and the exponential term becomes

$$\exp(AP_{i,m}) = \frac{\exp\left(\frac{1}{2} A\Delta X Q_{i,m}\right)}{\prod_{n=1}^{N-1} \exp(A\Delta X Q_{n,m})} \tag{61}$$

The governing equations (54) and (55) now become

$$H_{i,m} = \frac{1}{2}(X_i^2 + \Theta_m^2) - 2\Phi + \frac{2\Delta X}{\pi} \sum_{j=1}^{N} L_j'(X_i, \Xi) Q_{j,m} \tag{62}$$

$$H_{i,m} = \frac{\exp\left(\frac{A\Delta X Q_{i,m}}{6}\right)}{\prod_{n=1}^{N-1} \exp\left(\frac{A\Delta X Q_{n,m}}{3}\right)} \times \left\{\frac{1}{Q_{i,m}}\left[-U\Theta_m X_i + \frac{1}{2}\Delta X \lambda_m(X_i) + \frac{1}{\pi}\omega_m(\Delta X)^2 \sum_{j=1}^{N} L_j''(X_i, \Xi) Q_{j,m}\right]\right\}^{1/3} \tag{63}$$

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References

1 Cheng, H. S., and Sternlicht, B., "A Numerical Solution for the Pressure, Temperature, and Film Thickness Between Two Infinitely Long, Lubricated Rolling and Sliding Cylinders, Under Heavy Loads," *Journal of Basic Engineering*, Trans. ASME, Series D, Vol. 87, 1965, pp. 695–707.

2 Dowson, D., and Whitaker, A. V., "A Numerical Procedure for the Solution of the Elastohydrodynamic Problem of Rolling and Sliding Contacts Lubricated by a Newtonian Fluid," *Proceedings of the Institution of Mechanical Engineers*, Vol. 180, Part 3, Series B, 1965, pp. 57–71.

3 Cheng, H. S., "A Refined Solution to the Thermal-Elastohydrodynamic Lubrication of Rolling and Sliding Cylinders," *Transactions of the American Society of Lubrication Engineers*, Vol. 8, 1965, pp. 397–410.

4 Cheng, H. S., and Orcutt, F. K., "A Correlation Between the Theoretical and Experimental Results on the Elastohydrodynamic Lubrication of Rolling and Sliding Contacts," *Proceedings of the Institution of Mechanical Engineers*, Vol. 180, Part 3, Series B, 1965, pp. 158–168.

5 Bell, J. C., "Lubrication of Rolling Surfaces by a Ree-Eyring Fluid," *Transactions of the American Society of Lubrication Engineers*, Vol. 5, 1962, pp. 160–171.

6 Loo, T-T., "Effect of Curvature on the Hertz Theory for Two Circular Cylinders in Contact," *Journal of Applied Mechanics*, Vol. 25, Trans. ASME, Vol. 80, 1958, pp. 122–124.

7 Wernick, R. J., "Some Computer Results in the Direct Iteration Solution of the Elastohydrodynamic Equations," Mechanical Technology Incorporated Report Number 62TR38, Feb. 1963.

8 Greenwood, J. A., and Williamson, J. B. P., "Contact of Nominally Flat Surfaces," *Proceedings of the Royal Society*, London, Vol. 295, Series A, 1966, pp. 300–319.

9 Christensen, H., "The Oil Film in a Closing Gap," *Proceedings of the Royal Society*, London, Vol. 266, Series A, 1961, pp. 312–328.

10 "Research in Surface Forces—Vol. I," authorized translation from the Russian, Deryagin, B. V., Consultants Bureau, New York, 1963, pp. 110–115.

11 Fein, R. S., and Kreuz, K. L., "Chemistry of Boundary Lubrication of Steel by Hydrocarbons," *Transactions of the American Society of Lubrication Engineers*, Vol. 8, 1965, pp. 29–38.

DISCUSSION

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The author has made a valuable contribution to the elastohydrodynamic aspect of the little explored problem of asperity collision in thin film contacts.

