Proceeding, sufficient relationships are provided to solve for the
such that the residues at upper half-plane poles are zero; con-
and $b_r$ are residues at $\omega_r$ of this function.

are upper half-plane poles of

\[ a_r \text{ are residues at } \omega_r \text{ of this function; } \omega_r \text{ are upper half-plane poles of } \]

and $b_r$ are residues at $\omega_r$ of this function.

Since $Q_i(\omega)$ and $Q_j(\omega)$ are not known $a_r$ and $b_r$ are undefined coefficients at this stage. Thus

\[
H_1(\omega) = \frac{1}{\Phi^+(\omega)} \left[ \frac{Q_i(\omega)\Phi_2(\omega) - Q_j(\omega)\Phi_3(\omega)}{\Phi^-(\omega)} e^{j\omega}\right]_+
\]

Also from Equation [19] using a similar line of argument

Substituting for $H_1(\omega)$ and $H_2(\omega)$ in Equation [18] now gives rise to an equation whose partial fraction development must be such that the residues at upper half-plane poles are zero; con-
sequently, sufficient relationships are provided to solve for the coefficients $a_r$ and $b_r$.

Discussion

Rufus Oldenburger. The example in the paper, showing that the "optimum" filters for two channels considered in-
dividually are different from the optimum filters when the out-
pute of these filters are added, is most enlightening. Neverthe-
less, as can be seen from Fig. 3 the difference is small; in fact, for
engineering purposes the break points occur at about the same
frequency. One is naturally led to ask whether or not this differ-
ence is always small, and in particular, whether corresponding
break points in simple cases can be separated by an order of
magnitude.

The factoring of $\Phi(\omega)$ into $\Phi^+(\omega)\Phi^-(\omega)$ may in practice be a most difficult one to do explicitly. Does the author recommend substituting simple approximations for $\Phi(\omega)$ which can be so fac-
tored?

Oyro J. M. Smith. The symbol $[\cdot]$ used in Equation [5] and
defined in footnote 6 in the paper is the realizability operator equal
to the Laplace transform of the inverse Fourier transform. The
characteristics of this $L^{-1}$ operator are derived and discussed in
"Separating Information from Noise," by Otto J. M. Smith. Transac-
tions of the Professional Group on Circuit Theory, Institute

Author's Closure

Professor Oldenburger raises a difficulty of long standing in this
type of work, namely, how may one factorize $\Phi(\omega)$ into its com-
ponent product terms $\Phi^+(\omega)$ and $\Phi^-(\omega)$ when $\Phi(\omega)$ is known
only for real values of $\omega$? This is the case when $\Phi(\omega)$ has been
obtained either by direct measurement or by Fourier transforma-
tion from its correlation function. A number of techniques for
doing this have been discussed in the literature. The simplest
are based upon curve-matching techniques using Bode plots and
a set of templates for different values of relative damping ratio $\zeta$.
Simple solutions obtained in this manner can be readily improved
upon using Linvill's method. A general discussion of this type
of procedure is given in the book by Truax. The most refined
method known to the author is due to Kautz and consists in the
use of a generalized orthogonal set of functions. In fact Kautz
offers a wide range of possibilities in choosing an orthogonal set,
so that an element of skill is still required in order to get the
simplest good approximation. Other methods of approximation
depend on comparing coefficients or choosing coefficients so as to
minimize a measure. A simple method of this latter sort is dis-
cussed by Schumacher although here it is difficult to see what
connection there is between the measure minimized and practical
performance. It is unlikely that the last word has been said on
this problem, but sufficient work has already been done for a
small catalog of methods to exist from which one can be selected
to suit the circumstances of the problem, and the accuracy re-
quired.

1 Associate Professor of Electrical Engineering, University of
California, Berkeley, Calif.
11 "The Selection of Network Functions to Approximate Pre-
scribed Frequency Characteristics," by J. C. Linvill, M.I.T. Research

12 "Automatic Feedback Control System Synthesis," by J. G.

13 "Network Synthesis for Specified Transient Response," by
W. H. Kauts, M.I.T. Research Laboratory of Electronics Technical
Report No. 209, April, 1952.

14 "A Method of Evaluating Aircraft Stability Parameters From