still valid although the values of $\alpha_j$ and $\beta_j$ may be different from those of the potential flow. (3) $\gamma_j$ presented in this paper is based on the potential flow theory. As long as the ratio of the tube displacement to tube diameter is small, the potential flow theory will give results with sufficient accuracy. When the tube displacement is large, other nonlinear effects become important and should be considered.

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References


DISCUSSION

R. D. Blevins

Dr. Chen is to be congratulated for developing a numerical potential flow solution for a geometry as complicated as a tube bank. However, I do feel that the title of his paper is somewhat misleading. Since the solutions are limited to an inviscid stationary fluid, they do not directly apply to operating heat exchanger tube banks; they apply in a limited sense only to shut down heat exchanger tube banks. Since viscosity of the fluid is neglected, the fluid damping cannot be computed and the solution is limited to added mass and inertial coupling effects. While these effects can be significant if the fluid density is the same order as the tube density (if gases are used as the coolant, these effects are usually negligible), they are generally only of practical interest in moving fluids and this case is unsolved. If fluid motion is introduced, then turbulence and flow separation become important parts of the fluid dynamics (reference [5]). These effects cannot be incorporated in an inviscid model. Indeed, attempts to apply potential flow theory to determine the forces on bluff bodies in moving fluids have not generally met with success. For example, the use of the inviscid model to estimate drag in a tube array (reference [6]) produces results considerably different from the measured drag (reference [7]). Thus I believe the inviscid solutions for tube arrays will remain primarily of theoretical rather than practical interest, except perhaps for added mass and inertial coupling effects.

Additional References


M. P. Paidoussis

This discusser should like to congratulate Dr. Chen on another important contribution to this field. The discusser has recently been conducting similar theoretical work, the results of which generally agree with those of this paper [8, 9]. In that work the cluster of cylinders is surrounded by a rigid cylindrical container, the presence of which modifies the modal shapes (cf. Fig. 6) destroying the symmetry of certain groups of modes in some cases (depending on the geometry of the cluster); also axial flow is taken into account [9]. The results shown in Fig. 8 are particularly interesting, agreeing with observed behavior. For instance, they can explain some early observations by Quinn [10], where vibration of adjacent cylinders were found to "transfer" from one cylinder to another. The occurrence of frequency bands of first modes, second modes, etc. is also of great interest, as it throws a completely different light to the response of such systems to either deterministic or stochastic excitation. If one neglects fluid coupling, one then finds that the system absorbs energy in very narrow frequency bands, as one would then have a single first-mode eigenfrequency; however, coupling spreads the range of this frequency band, as there are found to be $2k$ first-mode frequencies for a system of $k$ cylinders. Flow further enhances this spread of the frequency band. If one also takes into account structural coupling through supports, etc., it is seen that the system behaves like anything but a narrow band filter, as was hitherto supposed.

Additional References


D. J. Gorman

This paper by Dr. Chen constitutes a significant step in the quest for an understanding of the dynamic behavior of tube banks in liquid cross flow. In effect it demonstrates mathematically that a band of frequencies and modal shapes are to be anticipated around each of the discrete frequencies which were hitherto considered to characterize the free vibration of tubes in uniform banks. A mathematical technique is provided for predicting tube response to prescribed deterministic or random driving forces.

The paper is likely to be of more interest to fundamental researchers than heat exchanger designers. The mathematical procedures are somewhat involved and require meticulous care and knowledge of matrix operations. The designer, for example, will be interested in the intensity of vibration but may not be concerned with whether response is at a discrete frequency or within a narrow band of frequencies.

The paper opens up ground for much further experiment to corroborate the theory. It is to be anticipated that there will be some scatter in tube frequencies in a real tube bank even without hydraulic coupling. It is not clear whether the range of scatter would equal or exceed the anticipated range due to hydraulic coupling.

As with all forced vibration problems it is necessary to be able...
to predict the forcing function if the response is to be predicted. It is not quite clear to this writer how this function is to be obtained. Does equation (20) presuppose some initial disturbance? Would it be possible to work with the old discrete frequencies and use a correction factor which would be a function of the tube bank geometry?

