American Journal of Physics

NOTES AND DISCUSSIONS | MAY 01 2015

Erratum: “The restoring force on a dielectric in a parallel plate capacitor” [Am. J. Phys. 82, 853–859 (2014)] FREE

L. P. Staunton

Am. J. Phys. 83, 475 (2015)
https://doi.org/10.1119/1.4914125

NOTES AND DISCUSSIONS

Erratum: "The restoring force on a dielectric in a parallel plate capacitor" [Am. J. Phys. 82, 853–859 (2014)]

L. P. Staunton
Department of Physics and Astronomy, Drake University, Des Moines, Iowa 50311

(Received 19 February 2015; accepted 23 December 2015)
[http://dx.doi.org/10.1119/1.4914125]

In a recent paper that appeared in this journal,[1] there is a serious error. The statement is made that since the voltage is maintained by the battery, the electric field is constant everywhere. Rather, it is the path integral of the electric field between the plates via any path that is constant. The two statements are equivalent in the usual approximation in which the fringing of the electric field is neglected. However, when the fringing field is explicitly included, as was the case in Ref. 1, then there are discontinuities in the electric field on all surfaces of the dielectric. Because the calculation specifically uses the electric field present before the dielectric is inserted, the calculation is not correct. At best it is a more accurate approximation than the one in which the fringing field is neglected.

[1]L. P. Staunton, "The restoring force on a dielectric in a parallel plate capacitor," Am. J. Phys. **82**, 853–859 (2014).

Comment on "Point charge in a three-dielectric medium with planar interfaces" [Am. J. Phys. 46, 1172–1179 (1978)]

Federico M. Pont[a)] and Pablo Serra[b)]
Facultad de Matemática, Astronomía y Física and IFEG-CONICET, Universidad Nacional de Córdoba, Ciudad Universitaria, X5000HUA Córdoba, Argentina

(Received 22 September 2014; accepted 17 January 2015)
[http://dx.doi.org/10.1119/1.4907259]

The solution for the electrostatic potential of a point charge immersed in a three-dielectric medium (ε_1, ε_2, ε_3) with planar interfaces was given in an article published in this journal.[1] The work shows how the use of the method of images leads to a closed form solution for the two-dimensional Fourier transform of the electrostatic potential and the induced surface charge on the interfaces. Here, we would like to emend the main results of the article, Tables I and II, because there are several errors that make their use problematic. We also comment on the physical units used in the article, since no discussion was presented in the work. The errors in the expressions of Tables I and II are a result of both typographical mistakes and some derivation flaws as we will show below.

The article considers a dielectric slab of permittivity ε_2 between two infinite dielectrics of permittivity ε_1 and ε_3. Figure 1 of Ref. 1 shows the geometry of the problem. The piecewise dielectric constant is given by

$$\varepsilon(z) = \begin{cases} \varepsilon_1 & z < -a \\ \varepsilon_2 & -a < z < a \\ \varepsilon_3 & a < z. \end{cases} \tag{1}$$

We will solve the problem in the same system of units used in Ref. 1, which, after our calculations, we confirmed to be the CGS system. The macroscopic equation for the electrostatic potential ϕ is derived from Gauss's law in dielectric matter[2]

$$\nabla \cdot [\varepsilon(\mathbf{r}) \nabla \phi(\mathbf{r})] = -4\pi \rho_f(\mathbf{r}), \tag{2}$$

where $\rho_f(\mathbf{r})$ represents the free charge density at position $\mathbf{r}$. In this problem, $\rho_f(\mathbf{r}) = Q\,\delta(\mathbf{r} - \mathbf{r}')$, where $\mathbf{r}'$ is the location of the point charge. We assume, as in Ref. 1, that the point charge has coordinates $\mathbf{r}' = (0, 0, z')$. In addition to satisfying Eq. (2), the electrostatic potential must also fulfill the following boundary conditions at the interfaces:[2]

$$\phi_1(x, y, -a) = \phi_2(x, y, -a), \tag{3}$$

$$\varepsilon_1 \frac{\partial \phi_1}{\partial z}\bigg|_{z=-a} = \varepsilon_2 \frac{\partial \phi_2}{\partial z}\bigg|_{z=-a}, \tag{4}$$

$$\phi_2(x, y, a) = \phi_3(x, y, a), \tag{5}$$

$$\varepsilon_2 \frac{\partial \phi_2}{\partial z}\bigg|_{z=a} = \varepsilon_3 \frac{\partial \phi_3}{\partial z}\bigg|_{z=a}, \tag{6}$$

where ϕ_i is the electrostatic potential in the region with dielectric constant ε_i.