The material parameter, $E'a$ (or A in the paper), assumes for steel materials lubricated with mineral oil, the value 5000. In the paper, the range of A is from 0 to only 1560. It is suspected that, if a realistic value of E' and of the pressure viscosity coefficient (a) is used, the analysis may yield a film pressure orders of magnitude higher than the Hertzian pressure at "microcontacts."

This analysis covers the case of sliding and not rolling. Since sliding (or viscous shear) in a very thin film causes heating, it will reduce the effective viscosity and one predicts that a thermal analysis will probably yield considerably lower contact pressure and thinner films than obtained in the present analysis. Also, the results are not applicable to rolling contacts, the most important EHD contacts in practice.

The discusser believes that the treatment would be more realistic if the analysis could consider a lubricant pressure other than zero before the asperities approach. In the present analysis the build-up of film pressure is entirely attributed to the approaching of the asperities. This disregards the fact that there exists a large EHD pressure in the entire contact due to the "sweep-in" motion at the inlet of a contact.

The most interesting result of the analysis is the showing that an EHD film can exist in an asperity contact. For a given degree of asperity "overlapping," the thickness of the film or the contact pressure increases with the sliding velocity, lubricant viscosity and the material parameter $E'a$. The contact pressure approaches the Hertzian case only if both the parameters U and A approach zero. In reality, the severity of the pressure build-up at an asperity in contact is reduced by the fact that (1) the asperities have curvature on the side so the "side leakage" can be significant and (2) plastic deformation will act to reduce the amount of overlap and to increase the asperity tip radius.

Little information is given regarding the possible cavitation between the two departing asperities after collision as well as the friction force when the center line distance is zero. Without considering the thermal effect, it is believed that the calculated tractive force will be exceedingly high.

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Author's Closure

The author thanks Dr. Chiu for his discussion. The highest value of A considered in the paper is indeed lower than is found in most practical systems. The choice not to consider more realistic values of A was made with the idea of compensating to some extent for the assumption that the system is isothermal. Preliminary results of a thermal analysis based on a real lubricant described in detail by Dowson and Whittaker (paper reference [2]) indicate that the use of the value 1560 for A actually overcompensates for the effect of temperature; the thermal analysis predicts considerably higher pressures and thicker films than does the present analysis.

Dr. Chiu believes that the treatment would be more realistic if the analysis could consider a lubricant pressure other than zero before the asperities approach. The analysis as presented can, in fact, do just this. The only parameter in the analysis which is dependent on the ambient pressure is the lubricant viscosity. Since the viscosity is assumed to vary exponentially with pressure, any change in the ambient pressure requires only that the ambient viscosity, η_0, be changed. This change affects only the conversion of the dimensionless sliding speed to its dimensional value. Thus, if it is remembered that the pressure predicted by this analysis is the pressure rise above ambient, then the analysis can be applied to systems with any ambient pressure provided that the correct value for the ambient viscosity is used to calculate the dimensionless sliding speed.

It was the problem of cavitation during the second half of the collision which restricted the analysis to positive values of Θ.

The author has since modified the analysis to include cavitation so that the entire collision process may be treated. Details of this modification with selected numerical results will shortly be submitted for publication in this journal as a Technical Brief.

The author agrees with Dr. Chiu that the severity of the pressure buildup during an asperity collision will be reduced by the fact that real asperities have curvature in three dimensions, so that significant side leakage can exist. However, the analysis of a three dimensional asperity collision would be considerably more difficult than the analysis of a macroscopic point contact EHD system, a feat which to the author's knowledge has yet to be fully accomplished.

Plastic deformation of the asperities during a collision will act to reduce the pressure buildup, but it will also act to increase the lubricant film thickness and must therefore be considered a beneficial effect. Whether plastic deformation will occur in a given collision is, however, a difficult question to answer. Certainly the pressures predicted in this paper do approach the yield stress of some bearing materials if they have not been work hardened. One might therefore assume that an asperity not previously involved in a collision might plastically deform, but this deformation would cause the asperity to work harden so that the likelihood of further plastic deformation in subsequent collisions will be reduced.