

Discussion: “A New Calculation Method of Hertz Elliptical Contact Pressure” (Tanaka, N., 2001, ASME J. Tribology, 123(4), pp. 887–889)

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The article by Tanaka describes a rapid and accurate method for the determination of contact pressure in the Hertz elliptical contact problem, without the need to use tables or graphs such as those given by Harris or Johnson in the original author's Refs. 2 and 3. Tanaka avoids the elliptic integral and proposes a numerical integration (Eqs. 7 and 8).

Readers may not be aware of another—much earlier—rapid and accurate method which was proposed by the late Alan Dyson. In Dyson's method the ellipse axial ratio, a/b is determined directly from the integral equation, (Tanaka's Ref. 3):

$$\frac{A}{B} = \frac{(a/b)^2 \mathbf{E}(e) - \mathbf{K}(e)}{\mathbf{E}(e) - \mathbf{K}(e)} \quad (1)$$

where $e = (1 - b^2/a^2)^{1/2}$ and \mathbf{E} and \mathbf{K} are complete elliptic integrals. The other required quantities, such as p_0 and δ then follow easily, as described by Johnson (Tanaka's Ref. [3]).

Appendices to the paper by Dyson et al. [1] provide iterative methods both for the inversion of Eq. (1) and for the determination of the elliptic integrals, the latter mysteriously attributed, without reference, to the (UK) National Physical Laboratory. The required iteration shows extremely rapid convergence, is easily adapted to any common computational platform (e.g., spreadsheets) and has been used in the discussor's laboratory for many years. However, Tanaka (and the referees) may perhaps be excused for not referring to Dyson's method in view of the obscurity of his chosen place of publication!

References

- [1] Dyson, A., Evans, H. P., and Snidle, R. W., 1989, “Wildhaber-Novikov Circular Arc Gears: Some Properties of Relevance to Their Design,” Proc. R. Soc. London, Ser. A, **425**, Appendices A2 and A3, pp. 360–361.