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ON THE POSSIBLE USE OF FRACTAL THEORY IN RAINFALL APPLICATIONS

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ABSTRACT

The rainfall data used in hydrological calculations are seldom sufficient to correctly represent the extreme variability of rainfall in time and space. Regarding for example traditional sewer design, the errors may be negligible, but the successively more complex hydrological models used today are much more sensitive to inadequate input data. The problem can be solved either by extensive data collection or by developing methods for extracting more information from limited data. An approach based on fractal theory has been shown to provide a suitable framework for this purpose. In this paper we apply two fractal-related analyzing methods to a two-year series of one-minute rainfall observations. The results indicate that the series exhibit statistical properties that are independent of scale, i.e., scale invariant. A possibility emerging from these findings is to make a shift from the scale defined by the available data to the scale needed for the hydrological problem in question, a prospect of tremendous practical importance.

KEYWORDS

Rainfall time series; scale invariance; fractal analysis; simple scaling; multiscaling.

INTRODUCTION

The spatial variability of precipitation, which is substantial even on very small catchments, can never be completely revealed by a network of gauges. Equally, the time variability and extremes, sampled in a set of data with limited duration, are seldom adequately represented. Thus, practitioners usually deal with rainfall data which are inadequate for the problem in question. This is most obvious in urban hydrology - hydrology of small time and space scales. Another aspect of the problem is the changing needs of the society, and the resulting altered objectives of hydrological calculations. When the main objective was simple design of conduits in towns, or dams and irrigation systems, simple approximations such as the "design storm" concept or probability distributions were sufficient. The recent shift of our focus to sustainability and environmental protection requires more complex systems to be analyzed. Modelling of processes such as pollution transport, rainfall-related pollution effects on treatment plants and runoff-induced wash-off from impermeable surfaces requires different approaches to be made and different rainfall data to be collected compared with traditional sewer design. Uncertainties in rainfall measurements should be evaluated in terms of acceptable errors for the user. This requires either dense networks of gauges, or the use of radar and satellites. However, dense networks are for practical reasons seldom available, and the reliability of radar and satellite measurements is still low (Einfalt and Deneoux, 1987). Therefore, development of a method enabling transformation of data from one time and space scale to another would lead to substantial improvements in many hydrological fields of application. Fractal theory and different related methods of data processing seem to provide a suitable framework for improving our understanding of the rainfall process and, thus, makes it possible to overcome the main problem of inadequacy of the available data. However, there is a need to bridge the gap between findings from on one hand theoretical studies based on fractal properties of the rainfall process, and the rainfall input used in practical applications on the other.

WHAT CAN BE LEARNED FROM FRACTAL CALCULATIONS

During recent years, a lot of research has been conducted aiming at testing the applicability of fractal theory to different natural phenomena. Regarding the temporal and spatial properties of convective mesoscale rainfall, the most exciting theory for practical hydrologists assumes that the convective rainfall fields possess no characteristic spatial scale, i.e., the rainfall process is scale invariant or scaling (e.g., Lovejoy, 1982; Lovejoy and Mandelbrot, 1985). According to this theory, the statistical variability of the rainfall process in time and space might be characterized by some parameters or functions which are valid over all scales. The existence of scale invariance would theoretically imply a possibility to make a shift from the scale defined by the data collection system (i.e., the specific time and space resolution) to any scale needed for the hydrological problem in question. There is no need to say how important such a possibility would be for practical applications of rainfall data in hydrological calculations (Olsson et al., 1992).

So far, most research in this field has been devoted to the development and successive refinement of a theoretical framework incorporating scale-invariant properties of the rainfall process. This work has spawned various techniques for analyzing if rainfall observations are characterized by a scale-invariant behaviour. However, the validity of these highly theoretical considerations has, as yet, only been tested on a small number of observed rainfall data sets and thus the theory has not had much impact on practical applications of rainfall data. There are only a few examples of research aimed at seeking physical and practically applicable interpretations of numerical and graphical outputs from fractal-related analyses.

One example of these practically oriented studies is a study conducted at the University of Lund, Sweden (Olsson et al., 1990, 1992). The main objective of this work is to try to find practical implications emerging from applications of established fractal-related analyzing methods to several sets of high-quality rainfall time series of various time resolution. Some results are interesting and encouraging. For example, the application of a simple box-counting procedure (functional box-counting, e.g., Lovejoy et al., 1987) to a one-minute rainfall time series produces results indicating scale invariance.

The time series is divided into non-overlapping time segments (boxes) of size L and N(L) is the number of boxes where the rainfall intensity at some point exceeds a specified threshold value. If the time series exhibits scale invariance, N(L) and L are related through the expression:

$$N(L) \propto L^{-D_B} \tag{1}$$

where D_B is the box dimension, an exponent originally believed to be a fundamental constant that constituted a direct link between statistical properties at all scales, thus completely characterizing the variability of the process (simple scaling). The validity of the expression can be evaluated by plotting N(L) as a function of L, where L varies from the resolution up to the total length of the series, in a double-logarithmic diagram. If this graph exhibits a straight-lined behaviour, scale invariance (characterized by D_B) is indicated.

