM*UM*WN)/(SQRT(2.0*PI)*S)*EXP(S*ZO-S**2*WN)

1*2/2)

Z=S**2

DO 300 N=1,3

Y1=OM*OM*CON(N+1)

W(N)=CMPLX(2,1)

300 TH(N)=CSQRT(W(N))

IF(D1.GT.1.0E+6) GO TO 301

IF(D2.GT.1.0E+6) QI=CMPLX(0.0,0.0)

IF(D3.GT.1.0E+6) GO TO 301

Q1=TH(2)-TH(3))/TH(2)+TH(3)) EXP(-2.*TH(2)*D2)

301 IF(D1.GT.1.0E+6) GO TO 302

Q1=(TH(1)+TH(2)+TH(3))/TH(2)+TH(3)) EXP(-2.*TH(2)*D2)

B=(O.0,-1.0)/(TH(1)*Q-I))

B1=(B-1.)/(B+1)

302 IF(D1.GT.1.0E+6) B1=TH(1)-TH(2))/(TH(1)+TH(2))

IF(Z2.LE.0.0) GO TO 304

A2=((A2+B2*CEXP(2.*TH(2)*D2)) / (A2*Q1*CEXP(TH(2)*D2))) EXP((TH(2)-TH(1)*D1))

B3=A3*Q1

IF(D2.GT.1.0E+6) GO TO 303

IF(D2.LT.D2) GO TO 305

A4=(A3+B3*CEXP(2.*TH(2)*D2)) EXP((TH(3)-TH(2)*D2))

GO TO 307

303 A4=A3

GO TO 307

304 IF(D1.GT.1.0E+6) B1=EXP(S*Z2) *CPRII

HY1=((0.0,-1.0)/(OM*UM)*S)/(EXP(-S*Z2)-B1*EXP(S*Z2)) *CPRII

H21=((0.0,1.0)*S)/(OM*UM)*E1

GO TO 308

305 IF(D1.GT.1.0E+6) B2=EXP((TH(1)+Z2)*CPRII

HY1=((0.0,-1.0)/(OM*UM)*TH(1)*A2*CEXP(-TH(1)*Z2)-B2*CEXP(TH(1)*Z2)) *CPRII

H21=((0.0,1.0)*S)/(OM*UM)*E1

GO TO 308

306 IF(D1.GT.1.0E+6) B3=EXP((TH(2)+Z2)*CPRII

HY1=((0.0,-1.0)/(OM*UM)*TH(2)*A3*CEXP(-TH(2)*Z2)-B3*CEXP(TH(2)*Z2)) *CPRII

H21=((0.0,1.0)*S)/(OM*UM)*E1

GO TO 308

307 E1=(A4*CEXP(-TH(3)+Z2)) *CPRII

HY1=(0.0,-1.0)/(OM*UM)*TH(3)*A4*CEXP(-TH(3)*Z2)) *CPRII

H21=((0.0,1.0)*S)/(OM*UM)*E1

GO TO 308

308 RETURN

END

Fig. 5. Subroutine CONST.

Fig. 6. Subroutine CONST (continued).
3.1 Elemental Gaussian

The final result depends greatly upon the accuracy of the elemental Gaussian solution. The important parameters here are \( B \), \( N5 \) and \( N8 \).

For half widths of approximately 10 km the source function \( C(s) \) is governed by the term \( \exp\left(s z_0\right) \). Therefore, \( B \) should be taken to be at least ten times the damping constant \( (1/|z_0|) \) for a source in free space (see Hermance & Peltier 1970). For a layered-earth model a value of \( 0.8 \times 10^{-4} \) or larger will be satisfactory.

A total of 4000 integration intervals \( (2*N5) \) is sufficient for a half width \( \left(HW\right) \) of 1–10 km. This value of \( N5 \) improves the accuracy of the apparent resistivity curves...
calculated from

\[ \rho_a = \frac{1}{\omega \mu_0} \left| \frac{E_x}{H_y} \right|^2 \]

(Cagniard 1953) since when calculating the apparent resistivity any error in the field values produces a greater error in the resistivity values. Satisfactory field values may be obtained by using approximately one-sixth this value of \( N5 \). These values of \( B \) and \( N5 \) were used in calculating the elemental Gaussian results of Tables 1 and 2 of Paper 1. In calculating the 240 km half width Gaussian the value of \( B \) was taken as \( 0.1 \times 10^{-6} \) there.

The solution for the field values at any particular level is calculated at a certain number of points and this is indirectly determined by \( N8 \). The values for intermediate points are determined by a linear interpolation between the calculated points. The
### Electromagnetic Fields of a Sheet Current Source

<table>
<thead>
<tr>
<th>xi</th>
<th>d1</th>
<th>d2</th>
<th>o1</th>
<th>z0</th>
<th>nw</th>
<th>n1</th>
<th>n8</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.5000E+05</td>
<td>0.5000E+05</td>
<td>0.1100E+07</td>
<td>0.1000E+01</td>
<td>-1.1100E+06</td>
<td>0.1000E+05</td>
<td>1</td>
<td>1000</td>
</tr>
</tbody>
</table>

**FREQ = 1.00000**  
**Upper Limit of Integration = 0.0000E-04**

**Conductivities =**  
0.0  
0.1000E-01  
0.1000E+00  
0.1000E+00

**Complex Field Values are Given at Intervals of 200 km, Over a Range of 0 km to 2000 km.**

**Level 1 Completed**

**Ex**

| 0.1157E-02 | -1.1324E-02 | -1.8798E-03 | -1.1038E-02 | -1.6028E-03 | -1.7521E-03 | -3.258E-03 | -4.661E-03 | -4.885E-04 | -1.602E-03 |
| 0.2282E-03 | 0.1058E-03  | 0.1910E-03  | 0.8845E-04  | 0.1538E-03  | 0.7112E-04  | 0.1167E-03 | 0.5380E-04 | 0.7954E-04 | 0.3647E-04 |
| 0.4238E-04 | 0.1915E-04  |              |              |              |              |              |              |              |              |