Using Eq. (1) in Eq. (2), we obtain a set of three Poisson equations for each of the three possible locations of the point charge (case I: $|z'| < a$; case II: $z' > a$; case III: $z' < -a$). In each case, the potential $\phi(z)$ in dielectric 1, 2, and 3 must be

Table I. Two-dimensional Fourier transform $V(k,z,z')$ of the electrostatic potential at z generated by a point charge located at z'. Here, k is the Fourier vector modulus, $\alpha = 2\pi Q/k$, $L_{i2} = (\varepsilon_i - \varepsilon_2)/(\varepsilon_i + \varepsilon_2)$ with $i = 1, 3$, and $\Delta(k,a) = (1 - L_{12}L_{32}e^{-4}ka)^{-1}$.

Region	$V(k,z,z')$
Case I: $-a < z' < a$ $(j=2)$	
$z > a$	$V_{3,2}(z,z') = \frac{2\alpha}{\varepsilon_3+\varepsilon_2}\Delta(k,a)\left(e^{-k(z-z')} - L_{12}e^{-k(z+z'+2a)}\right)$
$-a < z < a$	$V_{2,2}(z,z') = \frac{\alpha}{\varepsilon_2}\left\{e^{-k\lvert z-z'\rvert} - \Delta(k,a)e^{-2ka}\left[L_{12}e^{-k(z+z')} + L_{32}e^{k(z+z')} - L_{12}L_{32}e^{-2ka}\left(e^{-k(z-z')} + e^{k(z-z')}\right)\right]\right\}$
$z < -a$	$V_{1,2}(z,z') = V_{3,2}(-z,-z')\rvert_{\varepsilon_3\leftrightarrow\varepsilon_1} = \frac{2\alpha}{\varepsilon_1+\varepsilon_2}\Delta(k,a)\left(e^{k(z-z')} - L_{32}e^{k(z+z'-2a)}\right)$
Case II: $z' > a$ $(j=3)$	
$z > a$	$V_{3,3}(z,z') = \frac{\alpha}{\varepsilon_3}\left(e^{-k\lvert z-z'\rvert} + L_{32}e^{-k(z+z'-2a)} - \frac{4\varepsilon_2\varepsilon_3}{(\varepsilon_3+\varepsilon_2)^2}L_{12}\Delta(k,a)e^{-k(z+z'+2a)}\right)$
$-a < z < a$	$V_{2,3}(z,z') = \frac{2\alpha}{\varepsilon_3+\varepsilon_2}\Delta(k,a)\left(e^{k(z-z')} - L_{12}e^{-k(z+z'+2a)}\right)$
$z < -a$	$V_{1,3}(z,z') = \frac{4\alpha\varepsilon_2}{(\varepsilon_1+\varepsilon_2)(\varepsilon_3+\varepsilon_2)}\Delta(k,a)e^{k(z-z')}$
Case III: $z' < -a$ $(j=1)$	
$z > a$	$V_{3,1}(z,z') = V_{1,3}(-z,-z') = \frac{4\alpha\varepsilon_2}{(\varepsilon_1+\varepsilon_2)(\varepsilon_3+\varepsilon_2)}\Delta(k,a)e^{-k(z-z')}$
$-a < z < a$	$V_{2,1}(z,z') = V_{2,3}(-z,-z')\rvert_{\varepsilon_3\leftrightarrow\varepsilon_1} = \frac{2\alpha}{\varepsilon_1+\varepsilon_2}\Delta(k,a)\left(e^{-k(z-z')} - L_{32}e^{k(z+z'-2a)}\right)$
$z < -a$	$V_{1,1}(z,z') = V_{3,3}(-z,-z')\rvert_{\varepsilon_3\leftrightarrow\varepsilon_1}$
	$= \frac{\alpha}{\varepsilon_1}\left(e^{-k\lvert z-z'\rvert} + L_{12}e^{k(z+z'+2a)} - \frac{4\varepsilon_2\varepsilon_1}{(\varepsilon_1+\varepsilon_2)^2}L_{32}\Delta(k,a)e^{k(z+z'-2a)}\right)$