The work undertaken by Dr. Chen represents a complicating but significant advance. It will be necessary to pursue it further, particularly with careful experiment, if an understanding of this perplexing problem is to be achieved.

It would be interesting if Dr. Chen would discuss the ability of his theoretical model to predict the onset velocities for hydroelastic instabilities in tube vibration. Such velocities are now known to constitute the design limits, rather than steady state response, for many tube banks of practical interest.

Author's Closure

R. D. BLEVINS

The author appreciates Dr. Blevins' interest and comments.

As we know, flow-induced vibration of heat exchanger tube banks is very complicated; it involves formation of vortex systems, turbulence within tube banks, unsteady fluid forces acting on the tubes, and interaction of tube motion and flow field. Ideally, one would like to solve the problem using the Navier-Stokes equations or Reynolds equations. Unfortunately, even for a single elastic tube subjected to a uniform cross flow, no theoretical solution is available in the “lock-in” regions. Although several mathematical models have been published (see reference [11] for a brief review), experimental data are needed to establish the parameters in the models. Then, it is obvious that using a simplified theory in conjunction with experimental data to quantify the important parameters is the logical approach. This is precisely the approach taken by the author trying to solve this complicated problem.

As pointed out in the paper, in addition to the fluid inertia force, there are other fluid force components, such as fluidelastic forces and fluid damping force. In operating heat exchanger tube bank, in which the fluid is flowing, other force components have to be properly accounted for. This aspect of the problem has been published in a recent paper [12].

In the past, the fluid inertia coupling has not been included in the analysis of tube banks and the vibration of a tube bank has been treated on a single frequency basis. However, for a closely-packed tube bank vibrating in a dense fluid, there are many natural frequencies within each frequency band. Consequently, using a single tube as a model for tube banks may result in error. The procedure developed by the author can be used to account for this inertia coupling easily. This procedure has been followed by others [13]. The author feels that including the fluid inertia coupling is a significant improvement. The multiple-tube model enables us to explain some of the phenomena which cannot be explained using a single-tube model.

The potential flow theory has its limitations. For example, consider two parallel circular cylinders normal to a flow. The results from the potential flow theory show that the cylinders attract each other, while the experimental data show the presence of a mean repulsive force between the cylinders [14].

The potential flow theory is probably less useful in calculating the steady fluid force and fluidelastic force except in the cases that flow separation does not play an important role. As far as the fluid inertia force is concerned, the experimental data and theoretical results are in good agreement [15]. As long as the tube vibration amplitude is small compared with tube diameter, the potential flow theory will yield results with sufficient accuracy. Fortunately, in many vibration problems, one is more interested in the inception of large unstable motions and small steady-state oscillations. Therefore, in most cases, the potential theory is adequate. The results presented in the paper are applicable to not only heat exchanger tube banks, but also other practical system components involving multiple tubes, such as nuclear fuel bundles. The author is inclined not to agree with Dr. Blevins' suggestion and considers that the solution is not only of theoretical interest, but also of practical significance.

Additional References


M. P. PAIDOUSSIS

The author would like to thank Professor M. P. Paidoussis for his kind and thoughtful observations and also for shedding additional light on this work.

As pointed out by Professor Paidoussis, the dynamic characteristics of a group of tubes surrounded by a rigid container are modified by the presence of the container. A systematic experimental investigation has been performed at Argonne National Laboratory to study the effects of various parameters, such as circular container, flat wall, fluid viscosity, and water depth [15]. The experimental data and theoretical results are found to be in good agreement. The effect of axial flow on the vibration of tube banks has also been analyzed [16], and it is found that fluid coupling tends to destabilize the system.

