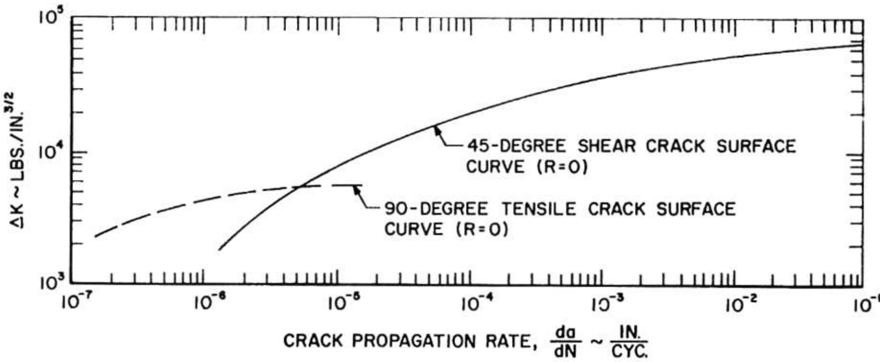

Fig. 8 Crack-propagation rates in 7075-T6 aluminum alloy for both 45-deg shear and 90-deg tensile-type crack surfaces

When changing from a high-load cycle to a low-load cycle, there is often a temporary arrest in the crack growth. The yield-zone model can be used to calculate the increase in the crack-tip radius owing to the high-load cycle. A fatigue crack is then assumed to reinitiate from the rounded crack tip.

The solution to item 7, the load cycles required to initiate a fatigue crack from a flaw, also appears possible. Since most flaws are relatively sharp, such as mechanical scratches and weld flaws, the stress-intensity factor can still be used to approximately define the stress field surrounding the flaw. Preliminary test data indicate that the ΔK concept can be used to predict cycles to fatigue-crack initiation with as much accuracy as exists for predicting crack-growth rates.

Conclusions

A new theory is proposed for predicting the crack-growth rate in cyclic-loaded structures. The theory is an improvement over other theories because the effect owing to the load ratio, R, and the crack instability at onset of fast fracture are taken into account. The theory shows excellent agreement with a wide range of test data. By use of the new theory and with numerical-integration techniques using a digital computer, a more accurate analysis is now possible for the crack-growth behavior of cyclic-loaded structures.

References

1 P. C. Paris and F. Erdogen, "A Critical Analysis of Crack Propagation Laws," Journal of Basic Engineering, Trans. ASME. Series D, vol. 85, 1963, pp. 528–534.

2 D. Broek and J. Schijve, "The Influence of the Mean Stress on the Propagation of Fatigue Cracks in Aluminum Alloy Sheet," National Aero- Research Institute, Amsterdam, The Netherlands, Report NLR-TN M.2111, January, 1963.

3 D R. Donaldson and W. E. Anderson, "Crack Propagation Behavior of Some Airframe Materials," *Proceedings of Crack Propagation Symposium*, College of Aeronautics, Cranfield, England, vol. 2, September, 1961, pp. 375–441.

4 A. J. McEvily and W. Illg, "The Rate of Crack Propagation in Two Aluminum Alloys," NACA TN 4394, September, 1958.

5 C. M. Hudson and H. F. Hardrath, "Effects of Changing Stress Amplitude on the Rate of Fatigue Crack Propagation in Two Aluminum Alloys," NASA TN D-960, September, 1961.

6 W. Weibull, "The Propagation of Fatigue Cracks in Light-Alloy Plates," Svenska Aeroplan Akliebalaget, Linkaping, SAAB TN 25, January, 1954.

7 W. Illg and A. J. McEvily, Jr., "The Rate of Fatigue-Crack Propagation for Two Aluminum Alloys Under Completely Reversed Loading," NASA TN D-52, October, 1959.

8 R. G. Forman, "Experimental Program to Determine Effect of Crack Buckling and Specimen Dimensions on Fracture Toughness of Thin Sheet Materials," AFFDL-TR-65-146, September, 1965.

