tion between the elastic and plastic modes of deformation." In the tests reported by this writer,[3] which were of course much more highly constrained, the superposed elastic vibrational amplitudes were very small (peak amplitude less than the beam thickness). It would be of interest to know peak elastic amplitudes from the authors' photographs, if they were measured.

Authors' Closure

The authors wish to thank Dr. J. S. Humphreys for his valuable comments.

There is no doubt that the axial tension brought about by the unavoidable friction at the beam supports causes some of the difference between experimental and theoretical deflections. To obtain in the simplest manner some idea of the sensitivity of the predicted central deflection to an axial tension, the rigid-plastic beam problems of the paper can be analyzed as in Refs. [1, 2] of the paper, but with a constant axial tension N. Results appear below, for clamped beams.

The axial tension is assumed to be much smaller than N_0, the maximum tensile force that can be developed in the beam. This assumption is based on the lack of longitudinal strain in the mid-surface of a beam (Experiment No. CA2) subjected to a high impulse. It allows the same yield condition to be used, that is, $M/M_0 = 1$ instead of $(N/N_0)^2 + (M/M_0) = 1$, and it allows the resulting simple analysis to show whether a small N produces a significant change in the central deflection, $\delta_{th}^{(1)}$.

For the clamped beam, the analysis gives the central deflection in the form

$$\delta_{th}/L = (2M_0/NL)[\{1 + (\delta_{th}^{(1)}/L)(NL/M_0)\}^{1/2} - 1] \quad (1)$$

The nomenclature of the paper is used.

With N as some small fraction of N_0, say $N = N_0/n$, and $N_0 = \sigma_0 d$, $M_0 = \sigma_0 d^2/4$ (beam of rectangular cross section), a three-term binomial expansion in expression (1) yields the approximation

$$\delta_{th}/L \approx (\delta_{th}^{(1)}/L)[1 - (\delta_{th}^{(1)}/L)(L/nd)] \quad (2)$$

Expression (2) is reasonably accurate for $0 < (\delta_{th}^{(1)}/L)(L/nd) < 1/8$.

For $\delta_{th}^{(1)}/L = 1/2$, $L = 9$ in., and $d = 1/4$ in., $(\delta_{th}^{(1)}/L)(L\ nd) = 1/8$ when $n = 144$ and (2) becomes

$$\delta_{th}/L = (7/8)(\delta_{th}^{(1)}/L)$$

For Al 6061-T6 with $\sigma = 40{,}000$ lb/in.^{2}, $n = 144$ means that the tensile force in the beam is only 70 lb.

Although the foregoing mathematical model does not constitute proof that the difference between experimental and theoretical deflections is mainly due to axial tension effect, it does indicate a high sensitivity to this effect.

In answer to the second point, the peak elastic amplitudes were of the order of the beam thickness. As mentioned in the paper, the elastic mode interrupted the smooth progression of the plastic hinges after they had traveled about two thirds of the half-span.

On the Solution to Transient Coupled Thermoelastic Problems by Perturbation Techniques[1]

MARTIN LESSEN.[2] It is a pleasure indeed for me to comment on the Soler-Brull paper.

Although the equations for linear, coupled thermoelasticity have been well known for many years, it is only quite recently that their properties have been investigated. The results of these investigations have indicated that, for small coupling, the phenomenon of thermoelastic shock is well described by a zeroth order coupling which is tantamount to the semicoupled theory, whereas thermoelastic damping can only be described in the fully coupled sense. The physical reasoning for the foregoing is clear; in the case of thermoelastic shock, one looks for the mechanical response to a thermal excitation and is usually content to neglect the mechanical modification to the temperature field. In the case of thermoelastic damping, however, it is precisely the mechanical modification to the temperature field which gives rise to the damping. The formulation of either problem in terms of the fully coupled theory, however, makes for an esthetically pleasing boundary value problem (from the point of view of matching normal modes of a dispersion relation or determinantal equation) but in the end, the complete inversion of a transform involving these modes cannot be effected without some approximation. It is for this reason that the authors are to be commended for explicitly and elegantly defining perturbation techniques that can accomplish practicable solutions which still retain some essence of the coupling.

[1] By A. I. Soler and M. A. Brull, published in the June, 1965, issue of the JOURNAL OF APPLIED MECHANICS, vol. 32, TRANS. ASME, vol. 87, Series E, 1965, pp. 389–399.

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A Contact Stress Problem for a Rigid Smooth Sphere in an Extended Elastic Solid[1]

H. B. WILSON, JR.[2] The author has presented an interesting application of dual Legendre series. However, some indication of the accuracy of his computed values of contact angle and boundary stresses would be desirable. The graphical results apparently demonstrate convergence, but the errors resulting from truncating the Legendre series for the normal stress are not readily assessed. The coefficients σ_n are defined by an infinite system of simultaneous equations having matrix coefficients which are infinite series. The effects of truncating the infinite system are important as well as the question of how the σ_n are influenced by the computational accuracy of the matrix elements. Even if a large number of the σ_n can be computed accurately by the method employed, a more significant consideration is how many terms must be retained in the Legendre expansion before the oscillation of this function near the edge of the contact surface is negligible. The contact angle is determined by the requirement that the normal stress must be compressive everywhere on the contact surface and must vanish at the edge of the contact surface. Any oscillation of the truncated Legendre series for the normal stress makes accurate evaluation of the contact angle difficult. Fig. 2 of the paper indicates pronounced oscillation of the normal stress curve even after 51 nonzero terms are retained; i.e., when $n = 100$.

The numerical results from the thesis forming the basis of the paper indicate slow convergence of the Legendre series. When $n = 100$, the ratio between the average magnitude of the last five nonzero values of σ_n and the average magnitude of the first five values is 0.229, while the ratio between the last and first coefficient to 0.709. It seems reasonable to question how accurate the accompanying values of contact angle and boundary stresses are.

An alternate approach which might reduce the accuracy problems associated with the series analysis is to express the σ_n in equation (31), which expresses the condition of zero normal displacement on the contact surface, in terms of the integral (32),

[1] By I-Chih Wang, published in the September, 1965, issue of the JOURNAL OF APPLIED MECHANICS, vol. 32, TRANS. ASME, vol. 87, Series E, pp. 651–655.

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