The Coupling of the Spin-Dipole Mode with the Dipole Oscillations

—E1 Radiative Widths of Isobaric Analog States in Pb\(^{207}\) and Bi\(^{209}\)—

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The coupling of the spin-flip dipole mode with the dipole oscillation is studied through the analysis of photoabsorption cross sections in light nuclei and also \(E1\) radiative widths from isobaric analog states in heavy ones. It is shown that this coupling is reasonably understood by the central force and the tensor force, where the latter works additively to the former. The role of the two-body spin-orbit force has only a minor role for this coupling, for the main coupling terms of this force vanish so long as the shell model wave functions are assumed. The anomalously large \(E1\) radiative width of the isobaric analog state in Bi\(^{209}\), which is deduced from the Bi\(^{209}\) \((\pi, e'p)\) Pb\(^{208}\) reaction at Tohoku University, cannot be explained from our studies. However, our results are consistent with the value obtained from the Pb\(^{208}\) \((p, \gamma)\) Bi\(^{209}\) reaction at the State University of New York at Stony Brook.

§ 1. Introduction

The vibrational modes in nuclei are usually classified by the carried angular momentum \(\lambda\), parity \(\pi\) and isospin \(\tau\), each of which can be classified further by a non-spin-flip and a spin-flip modes. The collective vibrations of the non-spin-flip modes have been studied extensively and their properties are fairly known, e.g., the giant dipole vibration \((\lambda^*=1^-, \tau=1)\), the quadrupole vibration \((\lambda^*=2^+, \tau=0)\) and the octupole vibration \((\lambda^*=3^-, \tau=0)\). On the other hand, the collective vibrations of the spin-flip modes are scarcely known due to the difficulty of excitations by electromagnetic interactions. The spin-dipole modes \((\lambda^*=0^-, 1^- \text{ and } 2^-; \tau=1)\) are, however, expected to have strong collectivities. Indeed hindrance phenomena in the 1st forbidden \(\beta\) transitions are reasonably understood if one assumes the collectivities for such modes.\(^5\) Among them, only the spin-dipole mode with \((\lambda^*=1^-, \tau=1)\) is possible to couple with the giant dipole vibration because of the same spin and parity. If the coupling interaction is strong, these two mode of oscillations mix each other and, as a result, a part of the dipole oscillator strength shifts to the spin-dipole state and the expected width of the giant dipole resonance becomes large.

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Recently radiative widths of isobaric analog states (hereafter abbreviated as IAS) in \(^{207}\text{Pb}\) and \(^{209}\text{Bi}\) were measured from \((e,e'p)\) reactions and the width of the ground state of \(^{209}\text{Pb}\) was reported to be 62 times the single particle unit. In order to explain this anomalously large width and the similar enhanced width of \(^{189}\text{La}\) Fujita et al.\(^4\) suggested that these enhanced \(E1\) transitions could be explained provided that the coupling interaction between spin-flip and non-spin-flip particle-hole pairs exists. Quite recently, however, another measurement of this radiative width has been done by the inversed \((p,\gamma_0)\) reaction\(^5\) and the result was less than 4 times the single particle width.\(^*\)

The purpose of this paper is to study the coupling of the spin-dipole to the dipole oscillations through photoabsorption cross sections in light nuclei and also \(E1\) radiative widths from IAS's in heavy ones. As the coupling is expected to be induced by noncentral forces, so in § 2 it is examined in \(^{16}\text{O}\) and \(^{40}\text{Ca}\) how much the distribution of oscillator strengths changes if the two-body spin-orbit and the tensor forces are introduced. It is found from these studies that the tensor force has an important role for the coupling, while the two-body spin-orbit force has only a minor role. In § 3 the coupling between the spin-flip particle-hole pair and the dipole oscillation is studied through the \(E1\) radiative width of the IAS in \(^{209}\text{Bi}\). It is found in this transition that the enhancement phenomenon instead of the well-known hindrance phenomena can be expected. Finally summaries are given in § 4.

