Inelastic Scattering of Negative Pion on C\textsuperscript{12}

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The inelastic scattering of pion on Carbon 12 is investigated under the impulse approximation by using a simple nuclear model. The cross sections are calculated for the excitation of the nuclear levels with 4.4 MeV ($T=0$), 9 MeV ($T=0$) and 15 MeV ($T=1$) in the energy range of incident pion from 120 to 280 MeV, and are compared with experiments. Even the simple model for the reaction process gives good results.

§ 1. Introduction

Recently measurements of the differential cross section for the elastic and inelastic scattering of negative pion on Carbon 12 at energies near the resonance $\Delta_{33}(1230)$ MeV have been reported by Binon et al.\textsuperscript{1) Due to the strong effects of the resonance $\Delta_{33}$, one may expect that the effective pion-nucleon potential is dominated by the strong non-local terms, that is, velocity-dependent in contrast with the ordinary nucleon-nucleon potential, so that the impulse approximation seems not to work well. At present two types of approximations exist to study the elastic pion scattering on nuclei: One of them is to use the optical potential of Kisslinger type\textsuperscript{2) which was recently improved by Ericson-Ericson,\textsuperscript{3)} and the other is to use the Glauber approximation.\textsuperscript{4)} Although applicability of these approximations, in principle, is questionable in the energy region near the resonance, their practical usefulness seems to be proved by success of these approximations\textsuperscript{5)} having obtained good fits to the experiments. In the inelastic scattering of pion on nuclei, one may expect that the Glauber approximation (GA) or Distorted Wave Impulse Approximation (DWIA) is a useful method if one reminds oneself of the success in reproducing the elastic scattering. Very recently there appeared some work along the above lines for the inelastic scattering on Carbon 12 with excitation of $2^+$ and $3^-$, in which agreement with experiment was obtained except at large angles by adopting nuclear shell model or collective model and by using spin non-flip scattering amplitude between pion and nucleon.\textsuperscript{7)} In literature, however, no one attempted to compare the approximation GA or DWIA with simple impulse approximation, namely, Plane Wave Impulse Approximation (PWIA). Does PWIA predict nothing reliable to the cross section? If so, how does the cross section depend upon the distortion of the pion wave?
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The purpose of the present paper is to investigate the above questions by comparing the calculated cross section under PWIA with experimental data or the calculated results under DWIA. To compare the DWIA or GA with the present treatment, we adopt the nuclear shell model of particle-hole type calculated by Gillet and Vinh Mau under Tamm-Dancoff approximation.\(^9\)

In § 2, the treatment of the PWIA is presented, and in § 3 the cross sections with excitation of 4.43 MeV ($2^+$), 9.63 MeV ($3^-$) and of the low-lying states of $T=1(1^+, 2^+, 1^-, 2^-)$ are calculated in the energy region of incident pion from 120 MeV to 280 MeV. In § 4 the calculated cross sections are compared with experiment and with the results of DWIA. The effects of distortion of pion wave and of spin-dependent interaction between pion and nucleon are discussed.

§ 2. Treatment of pion-nucleus interaction under impulse approximation

In this section the impulse approximation for the pion scattering from nuclei is summarized.\(^9\) The single scattering approximation is formulated by the $T$-matrix $T(k_f; k_i)$ taken as

$$T(k_f; k_i) = \sum_{j=1}^{A} t(j; k_f, k_i),$$

(2.1)

where $A$ is the number of nucleons in target nucleus, and $t(j; k_f, k_i)$ is $T$-matrix describing the scattering between pion and off-shell $j$-th nucleon. The momenta $k_i$ and $k_f$ are those of pion in initial and final states, respectively. The impulse approximation assumes that the two-body scattering amplitude $t(j; k_f; k_i)$ is given by the “free” $T$-matrix of pion and off-shell nucleon, i.e., the structure of the target nucleus has no dynamical effect on the pion-nucleon scattering process. Further the pion-nucleon scattering amplitude off-energy shell is assumed to be equal to the free $T$-matrix on-energy shell, which, for example, is taken from the work of Roper et al.\(^11\)

According to CGLN,\(^13\) the pion-nucleon $T$-matrix $T^*$ is defined through $S$-matrix as

$$S_{fi} = -i(2\pi)\delta(p_f + k_f - p_i - k_i) \frac{M}{\sqrt{4\omega_f\omega_i E_f E_i}} \chi^*_f T^* \chi_i,$$

