ing the study of the nonlinear dynamic vibration absorber with a differential equation of the Duffing type previously studied by Arnold [3], Atkinson [4], and Robertson [2].[4]

Apparently there is a discrepancy in Fig. 1 since the entire paper is based upon a physical system different from that indicated in the illustration. If a viscous damper (with viscous damping coefficient B) is inserted between the fixed wall and the mass m_1 in Fig. 1 as printed, and if the damper between masses m_1 and m_2 is deleted, the resulting physical system appears to be that investigated by the authors. It is on this assumption that the following remarks are made:

Evidently, also through a typographical error, a cube has been omitted in the last nonzero term of equation (8). This term should read $\beta^3A^3 \cos^3 \omega t$ and not $\beta^3A \cos^3 \omega t$.

In order for equation (15) to be correct as printed and consistent with previous equations, the terms ν, ν_1, and λ^2 must be defined as follows:

$$\nu = \frac{\beta^2 F_m{}^2 k_2}{k_1{}^3}$$

$$\nu_1 = \frac{\gamma^2 F_m{}^4}{k_1{}^2}$$

$$\lambda^2 = \frac{k_2}{\mu k_1}$$

This definition of λ^2 differs from that given in the nomenclature of the paper. If the foregoing three definitions are followed, then η^2/μ should be substituted for the $\mu\eta^2$ term appearing in the equations (18), (21), (22), (23), (24), (25), and the equation for $R(A_2/s)$ appearing before equation (20). In addition, the η^2-term of equation (26) should be revised to η^2/μ.

It is hoped that the data for the curves in Figs. 2 and 3 and the digital-computer results were obtained from correct equations and that the incorrect equations appearing in this paper resulted from typographical errors. Although the writer did not do the calculations necessary to verify it, the curves in Figs. 2 and 3 appear to be correct for $\mu = 10$ rather than $\mu = 0.1$ as stated in the figures. Apparently the authors have employed two different definitions of μ, one being the reciprocal of the other. With $\mu = m_1/m_2$ as defined in the nomenclature, then the suggested revisions in several equations and the indication in Figs. 2 and 3 that $\mu = 10$ appear desirable to make the paper completely consistent. In closure, this paper makes a significant contribution to the analysis of the Duffing-type nonlinear dynamic absorber.

Author's Closure

The authors wish to thank Professors Arnold and Kreisle for their comments. As both of them have noted, Fig. 1 of the paper was in error—the damper should have been between ground and m_1. The authors agree with Professor Arnold that there should be at most only three amplitudes for each disturbance frequency.

Stress Distribution in a Rotating Spherical Shell of Arbitrary Thickness[1]

H. PORITSKY.[2] The problem of stresses in a rotating spherical shell appears to be a popular one nowadays. In addition to the three authors, the writer, too, has had occasion recently to solve the elastic equations for the rotating spherical shell. However, his results have not been published even as an internal company report, since the writer discovered[3] that this problem had been solved over 70 years ago by C. Chree. Chree's paper, entitled "The Equations of an Isotropic Elastic Solid in Polar and Cylindrical Co-ordinates, Their Solution and Application," was presented to the Cambridge Philosophical Society on October 31, 1887.[4] After obtaining general solutions of the elastic equations in spherical co-ordinates and treating several problems of a solid sphere, Mr. Chree solves the problem of a rotating spherical shell.[5]

ROBERT SCHMIDT.[6] This paper is a significant addition to the collection of solutions of the axially symmetric problems in the theory of elasticity. It is also of interest to the investigators in the theory of shells.