computed independently, which renders a total of nine expressions to describe the electrostatic potential everywhere for all three cases. Table I of Ref. 1 gives the expressions for the Fourier transform of the potential in the xy-plane, which depends on the modulus of the two-dimensional Fourier vector $k = \lvert\mathbf{k}\rvert$. Regrettably, the expressions given in Ref. 1 do not fulfill the boundary conditions in Eqs. (3)–(6) in all cases and regions. Moreover, excessive use of the absolute value function in cases where the argument has a well defined sign is confusing.

Table II. Two-dimensional Fourier transform of the surface charge density $\Sigma(k,\pm a,z')$ induced at the interfaces $z = \pm a$ generated by a point charge located Q at z'.

$\Sigma(k,\pm a,z')$
Case I: $-a < z' < a$
$\Sigma(a,z') = \frac{Q}{\varepsilon_2}\left(-\Delta(k,a)L_{32}e^{-k(a-z')} + (\Delta(k,a)-1)\,e^{k(a-z')}\right)$
$\Sigma(-a,z') = \frac{Q}{\varepsilon_2}\left(-\Delta(k,a)L_{12}e^{-k(a+z')} + (\Delta(k,a)-1)\,e^{k(a+z')}\right)$
Case II: $z' > a$
$\Sigma(a,z') = -Q\left(\frac{1}{\varepsilon_3} - \frac{2\Delta(k,a)}{\varepsilon_3+\varepsilon_2}\right)e^{-k(z'-a)}$
$\Sigma(-a,z') = -\frac{2QL_{12}\Delta(k,a)}{\varepsilon_3+\varepsilon_2}\ e^{-k(z'+a)}$
Case III: $z' < -a$
$\Sigma(a,z') = -\frac{2QL_{32}\Delta(k,a)}{\varepsilon_1+\varepsilon_2}\ e^{k(z'-a)}$
$\Sigma(-a,z') = -Q\left(\frac{1}{\varepsilon_1} - \frac{2\Delta(k,a)}{\varepsilon_1+\varepsilon_2}\right)e^{k(z'+a)}$

We note, for example, that case I of Table I ($-a < z < a$), the only expression that is derived explicitly in Ref. 1, should be the same as Eq. (9a), which is obtained form Eq. (7), though this is not the case. It turns out that Eq. (9a) is incorrect: the last term $e^{-\lvert z-z'\rvert}$ should be $e^{-(z-z')}$. It appears that the authors used the signs of some quantities in Eq. (7) to obtain the result, but then reinstated the absolute value functions, giving three different expressions for the same potential [Eq. (7), Eq. (9a), and Table I]. We also note that in the expression for the potential in case II with $z < -a$, the cancellation of the first two exponential terms is hidden by the use of the absolute value functions. Moreover, there is also a factor of $2\pi\kappa$ missing in this expression.

In Table I, we show the correct formulae for the electrostatic potential (in CGS units) that solves Eq. (2) subject to boundary conditions (3)–(6). We introduce the notation $V_{i,j}(k,z,z')$ to identify the 9 expressions for the potential, where $i = 1$, 2, and 3 gives the dielectric region where the potential is given and $j = 1$, 2, and 3 specifies the region where the point charge is located. In Table II, we give expressions for the two-dimensional Fourier transform of the induced surface charge density $\Sigma(k, z = \pm a, z')$, as defined in Eq. (18) of Ref. 1.

F.M.P. and P.S. acknowledge financial support by Consejo Nacional de Investigaciones Científicas y Técnicas (CONICET).

[a]pont@famaf.unc.edu.ar
[b]serra@famaf.unc.edu.ar

[1]R. G. Barrera, O. Guzmán, and B. Balaguer, "Point charge in a three-dielectric medium with planar interfaces," Am. J. Phys. **46**(8), 1172–1179 (1978).

[2]L. D. Landau, L. P. Pitaevskii, and E. M. Lifshitz, *Electrodynamics of Continuous Media*, Course of Theoretical Physics, Vol. 8, 2nd ed. (Butterworth-Heinemann, Oxford, England, 1979).