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DISCUSSION

F. T. Brown[3]

Professor Streeter has given a fine summary of the basic numerical techniques for unsteady flows, presuming that equation (7) is an adequate statement of the conservation of momentum. Some exceptions, likely omitted because of overly inflexible ASME length limitations, ought nevertheless to be cited.

One exception, well known to Professor Streeter and included in several of his references, is the simpler case of laminar rather than turbulent friction for low frequency excitation. Only minor variations in the equations are necessary.

A much greater departure from equation (7) occurs at high frequencies (abrupt transients) when the wall shear depends on the instantaneous velocity profile or, often equivalently, on the *history* of the mean flow across the tube. Zielke [18][4] introduced the concept of a historetic *weighting function*, which he evaluated for laminar flow, to permit the method of characteristics to apply to such cases. This discusser generalized this concept as a "quasi method of characteristics" [19] applicable to a wider class of problems, including the effect of heat transfer on the unsteady laminar flow of perfect gases. Another paper [20] shows that equation (7) applies reasonably well in turbulent flow only for frequency components below certain specific limits. A more recent report [21] (unavailable when this paper was prepared) gives weighting functions for a variety of initial turbulent Reynolds numbers for transient disturbances which are so brief only frequency components above certain higher specific limits are involved. At such frequencies the applicability of one-dimensional models can be questioned, however (see, for example, the alternative methods and calculations of Jayasinghe and Leutheusser [22] and Tsao [23] which could be adapted to turbulent flow). The report also reveals a dramatic phenomenon at intermediate frequencies in turbulent flow. Apparently because of a little-understood resonance of ring vortices, the step response of a tube may contain significant oscillations. Wavelengths of the complicated patterns are about 25 and 50 diameters. (Further information is forthcoming in a thesis by Margolis.) The report also discusses the details of numerical application of the quasi method of characteristics to large amplitude transients, with illustrations.

Readers should know that the paper and this discussion represent a highly selected rather than comprehensive review of the important literature on numerical methods for unsteady flow calculations in channels and tubes.

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T. P. Propson[5]

The author has conducted a thorough review of the most popular techniques currrently employed to numerically evaluate the effect of transient flows in liquid piping systems. His discussion of the relative advantages and disadvantages of both the characteristics (explicit) and centered implicit method is excellent; of particular interest to the writer were the author's comments relative to the occurrence of instabilities and inaccuracies occasionally encountered during application of the implicit techniques. Recent unpublished work by the writer has confirmed these problems.

When frictional effects are very important, the writer would suggest that equations (30) and (31) be altered in the following manner:

$$C^{+}: H_P - H_A + \frac{a}{gA}(Q_P - Q_A) + R\Delta x\left(\frac{Q_A + Q_P}{2}\right) \times \left|\frac{Q_A + Q_P}{2}\right|^m = 0 \quad (63)$$

$$C^{-}: H_P - H_B - \frac{a}{gA}(Q_P - Q_B) - R\Delta x\left(\frac{Q_B + Q_P}{2}\right) \times \left|\frac{Q_B + Q_P}{2}\right|^m = 0 \quad (64)$$

It may be shown that the error introduced into the integration of the friction term by these finite-difference equations is usually about one-half of that introduced by equations (30) and (31). From this point of view, application of this latter form of the

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[3] Professor of Mechanical Engineering, Lehigh University, Bethlehem, Pa.

[4] Numbers in brackets designate Additional References at end of discussion.

[5] Assistant Professor of Civil Engineering, South Dakota School of Mines and Technology, Rapid City, S. Dak.

characteristic finite-difference equations then would be more competitive with the implicit methods.

The interior-point and boundary-condition equations resulting from utilization of equations (63) and (64) are frequently nonlinear algebraic equations of the form

$$x = F(x)$$

and can be solved efficiently by an iterative procedure based upon the method of successive approximations [24].[6] This technique previously has been employed [25] very satisfactorily in the solution of the comparable interior-point and boundary-condition equations derived from equations (30) and (31).

Additional References

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A. R. D. Thorley[7]

Professor Streeter is to be congratulated for providing such a useful summary of what must be the principal methods of surge analysis in common use today. There are two points that may be made in support of his section on column separation.

The first point concerns the assumption that when a cavity opens, it is assumed to stay at the section where it first appears. In a recent study[8] of column separation in aviation kerosene, some high-speed cine film was taken of the cavity which formed on the downstream side of a closing valve in a horizontal pipeline. This illustrated quite clearly that a cavity opened at the section adjacent to the valve itself and remained there. A pressure transducer mounted further downstream verified the assumption that pressure waves traveled to and fro within the liquid column with the cavity acting as a constant pressure boundary condition, prior to its collapse.

The second point concerns the pressure rise in the event of the cavity collapse. Swaffield found that the release of dissolved air from the kerosene was significant and allowed for this in his theoretical expression for the cavity pressure in terms of its partial pressure. He developed a series of programs employing this released air as the valve boundary condition, since it did not redissolve in the liquid. These programs predicted the values of the minimum and maximum pressures and their event times during and following separation to within 10 percent at the worst. On the upstream side of the valve, agreement was much better. The observed peak pressures were consistently less than the predicted values.

Author's Closure

The writer thanks F. T. Brown, T. P. Propson, and A. R. D. Thorley for their discussions which greatly strengthen the paper. Professor Brown has nicely qualified the range of transients covered by the paper, and has indicated some of the newer methods for refining transient calculations. Masaake Hirose, in a Master's thesis at Massachusetts Institute of Technology (September, 1971, Mechanical Engineering Department) has made an interesting study of frequency-dependent turbulent flow in pipes using an empirical relationship for the historetic weighing function.

Professor Propson's equations (63) and (64) do contain more accurate expressions for the friction-terms, and do not unduly complicate the programming. If the flow varies linearly over the pipe length, and $m = 1$, the error is reduced by one half (as obtained by integration over the pipe length).

Mr. Thorley has introduced the concept of release of dissolved air from a cavity during column separation. This is a new and interesting approach which considers the opening to start developing as the pressure reduces, but before vapor pressure is reached. It may lead to better methods for calculating maximum pressure rise after collapse of the cavity.

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[6] Numbers [24–25] in brackets designate Additional References at end of discussion.

[7] Senior Lecturer, Department of Mechanical Engineering, The City University, London, England. Mem. ASME.

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