The concept of frequency band in coupled vibration of tube bank is very important. This concept can be applied to both single-span and multi-span tubes. In the paper, only single-span tubes are considered; however, the method can be applied to multi-span tubes. This can be seen from equation (6). If $\Phi_0(z)$ is taken to be the $n$th orthonormal function of the multi-span tubes the result obtained for multi-span tubes will be similar to that for single-span tubes. More specifically, if one follows through the same type of analysis for periodically supported tube banks, it can be shown that the natural frequencies of coupled modes can be calculated as follows:

1) Natural Frequencies of a Periodically Supported Tube in Vacuo—Various methods are available for calculating the natural frequencies of continuous tubes. One of the methods is based on the concept of frequency band [17]. Let those frequencies be denoted by $\Omega_i$, where $i = 1, 2, 3, \ldots \infty$ and $j = 1, 2, 3, \ldots N$. $N$ is the number of spans. The detailed method of analysis for calculating $\Omega_i$ is presented in reference [17].

2) Eigenvalues of Added Mass Matrix—Since the added mass matrix is symmetric for a group of $M$ tubes, there are $2M$ eigenvalues, which are denoted by $\mu_k, k = 1, 2, 3, \ldots, 2M$.

3) Natural Frequencies of Periodically Supported Tube Banks—The natural frequencies of continuous tube banks are

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Numbers 11-15 in brackets designate Additional References at end of closure.
then given by \( \omega_{i,j,k} \);

\[
\omega_{i,j,k} = \frac{\Omega_{ij}}{\left(1 + \frac{\mu_k}{m}\right)^{1/2}}
\]

\(i = 1, 2, 3, \ldots \)

\(j = 1, 2, 3, \ldots N\)

\(k = 1, 2, 3, \ldots 2M\)

where \(m\) is tube mass per unit length.

It is seen from equation (1) that corresponding to a single frequency for an isolated tube of single span, there are \(2MN\) natural frequencies of coupled modes for a group of \(M\) tubes with \(N\) spans vibrating in a liquid. Then, it is not difficult to imagine that a multi-span tube bank will respond like anything but a narrow band filter.

Additional References


D. J. GORMAN

The author appreciates Professor Gorman’s comments and close reading of the paper.

The author agrees with Professor Gorman that it is necessary to pursue the subject matter further, particularly with systematic experiments. There are many questions to be answered and many parameters to be quantified before a thorough understanding of the complex phenomena can be achieved. An experimental/analytical study is being conducted to achieve this goal.

Other points raised by Professor Gorman are discussed as follows:

1. The bandwidth of each frequency band for a tube bank consisting of identical tubes can be calculated by equation (17).

The bandwidth depends on the tube properties, tube spacing, and fluid density. For a tube bank in which the tubes are not in tune, there will be some scatter in tube frequencies. The fluid coupling will enlarge the scatter. Therefore the frequency bandwidth including fluid coupling will be always larger than that without fluid coupling.

2. Fluid forces acting on the tubes can be classified as two groups: (1) motion-dependent forces—forces induced by tube motion; and (2) fluid excitation forces—forces independent of tube motion. Equation (20) represents the motion-dependent forces including fluid inertia force, fluid damping force and fluidelastic force; those terms are obtained based on the potential flow theory [18].

3. It would be nice if we could always work with the discrete frequencies associated with the single tube. But this may not be possible for closely packed tube banks even using a correction factor. As a matter of fact, it has been pointed out in the response to Professor Paidoussis’ comments that closely-packed tubes will respond like anything but a narrow band filter while the single tube responds as a narrow band filter.

4. What a heat exchanger designer needs is a simple formula that can be used to predict whether vibration is a problem. It would be very useful if such a simple formula could be obtained for heat exchanger tube banks. Although several methods of analysis based on different excitation mechanisms are available, no single formula is satisfactory in predicting all tube vibration problems. This is because some of the phenomena are not well understood and some of the important parameters are yet to be quantified.

5. The application of the theory to cross flow was published recently [12]. The model can be used to predict flutter flow velocity and instability mode once the motion-dependent force is known.

Additional Reference


*Number 18 in brackets designates Additional Reference at end of closure.