§ 2. A search for the coupling between the spin mode and the non-spin mode of oscillations

The operators which generate the non-spin-flip and the spin-flip modes with the same spin and parity are given as

\[ \bar{\phi} = r^\lambda Y_{\lambda\rho} \]  
\[ \bar{\phi}_s = r^\lambda [\sigma \times Y_{\lambda\rho}] \]  

(2.1)

(2.2)

For a spin mode of oscillation of rank \(\lambda (\lambda^* = 1^-, 2^+ \text{ and } 3^-)\), the strengths are expected to be scattered more than those of the non-spin mode of the same rank, since the spin-orbit interaction in the single particle field mixes the spin angular momentum of a particle-hole state.

To exhibit the dispersion of strength of each mode, we use the following schematic model. We suppose that there are two single particle orbitals with the orbital quantum number \(l\) and \(l' = l + \lambda\), each of which splits into two states due to the spin-orbit interaction. The former two states are assumed to be closed.

\(*\) These two experiments are consistent in so far as the cross sections of \(\sigma(r,p_0)\) and \(\sigma(p,\gamma_0)\) are not observed, respectively. The large radiative width obtained at Tohoku University is due to the enhanced proton decay from the region of the IAS of \(^{209}\text{Bi}\) to the excited states of \(^{208}\text{Pb}\).
and the latter two are empty. We can calculate the reduced transition probabilities between any of these two states with respect to the operators given in (2·1) and (2·2). Figure 1(a) shows the reduced transition probabilities for the non-spin mode of rank 1 and in Fig. 1(b) those of the spin mode are shown. It is assumed that the radial dependence is ignored and $l \gg 1$. For the $\lambda = 2$ mode the reduced transition probabilities become $2l$, $2/l$ and $2l$, which correspond to the left, the middle and the right transitions in Fig. 1(a), while those in Fig. 1(b) are $(4/3)l$, $(4/3)l$ and $(4/3)l$. For the $\lambda = 3$ mode the transitions in Fig. 1(a) are $2l$, $3/l$ and $2l$, while those in Fig. 1(b) are $(3/2)l$, $l$ and $(3/2)l$. It is easily seen that the dispersion of the strength of a spin mode is much larger than that of the non-spin mode, especially for $\lambda = 2$ and 3.

![Fig. 1. Ratios of $B(E1)$, where the radial dependence is ignored and $l \gg 1$.](image)

![Fig. 2. Distributions of $B(E1)$ for Ca$^{40}$: (2a) the ($\lambda^* = 2^+$, $\tau = 0$) mode and (2b) the ($\lambda^* = 3^-$, $\tau = 0$) mode. Black and light histograms denote those of the non-spin and the spin modes, respectively.](image)
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Table I. Single particle and hole energies.

<table>
<thead>
<tr>
<th></th>
<th>0s_{1/2}</th>
<th>0p_{1/2}</th>
<th>0p_{3/2}</th>
<th>0d_{3/2}</th>
<th>1s_{1/2}</th>
<th>0f_{7/2}</th>
<th>1p_{1/2}</th>
<th>1p_{3/2}</th>
<th>0f_{5/2}</th>
</tr>
</thead>
<tbody>
<tr>
<td>C^{12}</td>
<td>-30.1</td>
<td>-13.8</td>
<td>0</td>
<td>3.85</td>
<td>3.09</td>
<td>8.34</td>
<td>12.0</td>
<td>13.6</td>
<td>19.6</td>
</tr>
<tr>
<td>O^{16}</td>
<td>-45.9</td>
<td>-17.7</td>
<td>-11.5</td>
<td>0</td>
<td>0.88</td>
<td>5.03</td>
<td>8.7</td>
<td>10.3</td>
<td>16.3</td>
</tr>
<tr>
<td>Ca^{40}</td>
<td>-13.1</td>
<td>-8.7</td>
<td>-6.7</td>
<td>0</td>
<td>1.9</td>
<td>4.1</td>
<td>6.4</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table II. Residual interactions: $P^{2T+1,2S+1}$ is the projection operator, $\mu$ is the potential range parameter where the range parameter is given by $\lambda = \mu(0.96/2A^{1/3})^{1/2}$ and $V_0$ is the potential depth. The force 3C has the Yukawa radial dependence, while the others have the Gaussian radial dependence. The potential depth of the central force is taken as $-40$ MeV for C^{12} and O^{16}.

