\[ l_j = s_{j+1} - s_j \]
\[ f_j = (s - s_j)/l_j \]

The 2N+2 unknowns \( \gamma_j \) and \( \delta_j \) (j = 0, 1, ..., N) can be obtained by solving a set of 2N+2 linear simultaneous equations as follows: The boundary condition which prescribes the normal component of the velocity to vanish at a control point in each interval yields N linear equations, the condition of the first derivative to be continuous at the junction point and the two end conditions (namely, the conditions of the spline fit approximation) give N+1 equations, and the condition which prescribes the flow rate through the venturi tube gives the last equation. Then, the surface velocity at any point is calculated by equation (A1).

The accuracy of the calculation can be determined by examining whether the velocity at the wall surface induced by all the ring vortices coincides with equation (A1). If the required accuracy is not obtained, the number of intervals N is increased. In this study, the calculation was performed so that the difference between the induced velocity and the vortex density (equation (A1)) is smaller than 0.1 percent at the throat tap.

**DISCUSSION**

D. Halmi

In view of the restricted space allowed for this discussion, subjects dealt with here are narrowed to points deemed most important for those who wish to use information from the paper or wish to refine its proposed discharge coefficient calculation method.

The paper presents a method by which \( C \) can be calculated accurately for "well designed" short Venturi Tubes. To gauge how well this goal is achieved, we ought to realize that due to the nature of the concept of \( C \), and due to the way \( C \) are used (also because of the means by which knowledge is obtained about them), we should consider their values, shapes, and the inevitable uncertainties of gathering knowledge about them.

The value of \( C \) is defined as:

\[ C = \frac{\text{True Actual Rate of Flow}}{\text{True Idealized Rate of Flow}} \]

\[ \text{Value} \]

The true shape of \( C \) describes the change in single \( C \) values in the function of some controlling parameters like

- \( C \) versus \( Re \);
- \( C \) versus \( Re \) versus \( \beta \) or
- \( C \) versus \( Re \) versus \( \beta \) versus \( D \), etc.

The uncertainty is a calculated \( \pm C \) value band that should contain the errors with the specified confidence level included in a determined \( C \) value or shape due to the execution of the method by which they were determined.

Utilizing above clarification, the discusser puts forth some of his observations:

- The calculations – for UVT's indicate no size effect (D) but some \( \beta \) and Re sensitivity. The UVT experimental data – to the extent of its uncertainty band – verifies the size insensitivity but contradicts the \( \beta \) and Re sensitivity. Further work would be required either to disprove the indications of the experimental data or improve the mathematical image of \( C \) to make it agree with the results of the experiments.
- \( C \) values and shapes have been determined by the calc method but no attempt is presented in the paper to calculate the uncertainties that belong to the determined \( C \) values and shapes. To develop such knowledge is indispensable if the calc method is intended to be used for \( C \) determination without flow calibration.
- The mathematical "image" could be (should be) refined by recognizing in it the energy loss that occurs as the flow moves through the tube. By adding this refinement to the calc method would eliminate the disagreement illustrated by Fig. 4(b) in the UVT's throat section. The test points indicate a lowering of the static pressure as dictated by the laws of nature in contrast to the calculated shape that, apparently, disregards this energy loss.
- A further refinement could be achieved on the mathematical image of the physical phenomenon that takes place in the UVT if the fact was recognized that the UVT paper – from which the test data was taken – indicates wall pressures as sensed by piezometer holes with their centerline perpendicular to the axis of the tube. These pressures, consequently, indicate true static pressure only if the flow moves in a cylindrical section (like throat section) through a sufficiently long distance after a tube contour change. If this is not the case, the contour of the tube shall effect the magnitude in which the true local static pressure appears at the wall.
- The paper seems to consider a Venturi Tube "well designed" if in it flow separation and vena contractae effects are small. A more proper term here would be "suitably designed" meaning a design that lends itself for accurate and reliable \( C \) calculation.
- At last, a few points are mentioned for correction and/or clarification:
- \( P – \text{Average static pressure.} \) The paper should indicate how the average static pressure should be calculated to satisfy the requirements of the calc method.
- Quote from Paper: "In general, an inlet tap can be located at a point where \( P_i = \dot{P}_i. \)" How can this be achieved?

