Dynamics of a Vehicle Containing Moving Parts

M. J. ABZUG. The author has extended to the case of a vehicle containing moving parts his earlier work (footnote 3 of the paper) on the rotational dynamics of a single rigid body. This extension is a useful addition to the literature on the rotational dynamics of multiple-part bodies. This literature has application to studies of the attitude control of spacecraft.

The author's choice of a reference frame that is fixed to the main body is a logical one, in view of this application. The sensing instruments of practical attitude-control systems are also fixed to the main body. A further generalization of the equation of motion (11) is suggested in which the origin of reference axes does not coincide with the main body mass center. The author refers to the elimination of time-dependent components of the moving-part inertia tensors as a result of his choice of moving-part reference axes that are embedded in each part. This is contrasted to the Roberston scheme in which the moving-part inertia tensors are referenced to axes which remain parallel to main-body axes. However, the author's formulation has an offsetting disadvantage. This appears in the equation for the time derivative of angular momentum for each part relative to its own mass center (12). This derivative applies to axes moving with the part. Additional coordinate transformations are required for each part to obtain components along a common set of axes fixed in the main body.

Author's Closure

We appreciate the comments of Dr. Abzug. In the first place, it gives us the opportunity to point out a notation error in the equation immediately preceding (13); the symbol 'n' which appears in several places in the former equation should be replaced by '1,'. Second, we may note that if there is only one moving part, there is a simplification in equation (11); the second and third terms on the left-hand side can be combined to yield \( m_0 \left( m + m_1 \right)^{-1} \dot{q}_1 \times \dot{q}_1 \). Regarding the comments themselves, we refer to the further generalization of equation (11) so that the reference origin is not coincident with the main vehicle mass center, of course, possible. However, we cannot see any advantage for this since the mass center is already fixed relative to the main vehicle axes on the hypothesis that the main vehicle mass is constant. If the main vehicle mass were time-dependent, then its mass center would be wandering; in this case it would be conceptually simpler to take the origin at some other point which is fixed in the main vehicle coordinate system. This has been done in another paper without considering, however, the dynamical effects of moving parts.

Dr. Abzug's second comment is quite pertinent; perhaps all that can be said is that whether our formulation is advantageous or not depends on the particular problem under consideration. We emphasize that equation (11) of the paper is completely general if one leaves the evaluation of the derivative (d\( I_1 / \dot{q}_1 \)) unspecified. That is, the angular momentum for any rigid body relative to its own mass center can always be written as \( \Pi = I_1 \dot{q} / \dot{q}_1 \), where \( I_1 \) is the inertia tensor for the body (see footnote 4 of the paper) referred to any set of Cartesian axes whose origin is at the mass center, \( \dot{q}_1 \).

Author's Closure

The author cannot disagree with Mr. Putnam on his point that the technique of observation can cause complications. I can only say that very small amounts of the additives were used. Further, the instability at very low Reynolds number was observed to be a ripple at the edge of the jet close to the orifice rather than ring vortices. A small disturbance would of course be necessary to observe instability. It may be that the chemical reaction could be such a disturbance. The question then becomes one of possible reduction of the minimum Reynolds number for instability caused by the observational method. It is observed that the latest theoretical calculations available indicate a lower minimum Reynolds number for instability than has been measured. If a three-dimensional mathematical solution for jet instability becomes available for free jets and indicates a higher minimum Reynolds number than has been observed, the observational method may well be questioned.

Author's Closure

Professor Sandor has noted that, by means of harmonic analysis, it has been established that the dynamic response of the spherical crank drive possesses certain advantages over its planar counterpart, the crank-and-rocker mechanisms.

It may be pointed out that the author's comparison of his wobble-plate and spherical crank-drive mechanisms with the specially proportioned planar crank-and-rocker mechanisms selected by F. Freudenstein is of particular interest in that it reveals certain advantages of spherical over plane crank-and-rocker drives.
are unit vectors along these axes, \( \omega_i = \dot{\xi}_i \), and \( \ddot{\omega} \) is the inertial angular velocity of the body. For the time derivative of this term write simply \( (d/dt) (\dot{\xi}_I \omega_i) \). To carry out the differentiation one must explicitly select the axes; they can be fixed in the body or otherwise. For each particular problem the choice is left to the analyst. Roberson (footnote 2 of the paper) discusses this at some length.

### Minimum-Weight Proportions of Pressure-Vessel Heads

**C. W. Bert.** The author is to be congratulated for his pioneering work on synthesis of varying-thickness pressure-vessel heads for minimum weight.

The writer would like to add the following comments:

1. On the basis of weight per volume enclosed, the hemispherical head still may be the most efficient.
2. Since, as the author stated, only membrane stresses were considered in his analysis, considerable care should be taken in the design of the junction of the head and cylinder so as to minimize the discontinuity stresses. This is particularly necessary if a less ductile material is used.

If the wall thicknesses of the head and cylinder at the junction are unequal, the discontinuity stresses can be minimized for a given nominal head-and-cylinder combination by properly arranging the “imbalance” of the junction. Also care must be taken to fillet the junction properly to reduce stress-concentration effects.

3. In a recent brief note, Johnston presented a closed-form expression for the weight of a varying-thickness ellipsoidal head. Written in the notation of the present paper, it is

\[
W = KpVw/2r,
\]

where, for a short ellipsoid (i.e., oblate spheroid),

\[
K = 1 + 0.5N^{-1}
\]

Since

\[
pVw/2r = (2/3)N,
\]

the foregoing expression can be rewritten as

\[
W/H = (2/3)(N + 0.5N^{-1})
\]

Setting equal to zero the derivative of the foregoing expression gives a minimum \( W/H \)-value of 0.943 at \( N = 0.707 \), which is in agreement with the author’s result.

Furthermore, it can be shown that the total weight, as defined by the author, associated with a varying-thickness ellipsoidal closure is a minimum when the value of \( N \) is selected to be

\[
N = (6a - 2)^{-1/2}
\]

This shows that for values of \( a \) larger than 0.5, varying-thickness ellipsoidal heads more shallow than a hemispherical head are advantageous.

### Author’s Closure

The author is most grateful to Dr. Bert for his kind comments and remarks. As he points out in his first comment, and as described in the paper under Section 4 and in Fig. 9, Case IV, the hemispherical head may indeed be the most efficient in many instances where the minimization of the weight per volume enclosed is the basis of design.

With regard to Dr. Bert’s second comment on the need for care when designing junctions so as to compensate for discontinuity stresses, the author again agrees emphatically. A treatment of this important topic is in the section entitled “Head Shapes That Compensate for Discontinuity Stresses” in Hoffman and is submitted as evidence of the author’s concern with this subject.

Dr. Bert, in his third and final comment, notes the remarkable agreement with results derived independently by another investigator. Such an agreement (down to the third or fourth significant figure) is certainly gratifying, and the author wishes to thank Dr. Bert for calling it to the readers’ attention.

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