(2.2)

where $M$ is a nucleon mass, $\omega_i$ and $\omega_f$ are total energies of pion in the initial and final states, and $\chi_i$ and $\chi_f$ are, respectively, two-component spinors for initial and final nucleons. In the center of mass system of pion and nucleon, the scattering amplitude $f$ is given by

$$f = -\frac{M}{4\pi W} \chi^*_f T^* \chi_i,$$

(2.3)
where $W$ is total energy of pion and nucleon. The $T$-matrix $T(k_f; k_i)$ in Eq. (2·1) is given by

$$T_{fi}(k_f; k_i) = \sum_{j=1}^{A} \mathcal{F}_j^+(p_1, \ldots, p_i, \ldots, p_A) T^*(j) \mathcal{F}_i(p_1, \ldots, p_i, \ldots, p_A)$$

$$\times \delta(p_i' + k_f - p_j - k_i) \prod_{i+j} dp_i dp_i' dp_j,$$  

(2·4)

where $\mathcal{F}_i$ and $\mathcal{F}_j$ are initial and final nuclear wave functions in the momentum space, and the momenta $p_s(s=1 \sim A)$ are those of nucleons. Since the nucleon-momentum dependence of $T^*$ is small as compared with that of nuclear wave functions, we can set $p_j=0$ in the $T$-matrix $T^*$. It means that we neglect the effect of Fermi motion of nucleons in the target nucleus. Then the $T$-matrix $T_{fi}(k_f; k_i)$ in Eq. (2·4) is simply given by the nuclear wave functions in the coordinate space, as

$$T_{fi}(k_f; k_i) = \sum_{j=1}^{A} \mathcal{F}_j^+(r_1, \ldots, r_A) e^{-ik_f r_f} T^*(j) e^{ik_i r_i} \mathcal{F}_i(r_1, \ldots, r_A) \prod_{i+j} dr_i.$$  

(2·5)

The operators in $T^*$ acting on individual nucleons in nucleus are, therefore, reduced to spin and isospin operators. Other quantities in $T^*$ should be those for the laboratory system of pion and nucleon.

The differential cross section for inelastic scattering of pion on nucleus with spin $J_i$ and isospin $T_i$ leading to an excited state with spin $J_f$ and isospin $T_f$ is given by

$$\frac{d\sigma}{d\Omega} = \left( \frac{W}{M} \right)^{k_f/k_i} \frac{1}{(2J_i+1)} \sum_{i+j} |F_{fi}|^2$$  

(2·6)

with

$$F_{fi} = \langle J_f, M_f, T_f, T_{fs}; \sum_{j=1}^{A} e^{-ik_f r_f} f(j) e^{ik_i r_i} | J_i, M_i, T_i, T_{is} \rangle,$$  

(2·7)

where $M_i$ and $T_{is}$, and $M_f$ and $T_{fs}$ are $z$-components of spin and isospin of the initial, and final nuclear states, respectively. Pion-nucleon scattering amplitude $f(j)$ is simply expressed, in C.M. system, as

$$f(j) = [A_0 + B_0 \sigma_j \cdot n] + [A_1 + B_1 \sigma_j \cdot n] \cdot (\tau_j \cdot I),$$  

(2·8)

where $\sigma_j$ and $\tau_j$ are spin and isospin operators of nucleon and $I$ is isospin operator of pion. The quantities $A_i$ and $B_i(i=0,1)$ are determined by the scattering angle and the phase shifts, and $n$ is a unit vector perpendicular to the scattering plane.

§ 3. Calculation of the inelastic-scattering cross section of pion on Carbon 12

The explicit formula of the cross section (2·6) can be written, in the spherical tensor representation, as
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\[
\frac{d\sigma}{d\Omega} = \left( \frac{W}{M} \right)^2 \frac{k_f}{k_i} \frac{1}{2J_i+1} \sum_{\mu_i,\mu_f} |F_{\mu_i\mu_f}|^2,
\]

(3.1)

with

\[
\sum_{\mu_i,\mu_f} |F_{\mu_i\mu_f}|^2 = 4\pi \left\{ (J_f; T_f, T_{fs}) |(A_0 - A_1)\tilde{j}_f(qr) Y_\theta(\tilde{r})| |J_i; T_i, T_{ts})|^2 \\
+ \frac{1}{2} |(J_f; T_f, T_{fs}) |(B_0 - B_1)\tilde{j}_f(qr) \{ \sqrt{\frac{J+1}{2J+1}} j_{j-1}(qr) [Y_{j-1}(\tilde{r}) \otimes \sigma]_{\mu_i} \}
\right. \\
- \sqrt{\frac{J}{2J+1}} j_{j+1}(qr) [Y_{j+1}(\tilde{r}) \otimes \sigma]_{\mu_f} | |J_i; T_i, T_{ts})|^2 \}.
\]

(3.2)

where the momentum $q$ is a momentum transfer $|k_i - k_f|$.

The nuclear wave functions adopted here are those calculated by Gillet and Vinh Mau in Tamm-Dancoff approximation.\textsuperscript{9} They calculated the nuclear wave functions in Random Phase Approximations and obtained better results, especially for the collective nature of 4.43 MeV level than those obtained in Tamm-Dancoff approximation.\textsuperscript{10} But for comparison with the other authors' results, we adopt the former. The harmonic oscillator model is adopted for the single particle wave function, in which the oscillator parameter $b$ is determined from the elastic electron scattering experiment on Carbon 12, as $b = 1.64$ fm.\textsuperscript{11}

The calculations are performed for the excitation of the levels with $2^+ (T=0)$, $3^- (T=0)$ and for the low-lying $T=1 (1^+, 2^+, 1^-, 2^-)$ levels. The calculated results are shown in Figs. 1, 2, 3 and 4. Note that half values of the cross sections of the present calculation shown in Figs. 1 and 2 for excitations of $T=0$ levels.