**Hy**

| -5.969E-01 | -1.477E-02 | -1.954E-01 | -1.117E-02 | -1.312E-01 | -8.783E-03 | -1.698E-01 | -5.789E-03 | -2.749E-02 | -2.796E-03 |
| 0.1149E-01 | 0.1977E-04  | 0.9617E-02  | 0.1673E-04 | 0.7746E-02 | 0.1370E-04 | 0.5876E-02 | 0.1067E-04 | 0.4005E-02 | 0.7634E-05 |
| 0.2135E-02 | 0.4600E+05  |              |              |              |              |              |              |              |              |

**Hz**

| 0.0  | 0.0  | 0.8815E-06 | -9.073E-06 | 0.1763E-05 | -1.815E-05 | 0.2648E-05 | -2.722E-05 | 0.3526E-05 | -3.629E-05 |
| 0.4907E-05 | -4.536E-05 | 0.3615E-05 | -3.721E-05 | 0.2822E-05 | -2.906E-05 | 0.2030E-05 | -2.291E-05 | 0.1237E-05 | -1.276E-05 |
| 0.4438E-06 | -4.604E-06 |              |              |              |              |              |              |              |              |

**Fig. 9. Sample run output for elemental Gaussian.**
### Table 1

<table>
<thead>
<tr>
<th>Level</th>
<th>Completed</th>
</tr>
</thead>
<tbody>
<tr>
<td>2001</td>
<td>1</td>
</tr>
<tr>
<td>3</td>
<td>10</td>
</tr>
<tr>
<td>2</td>
<td>10</td>
</tr>
<tr>
<td>10</td>
<td>10</td>
</tr>
</tbody>
</table>

**Complex Field Values**

Levels 1 completed are given at intervals of 200 km, over a range of -2000 km to 2000 km.

**Fig. 10.** Sample run output from RECTAZOID for the $H_x$ field values for the rectangular current distribution.
quantity \( (N8-1) \) represents the number of points between calculated values. In all cases except the sample run \( N8 \) was set equal to 10. If this value is made too large the solution will be less accurate, especially when the field solutions are shifted and added.

3.2 \textit{RECTAZOID}

The parameters in \textit{RECTAZOID} which most affect the results are \( N\text{SOUR} \) and \( N\text{SHIFT} \). When synthesizing the rectangular source the best approximation is obtained when the adjacent elemental Gaussians are one half width apart. Because of this, \( N\text{SHIFT} \) is equal to the half width in kilometres. When arbitrary source currents are approximated \( N\text{SHIFT} \) is equal to the width of the rectangles. The width of the rectangular source specified by \( N\text{SOUR} \) can also affect the accuracy of the solution. The normalized values of \( H_z \) are most affected by the source geometry and coarse rectangular approximations to the arbitrary source configuration can noticeably affect these profiles.

4. Sample run

Sample calculations have been made and the results of this run are given in Figs 9, 10, 11 and 12.

The sample run has been designed to provide an easy verification of the program. The total running time to obtain the results given here is approximately 40 s on the IBM 360/67 at the University of Alberta. In the \textit{ELEMENTAL GAUSSIAN} program for this example only three points are calculated and the remaining ones are obtained through linear interpolation. A single value of 50 must be supplied for the \( Z \) array since the fields are obtained for the surface level. The value of \( NT \) was 2001 and \( N5 \) was taken to be 20. The other parameters are as specified in Fig. 9. Fig. 10 gives the parameters used in \textit{RECTAZOID} to construct the 30-km rectangular source. Since only vertical magnetic field data are produced in this run \( NHZ \) must be set equal to 1 to ensure the non-symmetry of the \( H_z \) field. The value of \( NHZ \) must be taken as 1 in construction of the rectangular source, but when synthesizing the arbitrary source from these rectangles \( NHZ \) must equal zero. The arbitrary source consists of fourteen rectangular sources weighted as indicated in the source array \( SA \) given in Fig. 11. The third rectangular current source is positioned at the origin as indicated by \( NSC \). The parameters of Fig. 12 produce the vertical magnetic field associated with this source and these field values are given as well in Fig. 12. If the electric field or the horizontal magnetic field is desired, the elemental Gaussian data for those fields could be operated upon by \textit{RECTAZOID} to produce the desired field. In these latter two cases all parameters for \textit{RECTAZOID} would remain the same as before, except for \( NHZ \) which would be set equal to zero.
### Table 1

<table>
<thead>
<tr>
<th>Level</th>
<th>Field Values at Intervals of 200 km, Over a Range of ~2000 km</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1.720-03 0.1789-03 0.4612-01 0.5259-01 0.2932-03 0.1132-01 0.165-01 0.2108-01 0.2306-01 0.2530-01</td>
</tr>
<tr>
<td>2</td>
<td>0.3672-01 0.4197-02 0.5132-00 0.3092-01 0.2108-01 0.0452-01 0.132-01 0.165-01 0.2108-01 0.2306-01</td>
</tr>
</tbody>
</table>

**Fig. 12**: Sample run output from **RECTAZOID** for the arbitrary current source. This shows the parameters used in **RECTAZOID** as well as the vertical magnetic field values.
Acknowledgments

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References