<table>
<thead>
<tr>
<th>Force</th>
<th>p_{13}</th>
<th>p_{15}</th>
<th>p_{16}</th>
<th>p_{11}</th>
<th>$V_0$ (MeV)</th>
<th>$\mu$ (fm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Central</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1C</td>
<td>-0.6</td>
<td>0.6</td>
<td>1.0</td>
<td>0.6</td>
<td>-50.0</td>
<td>1.67</td>
</tr>
<tr>
<td>2C</td>
<td>0.0</td>
<td>1.0</td>
<td>1.0</td>
<td>0.0</td>
<td>-50.0</td>
<td>1.73</td>
</tr>
<tr>
<td>3C</td>
<td>-0.33</td>
<td>0.6</td>
<td>1.0</td>
<td>-1.8</td>
<td>-50.0</td>
<td>1.40</td>
</tr>
<tr>
<td>Tensor</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1T</td>
<td>-0.14</td>
<td>0.0</td>
<td>1.0</td>
<td>0.0</td>
<td>-40.5</td>
<td>1.41</td>
</tr>
<tr>
<td>2T</td>
<td>-0.10</td>
<td>0.0</td>
<td>1.0</td>
<td>0.0</td>
<td>-99.3</td>
<td>1.41</td>
</tr>
<tr>
<td>3T</td>
<td>-0.14</td>
<td>0.0</td>
<td>1.0</td>
<td>0.0</td>
<td>-159.4</td>
<td>0.95</td>
</tr>
<tr>
<td>L-S</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1L</td>
<td>1.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>-30.0</td>
<td>0.95</td>
</tr>
</tbody>
</table>

When the particle-hole interaction is introduced, the dipole strength is concentrated in one state and the spin-dipole strength splits into two, because the matrix elements between spin-flip and non-spin-flip particle-hole pairs are small.

The above arguments will be clarified in Figs. 3(b), 2(a) and 2(b) where distributions of strengths of a non-spin mode and a spin mode for the case of Ca^{40} are given by black and light histograms, respectively.

These are calculated in the Tamm-Dancoff approximation including the isospin, where the ground state assumed to be closed. The particle and the hole energies and the residual interactions used in this paper are listed in Tables I and II. Here the interaction of Gillet's exchange character (Force 1C) with the potential depth of $-50$ MeV is used. Similar tendencies remain unchanged for C^{12} and O^{16} and the same things are also true in the calculation of the random phase approximation and for the residual interaction of the Serber or the Rosenfeld mixture.

In order to see more clearly the dispersions of reduced transition probabilities of rank $\lambda$ for the non-spin mode and the spin mode of oscillations, the root mean square deviation $\Delta$ can be a useful measure. This is defined as

$$
\Delta = \frac{\sum_n (E_n - \bar{E})^2 B(E\lambda; 0\rightarrow n)}{\sum_n B(E\lambda; 0\rightarrow n)}
$$

(2.3)
Table III. Mean energies and root mean square deviations of $B(E\lambda)$ for various vibrational modes. The suffix $S$ denotes those of the spin modes. Figures are given in units of MeV.