Fig. 1 shows the result of applying the above described procedure to a one-minute rainfall time series of two years length using the threshold values 0, 0.1, 0.2, and 0.3 mm/min. The presence of straight lines on these graphs confirms the possibility of scale invariance for the analyzed series. However, since the curves do not display a unique straight-lined behaviour (i.e., independent of scale and intensity), the series cannot be characterized by one single dimension. Firstly, several straight-lined sections must be used to describe a curve obtained from a certain threshold (see Fig. 1, the sections are numbered 1-3). Secondly, curves obtained from different thresholds have different slopes. These findings suggest that the scale invariance, expressed in terms of single dimensions, has a limited range of validity.

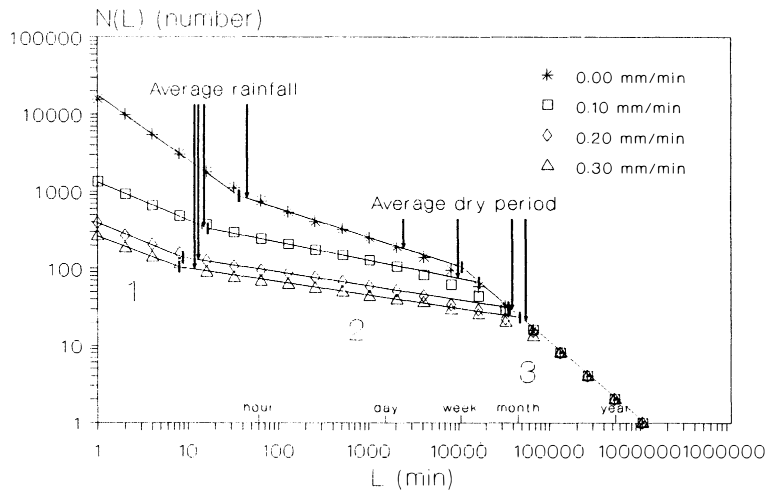

Fig. 1. The resulting graph from applying box-counting to a two-year series of one-minute rainfall intensity observations for the intensity thresholds 0, 0.1, 0.2, and 0.3 mm/min. The average values of rainfall event duration and dry period length for each threshold, obtained from frequency analysis, are marked with arrows (after Olsson *et al.*, 1992).

Another interesting finding, although difficult to translate to physical terms, is that the bending points on the graphs (between two straight-lined sections) are clearly related to characteristic durations of the rainfall process that can be obtained from traditional statistics of duration frequencies (see Fig. 1 where the average values of rainfall event duration and dry period length are marked with arrows).

To overcome the limitations associated with simple scaling, a new concept called multiscaling was introduced where the scale-invariant properties can be characterized by a dimension function related to the intensity level, rather than a single dimension (Schertzer and Lovejoy, 1987; Lovejoy and Schertzer, 1990; Gupta and Waymire, 1990; Tessier et al., 1993). An advantage of this recent development is that the multiscaling properties of the rainfall fields are related to a theory concerning the underlying physics. This theory says that the rainfall producing mechanism can be described as a so-called cascade process where fluxes of water are concentrated into successively smaller units in the atmosphere. The fact that this approach in a natural way incorporates typical features of rainfall such as the mesoscale "cell-structure" and the extreme small-scale variability, makes it intuitively attractive.

The application of a newer technique called Probability Distribution/Multiple Scaling, PDMS (Lovejoy and Schertzer, 1990), developed within the framework of multiscaling, to the one-minute rainfall time series also brings interesting results. Without going into details of the theory, the principal procedure of PDMS is, similarly to box-counting, to divide the series into boxes of sizes ranging from the resolution up to the length of the series. The total number of boxes for each size is denoted N. Then the average rainfall intensity for each box is calculated, after which the probability Pr that the averages exceed a certain threshold level is estimated. The threshold is specified as the number of boxes, N, raised to the exponent β. This means that the threshold, contrary to box-counting, changes with the box-size. If the series is characterized by multiscaling, the relationship between Pr and N is:

$$\Pr \propto N^{-C(\beta)} \tag{2}$$

where $C(\beta)$ is called the codimension. Fig. 2 shows the resulting graph from applying this procedure to the one-minute rainfall time series. The straight-lined behaviour over a fairly large range of scales indicates that multiscaling is respected.

