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Discussion

The Buckling of Thin-Walled Open-Profile Bars[1]

Raul Rosas e Silva.[2] The paper by Ojalvo has the merit of raising due concern with respect to the calculation of buckling loads using classical thin-walled beam theory. His results differ from the conventional ones for torsional buckling under compression because he used the average longitudinal displacement of the section w_c in equation (4), whereas in equation (6) he introduced a nonlinear term in w_c as a function of the squares of the derivatives of the transverse centroidal displacements u_c and v_c. It can be shown that if a similar averaging process had been used in defining quantities $(u_c')^2$ and $(v_c')^2$, the results would agree with the classical theory.

The average value of the square of a linear function (displacement in the cross-section) is not the same as the square of the average of the same function, except if the function is itself a constant. Therefore, the author's analysis essentially considers the transverse displacements derivatives to be constant in the cross-section; such an approximation fits the case of biaxial bending but contradicts the kinematic assumptions for torsion, which were indeed used in equation (1*a*) and (1*b*). Consequently, the end tractions appear to act through rollers which slide under torsional warping but follow the displacements of the centroid.

This result serves as a reminder that the potential of applied loads is dependent on the particular traction distribution whenever second-order terms are being considered. In other words, modifying quadratic terms in the potential functions may inadvertently introduce unwanted load behavior upon buckling (Silva, 1984).

The writer wishes to suggest that there is a common ground between the conventional and Ojalvo's theories, without resort to the so-called Wagner hypothesis, if the averaging procedure is adopted for the loaded area of the end cross-section. Consider acting over a certain area of the end face of the bar a uniform longitudinal stress traction distribution, its resultant being a force passing through the centroid of the section. If the area of distribution of such tractions is the total cross-section, the conventional theory applies. If the area of traction distribution is infinitesimal (concentrated force applied at the centroid), then the author's theory is to be used. For intermediate cases, replace the ratio between the centroidal polar moment of inertia and the area of the total cross-section (present in the conventional theory) by the same ratio computed for the area of traction distribution. Acting end moments received a different treatment in the paper, but it appears that a similar argument would apply.

[1] By Morris Ojalvo and published in the September 1989 issue of the ASME JOURNAL OF APPLIED MECHANICS, Vol. 56, pp. 633–638.

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Such theories retain the traditional kinematical constraints for thin-walled open-profile bars, with no provision for large displacements, local buckling, distortion of the cross-sections in their planes, or inelastic material behavior. Further refinements would require displacement fields and constitutive relationships of a more complex nature.

Reference

Silva, R. R., 1984, "Effect of Load Behavior and Kinematic Relations on Buckling of Curved Members," *Proceedings of the Annual Technical Session*, Structural Stability Research Council, pp. 83–90.

Author's Closure

We wish to thank the writer for his thoughtful remarks which are much appreciated. We respond as follows:

It is necessary to emphasize that, in mechanics, the object analyzed is never the prototype itself but, rather, a mathematical model of it. The foregoing is true for the simplest of bodies with the simplest of loadings and restraints, and continues to hold whether a technical or an elasticity theory is used.

With this in mind, we defined our model in the section labeled "The Bar." The model conforms to the "classical" representation in all respects and it is this model which dictates, for w_c, the form of equation (6) when the distortions are those which characterize lateral-torsional buckling. If another expression is suggested for w_c, it must conform to another model.

When a model for the bar is chosen it must be adhered to for all stages of the analysis and not solely to vindicate a particular part of the results such as the Wagner hypothesis. Most importantly, it must be possible to use the same model in an equilibrium method analysis and to achieve, in a totally acceptable way, the results of the variational method analysis. This is so because the variational theorem is derived from the virtual work theorem, which in turn is derived from the same concepts of continuity, equilibrium, and material behavior as form the foundation for an equilibrium method analysis (Fung, 1965, pp. 284–288).

The writer states we have considered the transverse displacement derivatives to be constant in a cross-section and that this contradicts the kinematic assumption of torsion. We disagree: The kinematic assumptions referred to are

$$U_g = u + (y_o - y)\,\theta \tag{20a}$$

and

$$V_g = v - (x_o - x)\,\theta \tag{20b}$$

where U_g and V_g are the transverse displacements of a general point in the profile plane parallel to the x and y-axes. Because x and y are centroidal axes for the profile area, the average values of U_g' and V_g' for the bar are

$$\frac{1}{A}\int (U_g')\, dA = u' + y_o \theta' = u_c' \qquad (21a)$$

and

$$\frac{1}{A}\int (V_g')\, dA = v' - x_o \theta' = v_c'. \qquad (21b)$$

We have used average values for the displacement derivatives of the bar in equation (6), although we obtained them more directly from equation (1).

The writer correctly implies that using $1/A\int (U_g')^2\, dA$ for $(u_c')^2$ and $1/A\int (V_g')^2\, dA$ for $(v_c')^2$ in equation (6) produces the conventional results. But this seemingly innocuous change requires us to deviate from the line model with which the theory is developed. In effect, it would require the adoption of a multifilament representation for the bar. Note how a longitudinal displacement at an end profile is determined from equation (4) for a single-filament representation for the bar. If one followed the conventional approach one would obtain longitudinal displacements by considering the sags of the individual filaments of a multifilament representation of the bar.

The writer suggests there may be room for both theories. Again, we disagree. We have shown that the theory based on our model for the bar results in potentials for the applied loads which are not dependant on a particular distribution of the tractions. There is, therefore, no reason to limit our theory to point loaded columns. The prototype, we believe, is similarly insensitive to the way the tractions are distributed, provided the resultants remain the same (H. Bleich, 1953). On the other hand, the conventional theory suffers from the theoretical deficiencies enumerated and is sensitive to the way tractions are distributed.

Reference

Fung, Y. C., 1965, *Foundations of Solid Mechanics*, Prentice-Hall, Englewood Cliffs, NJ.