9 R. G. Forman and A. S. Kobayashi, "On the Axial Rigidity of a Perforated Strip and the Strain Energy Release Rate in a Centrally Notched Strip Subjected to Uniaxial Tension," Journal of Basic Engineering, Trans. ASME, Series D, vol. 86, 1964, pp. 693–699.

10 D. Broek, P. DeRijk, and P. J. Sevenhuysen, "The Transition of Fatigue Cracks in Alclad Sheet," National Aero- Research Institute, Amsterdam, The Netherlands, Report NLR-TR M.2100, November, 1962.

11 D. S. Dugdale, "Yielding of Steel Sheets Containing Slits," *Journal of Mechanics and Physics of Solids*, vol. 8, 1960, pp. 100–104.

DISCUSSION

D. R. Miller[4]

The authors deserve commendation for development of a crack-propagation equation which accounts for the effects of mean stress and the effect of the approach to the critical flaw size for unstable fracture propagation. The writer will be interested in seeing checks of the applicability of this equation to materials which are used in pressure vessels in the thermal-power industry. This equation may provide a more rational basis than now exists for evaluation of the low-cycle fatigue resistance of bolts, in which cracks may initiate early in life, and in which the total life is determined primarily by crack propagation and terminal fracture considerations.

Two interesting questions arise in regard to the authors' equation and its application. Should the equation have the form

$$\frac{da}{dN} = \frac{C(\Delta K)^n}{K_c - \dfrac{\Delta K}{1 - R}} \tag{16}$$

for materials in which the crack-propagation rate is insensitive to mean stress as was observed by Frost [12][5] for mild steel and copper? Should the value of K_c in the equation be lower than the value for the virgin material because of embrittlement which is produced by cyclic strains in the material ahead of the advancing crack?

The writer has used equation (7) as a basis for deriving an equation which can be used to determine, for a particular number of cycles to complete fracture, the relation between the amplitude of alternating stress and the maximum tensile stress during the cycle. This relation has a form which is somewhat similar to that which has been observed by Trapp [13] in room-temperature fatigue tests under axial loading on specimens of SAE 4340 steel. The yield and ultimate strength of unnotched specimens was 147,000 and 159,000 psi, respectively. The specimen diameter in the unnotched region was 0.45 in., the circumferential notch was 0.025 in. deep, the notch root radius was 0.010 in., and the notch had an included angle of 60 deg.

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[5] Numbers in brackets designate Additional References at end of discussion.

If we assume a particular shape of crack and assume that the crack depth is small in comparison with the section thickness, we can write

$$\Delta K = 2\sigma_a A \sqrt{a} \tag{17}$$

where σ_a is the amplitude of the alternations of nominal stress, A is a constant which depends upon crack shape ($A = \sqrt{\pi}$ in equation (3)) and a is the maximum depth of the crack.

Substitution of equation (17) in equation (7), integrating and evaluating the constant of integration B, gives

$$\frac{2(1-R)K_c a^{\frac{2-n}{2}}}{2-n} - \frac{4\sigma_a A a^{\frac{3-n}{2}}}{3-n} + B = 2^n C \sigma_a{}^n A^n N \tag{18}$$

and

$$B = \frac{4\sigma_a A a_o{}^{\frac{3-n}{2}}}{3-n} - \frac{2(1-R)K_c a_o{}^{\frac{2-n}{2}}}{2-n} \tag{19}$$

If we denote the crack depth for unstable crack growth by a_c, we can write

$$K_c = \frac{2\sigma_a A\sqrt{a_c}}{1-R}$$

whence

$$a_c = \left[\frac{K_c(1-R)}{2\sigma_a A}\right]^2 \tag{20}$$

Substitution of equation (20) in equation (18) and using N_c to denote the number of cycles to complete fracture gives

$$\frac{2^{n-1}(1-R)^{3-n}(A\sigma_a)^{n-2}K_c{}^{3-n}}{(2-n)(3-n)} + B = 2^n C\sigma_a{}^n A^n N_c \tag{21}$$

