Finite Field Theory in Solvable Models

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Finite local field theories, known up to now, need fictitious fields in addition to original fields. These fields play a role of cancelling divergent quantities coming from original fields. Such fields have influence not only on divergent quantities but also on finite ones which can be compared with experiment.

The great success of the present renormalization theory suggests that correct finite theory should be the one giving no influence on observable quantities. Usually we use the value of masses of fictitious fields introduced large enough to reflect this fact. However, it seems to us that this is only an allopathic excuse.

Recently, a new finite local theory, requiring no additional field, has been presented. In this note, we show that, by direct calculation in solvable models, the new theory is just the one we are looking for in the sense that it gives the same results as those of the present renormalization theory, except for finiteness of renormalization constants.*

First we discuss neutral scalar meson interacting with a fixed nucleon. Our Hamiltonian is

\[ H = (m + \delta m) N^*N + \sum_k \omega_k a_k^*a_k \]

where \( \delta m \) is a finite vector with an arbitrary direction, \( \epsilon_i \)'s are odd polynomials of \( \epsilon \), \( b_i \)'s real adjustable parameters with the normalization condition: \( \sum b_i = 1 \). We get finite results by suitable choice of \( \epsilon_i \)'s [see (12)]. Our Hamiltonian has the property \( \lim H^*(-\epsilon) = \lim H(\epsilon) \) instead of hermiticity. Then we must define the norm of a state \( |\Phi(\epsilon)\rangle \) by

\[ \lim \langle \Phi^*(-\epsilon) | \Phi(\epsilon) \rangle \]

and the probability of finding the state \( a \) in the state \( b \) by

\[ \lim \langle \Phi_a^*(-\epsilon) | \Phi_b(\epsilon) \rangle \times \langle \Phi_b^*(-\epsilon) | \Phi_a(\epsilon) \rangle / \langle \Phi_b^*(-\epsilon) | \Phi_b(\epsilon) \rangle . \]

The mass renormalization constant \( \delta m \) is determined from the condition that the eigenvalue of \( H \) for one nucleon state is \( m \). The result is

\[ \delta m = \lim g^2 \sum_{ij} \frac{1}{N} \sum_k (2\omega_k^2V)^{-1} b_i b_j \times \exp \{ik \cdot \Delta (\epsilon_i + \epsilon_j) \} . \]

Consider the case that only one nucleon presents. The Hamiltonian in this situation has a simple form:

\[ H = m + \lim \sum_k \omega_k C_k^*(\epsilon) C_k(-\epsilon) \]

with

\[ C_k^*(\epsilon) = a_k^* + g \sum_j (2\omega_k^2V)^{-1/3} b_j \times \exp \{ik \cdot \Delta \epsilon_j \} . \]

We have the commutation relation \([C_k(-\epsilon), C_k^*(\epsilon)] = \delta_{kk'}\). We construct the complete set of eigenvectors of \( H \) by using \( C_k^*(\epsilon) \) and \( C_k(-\epsilon) \). For example, one nucleon state is \( |N\rangle = \lim |N, \epsilon\rangle \), where \( |N, \epsilon\rangle \) is the state vector defined by \( C^*_k(-\epsilon) |N, \epsilon\rangle \) and the perturbation series is given by

\[ + \lim \sum_{i=0} b_i H_i(\epsilon_i), \]

\[ H_i(\epsilon_i) = \sum_k gN^*N (2\omega_k^2V)^{-1/3} \times (a_k^* + a_k) \exp \{ik \cdot \Delta \epsilon_i \} . \]
1632

Letters to the Editor

Then

\[ |N\rangle = Z_2^{1/2} \lim_{n \to \infty} \sum_{n=1}^\infty \frac{(-1)^n/n!}{n!} \times \left[ \sum_k \sum_i g b_i (2\omega_k^3 V)^{-1/2} \times \exp \{ ik \cdot A (e_i + e_j) \} \right] \]

with

\[ Z_2 = \lim_{n \to \infty} \exp \left[ - \sum_k \sum_i g^2 b_i b_j (2\omega_k^3 V)^{-1} \times \exp \{ ik \cdot A (e_i + e_j) \} \right]. \]