The physical meaning of surface velocity ought to be explained (*).
- The diverging recovery cone angle is 5 deg (Not 50,.) See Pg. 6, Line 15.
- On Page 7 in lines 14 and 15, the terms laminar and turbulent regions are used without elaborating on what is meant by them here.

R. P. Benedict

The author has contributed a useful paper on fluid flow meters which considers for the first time the effect of
nonuniform static pressures at the venturi throat on the discharge coefficient.

Two questions are raised concerning Professor Inoue’s analysis. The first concerns the author’s equations (10). The Benedict-Wyler equations (6) show no $\beta$ factor in the third terms of the denominators. Does the author have a basis for these additional terms? The second concerns the throat taps used in the experimental verification data. Were static tap corrections applied before comparing theoretical discharge coefficients with measured discharge coefficients? The effect of such corrections is on the order of 0.5 percent on the discharge coefficient (see for examples: [4, 10, and 11]).

Again, let me congratulate the author for his worthy contribution to the open literature.

Additional References


Author’s Closure

The author wishes to thank Mr. Benedict for his bringing up key questions and to thank Mr. Halmi for his much appreciated comments.

The first question of Mr. Benedict is linked to definition of Reynolds number which refers to inlet condition in this paper. Namely, the equation (10) is derived by substituting the relation of

$$R_e = \frac{\bar{u} d}{v} = \frac{\bar{u} D}{v} = \frac{\bar{u}_t}{\bar{v}} D = \beta$$

into the original equation.

The author agrees with Mr. Benedict that the tap size corrections are not negligible especially in the case of high Reynolds number if $C_D$ is calculated by a theory with zero tap size. However, one should remember that the present method is based on a combination of a potential flow consideration and an empirical viscous flow consideration. As the empirical relation was obtained in the flowmeters with finite tap size, the tap size corrections are included to some extent in this method. Therefore the corrections were not applied.

The purpose of the paper is to consider the effect of nonuniform static pressure at the throat in the prediction of $C_D$, but not criticize the universal value of the UVT proposed by Mr. Halmi. The $\beta$ insensitivity of the UVT has been verified in this paper (see the description iii in relation to Fig. 5). The $R_e$ sensitivity may be caused by using the generalized empirical equation (10), in spite of the existence of the artificial vena contractae in the UVT. However, it should be noted that the differences between the predicted and the universal values are small in comparison with the uncertainty band. The availability of the universal value of the UVT is accepted.

The uncertainties can be estimated from the law of propagation of errors, equation (9) and equation (7).

$$\epsilon_D = \sqrt{\left( \frac{\partial C_D}{\partial C_I} \right)^2 \epsilon_I^2 + \left( \frac{\partial C_D}{\partial C_V} \right)^2 \epsilon_V^2}$$

$$= \sqrt{\left( \frac{C_D}{C_I} \right)^2 \epsilon_I^2 + \left( \frac{C_D}{C_V} \right)^2 \epsilon_V^2}$$

where

$$\epsilon_D: \text{uncertainty of } C_D$$

$$\epsilon_I: \text{uncertainty of } C_I$$

$$\epsilon_V: \text{uncertainty of } C_V$$

$$\epsilon_I = \left| \frac{\partial C_I}{\partial \gamma_I} \right| \epsilon_I = C_I^2 \gamma_I \beta^2 (1 - \beta^2) \epsilon_I = \beta^2 (1 - \beta^2) \epsilon_I$$

$$\epsilon_V: \text{uncertainty of } \gamma_I$$

In this method $\epsilon_I$ becomes 0.00006 – 0.0008 for $\beta = 0.25 – 0.75$ providing that $\epsilon_I = 0.1$ percent (see Appendix). Therefore, the $\epsilon_D$ is mainly dependent upon $\epsilon_V$ which is obtained experimentally.

As to the other comments of Mr. Halmi, the author would like to emphasize again the basis of present method. One may use a more refined empirical or theoretical equation related to the energy loss as a viscous consideration.

For the last points for clarifications, the author’s answers are as follows:

- it is not necessary to calculate $\bar{p}$,
- the term $p = \bar{p}_I$ means the uniform inlet pressure,
- the meaning of $u^*$ should refer to reference [5],

and

- the terms laminar and turbulent regions are on the basis of the equation (10).

(• ‘50–diverging recovery cone angle’ was a mistyping.)