We now let σ_m denote the maximum nominal tensile stress, and substitute $2\sigma_a/\sigma_m$ for $(1-R)$ in equations (19) and (21). We then combine the two new equations to eliminate B and obtain

$$\frac{4A^{n-2}K_c{}^{3-n}}{(2-n)(3-n)\sigma_m{}^{3-n}} + \frac{4Aa_o{}^{\frac{3-n}{2}}}{3-n} - \frac{4K_c a_o{}^{\frac{2-n}{2}}}{(2-n)\sigma_m} = 2^n C\sigma_a{}^{n-1}A^n N_c \tag{22}$$

We now use $n = 4$ as suggested by Paris, and equation (22) becomes

$$\frac{2A^2\sigma_m}{K_c} - \frac{4A}{a_o{}^{1/2}} + \frac{2K_c}{\sigma_m a_o} = 16C\sigma_a{}^3 A^4 N_c \tag{23}$$

We now introduce the symbol σ_o denoting the maximum nominal tensile stress which would have caused unstable crack propagation before the initial crack was extended by cyclic loading. This stress is

$$\sigma_o = \frac{K_c}{A\sqrt{a_o}} \tag{24}$$

Substitution of equation (24) in equation (23) gives

$$\frac{\sigma_m}{\sigma_o} + \frac{\sigma_o}{\sigma_m} - 2 = 8C\sigma_a{}^3 a_o{}^{1/2} A^3 N_c \tag{25}$$

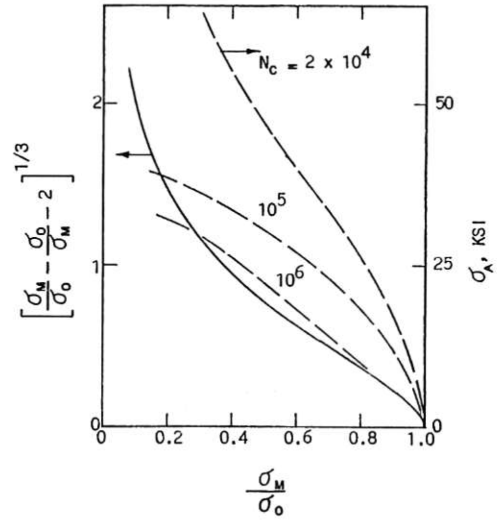

Fig. 9 Comparison of theoretical shape of stress amplitude-vs-maximum stress relation (solid line) with experimental data

This equation indicates that the value of σ_a would decrease monotonically from positive infinity when $\sigma_m/\sigma_o = 0$ to zero when $\sigma_m/\sigma_o = 1$. However, if we restrict use of equation (25) to situations where the mean nominal stress is tensile, the values of σ_a remain finite. Values of σ_a are proportional to the cube root of the left-hand side of equation (25).

The solid line in Fig. 9 shows how this quantity varies with the ratio of σ_m/σ_o. The broken lines represent Trapp's data. Their similarity to the theoretical curve is most notable for the shortest cyclic life, in which crack propagation occupies the greatest fraction of life. The ratio of alternating stress amplitude for a life of 2×10^4 cycles to that for a life of 10^5 cycles is about 1.53 instead of the value of 1.71 which is indicated by equation (25). A better correspondence between theory and experiment might have been obtained in larger specimens in which the crack depth at instability is a smaller fraction of the diameter.

Additional References

12 N. E. Frost, "Effect of Mean Stress on the Rate of Growth of Fatigue Cracks in Sheet Materials," *The Journal of Mechanical Engineering Science*, vol. 4, no. 1, 1962, pp. 22–35.

13 W. J. Trapp, "Elevated Temperature Fatigue Properties of SAE 4340 Steel," Wright Air Development Center, Report no. WADC-TR-52-325, part 1, 1952.

Authors' Closure

The authors wish to thank Mr. Miller for his valuable discussion which offered further verification of the improved crack propagation theory. In reply to Mr. Miller's questions, the authors can only state that equation (7) still requires further study and comparison with experimental results. Mainly, more has to be known about the behavior of the material constants K_c, C, and n. For instance, does material embrittlement due to strain cycling or temperature effects cause changes in all three material constants or in only the fracture toughness, K_c?