Using eigenvectors thus obtained, we find that the nucleon propagation function is expressed as

\[ S'(t-t') = \langle \text{vac} | N(t) N^*(t') | \text{vac} \rangle \theta (t-t') \]

\[ = Z_2 \theta (t-t') \lim \exp \{ - im (t-t') \} \times \exp \{ g^2 \sum_k b_i b_j (2\omega_k^3 V)^{-1} \}
\times \exp \{ ik \cdot A (e_i + e_j) \}
\times \exp \{ - i \omega_k (t-t') \} \}

(10)

\[ = Z_2 \theta (t-t') \exp \{ - im (t-t') \} \times \exp \{ g^2 \sum_k (2\omega_k^3 V)^{-1} \}
\times \exp \{ - i \omega_k (t-t') \} \} \}

(11)

Now let us show that \( \delta m \) and \( Z_2 \) are finite. In \( V \to \infty \), after the integration on \( k \), we see that

\[ \delta m = \lim g^2 \sum_{t,j} \langle 8\pi A \rangle^{-1} b_i b_j |\epsilon_i| \]

\[ + |\epsilon_j|^{-1} - g^2 \mu / 8 \pi, \]

\[ Z_2 = \lim \exp \{ g^2 (2\pi)^{-1} \sum_{t,j} b_i b_j \log |\epsilon_i + \epsilon_j| \}
\times \log (\gamma A \mu / 2) \}, \]

where \( \gamma \) is Euler's constant. If we take \( \epsilon_i \)'s and \( b_i \)'s (\( i = 1 \sim 4 \)) as

\[ \epsilon_1 = \alpha \epsilon, \epsilon_2 = -\beta_1 \epsilon + \beta_2 \epsilon^3, \]

\[ \epsilon_3 = -\beta_1 \epsilon + \beta_3 \epsilon^3, \epsilon_4 = \beta_1 \epsilon, \]

\[ 4b_1 = 4b_2 = -2b_3 = b_4 = 1, |\beta_1| = 2|\beta_2| \]

\[ 1/ (32|\alpha|) - 1/ (8|\alpha - \beta_1|) + 1/ (2|\alpha \]

\[ + \beta_1|) + 17 / (32|\beta_1|) = 0, \]

we easily see that

\[ \lim \sum_{t,j} b_i b_j |\epsilon_i + \epsilon_j|^{-1} = 0, \]

\[ \lim \sum_{t,j} b_i b_j \log |\epsilon_i + \epsilon_j| = \text{finite const.} \]

Then, \( \delta m \) and \( Z_2 \) become finite. In spite of finiteness of renormalization constants, \( \delta m \) is the mass shift due to the interaction. Furthermore both Eqs. (8) and (10) show that \( Z_2 \) is the probability of finding the bare nucleon in the state of the physical nucleon in the sense of (4). As is shown by (11) the propagation function is, apart from finiteness of \( Z_2 \), exactly the same as that of the usual theory. Our choice (12) of \( \epsilon_i \)'s and \( b_i \)'s is only an example for getting finite results. Instead of (12) we can choose them in such a way that Eqs. (13) and (14) have finite special values without any modification of \( S'(t-t') / Z_2 \). Therefore \( \delta m \) and \( Z_2 \) cannot be observable quantities.

In the same way we can discuss the Lee model. Again, our results are identical with those of the usual renormalization theory. Since \( S'_V(t-t') / Z_2 \) is identical with that of the renormalization theory, we cannot remove a ghost state for which (3) is negative.

2) As to the importance of finiteness of renormalization constants, see for example, Y. Kato and N. Mugibayashi, Prog. Theor. Phys. 30 (1963), 409; 31 (1964), 300.