The writer believes that there is a simpler and more direct approach to this problem. It can be shown that the problem may be formulated by means of a harmonic function

$$\psi = \sum (A_n r^n + B_n r^{-n-1}) P_n(p) \tag{1}$$

and a biharmonic function

$$\Omega = \chi + \frac{(1+\nu)(11-17\nu)}{360(1-\nu)E}\gamma\omega^2 r^4, \tag{2}$$

where

$$\chi = \sum \left[C_n r^n + D_n r^{-n-1} - \frac{5-4\nu+n}{2(2n+3)(1-\nu)} A_n r^{n+2} + \frac{4-4\nu-n}{2(2n-1)(1-\nu)} B_n r^{-n+1} \right] P_n(p) + \frac{(1+\nu)\gamma\omega^2 r^4}{360E}\left[50p^4 - \frac{3(7-9\nu)}{1-\nu}\right] \tag{3}$$

is a solution of the equation

$$\nabla^2\chi = -\frac{1}{1-\nu}\frac{\partial}{\partial r}(r\psi) - 4\psi + \frac{(1+\nu)\gamma\omega^2 r^2}{3E}\left[\frac{5}{2}(p^2-\bar{p}^2) - \frac{1-2\nu}{1-\nu}\right] \tag{4}$$

The stress components are calculated by the formulas

$$\sigma_r = \frac{E}{1+\nu}\frac{\partial}{\partial r}\left(\frac{\partial\chi}{\partial r} + \frac{2-\nu}{1-\nu}r\psi\right) - \gamma\omega^2 r^2\left[\frac{\nu}{3(1-\nu)} + \frac{1}{2}(p^2-\bar{p}^2)\right], \tag{5}$$

$$\tau_{r\theta} = \frac{E}{1+v}\frac{\partial}{\partial\theta}\left(\frac{1}{r}\frac{\partial\chi}{\partial r} - \frac{\chi}{r^2} + \psi\right) + \frac{1}{3}\gamma\omega^2 r^2 p\bar{p}, \tag{6}$$

and so on. These results are similar to those obtained by Hampl.[7]

[4] References in paper.

[1] By M. A. Goldberg, V. L. Salerno, and M. A. Sadowsky, published in the March, 1961, issue of the JOURNAL OF APPLIED MECHANICS, vol. 28, TRANS. ASME, vol. 83, Series E, pp. 127–131.

[2] Consulting Engineer, General Electric Company, Schenectady, N. Y., Mem. ASME.

[3] A. E. H. Lore, "Treatise on Mathematical Memory of Elasticity," McGraw-Hill Book Company, New York, N. Y., 4th ed., p. 255.

[4] Published in *Transactions of the Cambridge Philosophical Society*, vol. 14, 1889, pp. 250–481.

[5] Ibid., section 6, pp. 295–302.

[6] Associate Professor of Civil Engineering, University of Arizona, Tucson, Ariz.

[7] Miloslav Hampl, "The Stress Problem of Axisymmetrically Loaded Thick Spherical Shells," *Der Stahlbau*, 1940, pp. 96–100.

Substituting (1), (3) in (5), (6) and using the boundary conditions

$$\sigma_r = \tau_{r\theta} = 0 \quad \text{on} \quad r = a, b, \tag{7}$$

we obtain a set of algebraic equations for determining the constants A, B, C, D by comparing the coefficients of $P_m(p)$ in (7). At the worst we must solve one 2 by 2 system and two 4 by 4 systems of algebraic equations. The work still could be simplified by certain manipulations of the equations. Thus the use of high-speed computers becomes avoidable.

Authors' Closure

The authors thank Dr. Poritsky for bringing Chree's[4] paper to their attention. However, Dr. Poritsky failed to notice that Chree treated in detail only the solution[8] for the limiting cases of a rotating spherical membrane and thin shell.

Professor Schmidt's statement, regarding a simpler method which would avoid the use of a high-speed computing device, is misleading. We note that Professor Schmidt's analysis would lead to three sets of simultaneous algebraic equations, totaling ten equations in all, for the arbitrary constants. These equations are similar to the system of ten linear algebraic equations obtained by the authors' equation (17).[1] We then obtained explicit representations for the seven nonvanishing coefficients of superposition, equation (18),[1] in terms of the geometric and physical quantities. At this stage, with equations (13)[1] and (18),[1] the problem is completely solved and the solution is amenable for calculations with a slide rule or desk calculator, depending upon the desired accuracy. However, in our opinion, the large volume of numerical results presented could be obtained most efficiently with a high-speed computing device. We are confident that Professor Schmidt would agree with our conclusion if he were to seek similar results using his method.

[8] Chree, Transactions of the Cambridge Philosophical Society, vol. 14, 1889, p. 297, par. 41.