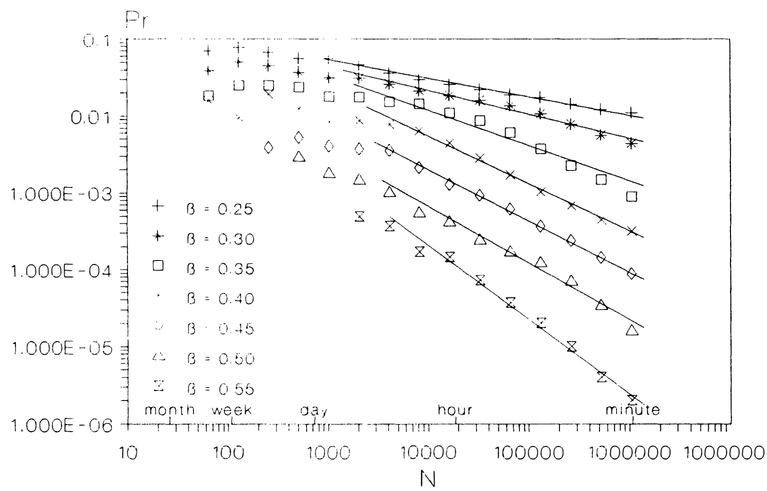

Fig. 2. The resulting graph from applying PDMS to a two-year series of one-minute rainfall intensity observations (after Olsson, 1992).

The main difference between PDMS and box-counting is that the aim of PDMS is not to calculate a single dimension, but the parameters of a scale-invariant codimension function $C(\beta)$ which is through β related to the rainfall intensity. The relationship between $C(\beta)$ and β is expressed by:

$$C(\beta) = C_1\left(\frac{\beta}{C_1\alpha'} + \frac{1}{\alpha}\right)^{\alpha'} \tag{3}$$

where C_1 is an "average codimension" and α ($0 \leq \alpha \leq 2$) is the Lévy index ($1/\alpha + 1/\alpha' = 1$). The result from fitting this theoretical relationship to the codimensions obtained from the time series (i.e., the slopes of the straight-lined parts of the curves in Fig. 2) is shown in Fig. 3 (parameter values: C_1=0.15, α=1.8, α'=2.25). The fitted line does not exactly coincide with the codimensions, but is very close to it. This raises new questions on how to explain these departures.

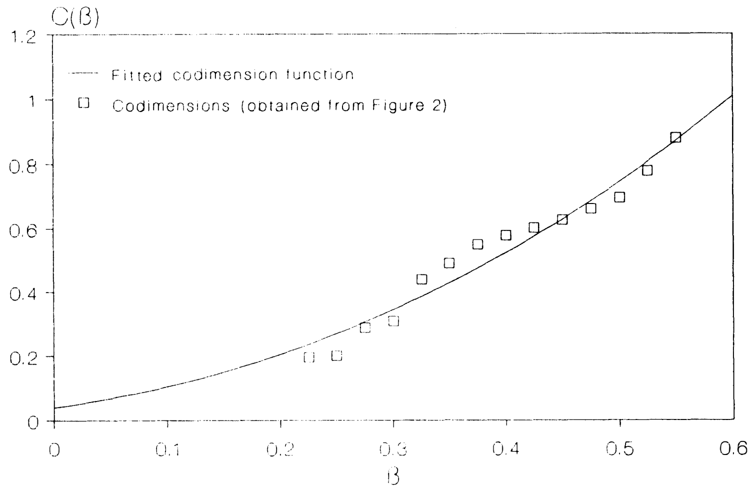

Fig. 3. Codimensions obtained as the slopes in Fig. 2, and the fitted codimension function (after Olsson, 1992).

SUMMARY AND CONCLUSIONS

The changing goals of society force us to consider smaller and smaller time and space scales of hydrological processes. The recent modification of the traditional urban drainage philosophy requires that the rainfall input used in practical applications must be brought closer to the physical reality of the rainfall process. The result of any hydrological calculation is highly dependent on the rainfall input used.

In most hydrological calculations, we are dealing with inadequate and/or insufficient rainfall data. Rainfall variability in time and space is substantial even for very small catchments. Rigorous requirements regarding the time and space resolution of gauge networks for rainfall measurements can never be satisfied for practical reasons. The reliability of radar and satellite measurements is still low.

Development of a method enabling transformation of data from one time and space scale to another would bring substantial improvement in many hydrological applications.

Research based on fractal theory may have a great potential to improve our understanding of the rainfall process and, thus, overcome the main problem of inadequacy of available data, but there is a need to bridge the gap between findings from theoretical studies of the rainfall process and the rainfall input used in practical applications.

In this paper we have shown that by direct application of fractal-related methods of analysis to rainfall time series, some interesting properties of the rainfall process are revealed. It may be concluded that these new findings bring some hope to the future possibilities of developing methods, based on fractal theory, for extracting sufficient information from limited data to perform scale shifts. However, simultaneously several new questions arise, and it seems that there is still some way to go before any powerful practical application is possible.

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