![Fig. 1(a) and 1(b)](https://example.com/fig1.png)

Fig. 1(a), (b). Figure caption is printed on page 192.
Fig. 1. Comparison between the calculated cross sections for excitation of $2^+$ (4.4 MeV) on Cl$^{18}$, and the measured cross sections of Binon et al. Broken curves represent half values of the cross sections including either spin dependent interaction (SD) or spin independent interaction (SI) between pion and nucleon. Solid curve represents half values of the cross section including both interactions. Dot-and -dashed curves (LM) and (ER) represent the cross sections calculated by Lee-McManus under GA and by Edwards-Rost under DWIA, respectively. In Figs. 1(a), 1(b), 1(c), 1(d), 1(e), 1(f) and 1(g) the cross sections are shown for the incident pion energies 120, 150, 180, 200, 230, 260 and 280 MeV.
Fig. 2. Comparison between the calculated cross sections for excitation of $3^-$ and the measured cross sections of Binon et al. In Figs. 2(a), 2(b), 2(c), 2(d), 2(e) and 2(f) the cross sections are shown for the incident pion energies 150, 180, 200, 230, 260 and 280 MeV. See the caption of Fig. 1.
Fig. 3. The calculated cross sections for excitation of $T=1$ levels, i.e., $1^+ (15.1\text{MeV})$, $2^+ (16.1\text{MeV})$, $2^- (16.6\text{MeV})$ and $1^- (17.2\text{MeV})$. In Figs. 3a), 3b), 3c) and 3d) the calculated cross sections for each level are shown for the incident pion energies 200, 230, 260 and 280 MeV.

§ 4. Discussion and conclusion

In spite of the crude approximation for the treatment of the reaction processes, it is very interesting to see that the general fits to the experimental data are unexpectedly good except the factor two for the absolute values of the cross sections. The deviations of the calculated cross sections from the experi-
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Fig. 4. Comparison between the sums of the calculated cross sections for excitation of \( T=1 \) levels (solid curve), and the measured cross sections of Binon et al. In Figs. 4a), 4b), 4c) and 4d) the cross sections are shown for the incident pion energies 200, 230, 260 and 280 MeV, respectively. Dotted curves represent the cross section for excitation of the single level \( 1^+ \).

Experimental data are large at backward angles, since the impulse approximation gives nothing reliable to the cross sections at backwards direction where the multiple scattering effects and perhaps the short range correlations between nucleons appear to be important.
For the excitation of $2^+$ and $3^-$, the cross sections near forward direction are dominated by spin-independent interaction between pion and nucleon. (See Eq. (2·8).) It should be noted that the other approximations, for example, GA by Lee-McManus$^b$ or DWIA by Edwards-Rost$^b$, in which the spin independent interactions are retained, give almost the same feature of the differential cross sections as those of the present calculation taking account of the spin-independent interactions. (See Fig. 1.) Then the distortion of the pion wave does not seem to affect the cross section appreciably although the absolute values of the cross sections depend upon the distortion of the pion wave about factor two. Contribution of the spin-dependent interactions appears appreciably large at the large scattering angles for $2^+$.

For $3^-$, the spin-dependent interaction contributes to the cross section in the dip point and seems to be unimportant, which is consistent with the results of Lee-McManus. (See Fig. 2.) Here again the distortion of the pion wave seems not to be so important.

So far no one paid attention to the $T=1$ levels since the experiment could not separate each level from others. The calculated cross sections for individual levels $1^+$ (15.1 MeV), $2^+$ (16.1 MeV), $2^-$ (16.6 MeV) and $1^-$ (17.2 MeV) are shown in Fig. 3, and the sum of the cross sections are compared with the experiment in Fig. 4. The fits are almost perfect. Sum of the cross section is dominated by $1^-$ and $2^+$. The effect of $1^+$ is very small. (Note that spin dependent interaction causes the excitation of the levels with spin $J$ and parity $p=-(−)^J$.)

There exists another important observation which may support present treatment: In experiment small excitation of $0^+$ at 7.66 MeV was observed. On the other hand, the calculated cross sections are zero under the impulse approximation by assuming that the $0^+$ level consists of two particle-hole and more complicated configurations, because the pion can excite only one nucleon in the target nucleus per collision. In other words, excitation of $0^+$ is caused by multiple scattering of pion with nucleus. Therefore the magnitude of the cross section for excitation of $0^+$ will suggest how much the multiple scattering affects the cross section. In the case of the present reaction small magnitude of the measured cross section for excitation of $0^+$ at 7.66 MeV might imply the approximate validity of application of the impulse approximation to the inelastic scattering of pions on nuclei.

Summarizing the above discussions, we obtain the following results:

i) Spin-dependent interaction between pion and nucleon cannot be neglected.

ii) The distortion of pion wave does not give large effect to the cross section except for those at large scattering angles.

iii) General fit to the experiment and small excitation of $0^+$ suggest the validity of the impulse approximation.

In conclusion one might say that the plane wave impulse approximation is more than usually believed.
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References

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