<table>
<thead>
<tr>
<th>$J^\pi$</th>
<th>$T$</th>
<th>$^{12}C$</th>
<th>$^{16}O$</th>
<th>$^{40}Ca$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>$E$</td>
<td>$\delta$</td>
<td>$E_S$</td>
</tr>
<tr>
<td>1$^-$</td>
<td>0</td>
<td>7.6</td>
<td>2.5</td>
<td>23.1</td>
</tr>
<tr>
<td></td>
<td>1</td>
<td>25.5</td>
<td>5.9</td>
<td>27.9</td>
</tr>
<tr>
<td>2$^+$</td>
<td>0</td>
<td>14.7</td>
<td>6.4</td>
<td>27.9</td>
</tr>
<tr>
<td></td>
<td>1</td>
<td>30.3</td>
<td>6.5</td>
<td>32.1</td>
</tr>
<tr>
<td>3$^-$</td>
<td>0</td>
<td>13.4</td>
<td>2.2</td>
<td>18.6</td>
</tr>
<tr>
<td></td>
<td>1</td>
<td>21.5</td>
<td>2.5</td>
<td>23.5</td>
</tr>
</tbody>
</table>

and

$$E = \frac{\sum E_n B(E\lambda: 0 \to n)}{\sum B(E\lambda: 0 \to n)} \quad (2.4)$$

where $E_n$ is the excitation energy of the $n$-th excited state. The root mean square deviation $\delta$ and the center-of-gravity $E$ for each mode are listed in Table III. The root mean square deviations for the spin modes of rank 2 and 3 are quite large which indicate also the large dispersions of strengths of these modes.

From these figures and Table III, the dipole and the spin-dipole states are expected to be the most appropriate candidates to see whether the coupling is strong or not, for the strong coupling occurs when both states are collective and locate near in energy.

Unfortunately, as photons cannot excite a spin-dipole state, the electric transverse form factor in the inelastic electron scattering is the powerful tool to see this state. This form factor is, however, smaller by an order than the longitudinal one and spin-dipole states so far observed are only in $^{12}C$. The shape of the photoexcitation cross section may give an alternative information for this coupling, for a part of the dipole strength shifts to the spin-dipole state provided that the coupling is fairly strong.

Microscopically this coupling is thought to be induced by noncentral forces, i.e., the two-body spin-orbit force and the tensor force. These interactions are examined in the following subsections. In the subsection 2.1, the role of the central force is summarized in order to study the role of the noncentral forces which will be discussed in 2.2 and 2.3.

2.1 The role of the central force

The cross section for the photoexcitation of a state $n$ is proportional to the dipole oscillator strength which is given as
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\[
f_{\text{em}} = \frac{4\pi}{9} \frac{2m(E_n - E_0)}{\hbar^2} |\langle \phi_n | \hat{O}_D | \psi_0 \rangle|^2 \tag{2.5}
\]

with
\[
\hat{O}_D = -\frac{1}{\hbar} \sum_{i} \tau_{it} r_i Y_1(i), \tag{2.6}
\]

where \( E_n, \psi_n \) and \( m \) are the excitation energy, the wave function of the \( n \)-th state and the mass of a nucleon. The spin-dipole operator is given as
\[
\hat{O}_{SD} = -\frac{1}{\hbar} \sum_{t} \tau_{t} r_t \left[ \sigma_t \times Y_1(t) \right]. \tag{2.7}
\]

In order to compare with the dipole oscillator strength, we define here the spin-dipole strength by Eq. (2.5), where \( \hat{O}_D \) is replaced by \( \hat{O}_{SD} \).

The calculated results for \( \text{O}^{16} \) and \( \text{Ca}^{40} \) are shown in Figs. 3(a) and 3(b), where dipole oscillator strengths and spin-dipole strengths are represented by black and light histograms, respectively and curves denote the photoabsorption cross section in arbitrary units.

For \( \text{O}^{16} \) (Fig. 3(a)) the peaks at 22\text{--}23 \text{MeV} and 24\text{--}25 \text{MeV} correspond to the calculated peaks, whose configurations are dominantly \( 0\rho_{3/2} \) \( 0d_{3/2} \) at 22.7 \text{MeV} and \( 0\rho_{1/2} \) \( 0d_{5/2} \) at 25.8 \text{MeV}. The latter is the spin-flip state and its photoabsorption strength is expected to be weak, while the observed one is comparable with

![Fig. 3. Distributions of the dipole oscillator strengths and the spin-dipole strengths for \( \text{O}^{16}(3a) \) and \( \text{Ca}^{40}(3b) \). The former and the latter strengths are given in black and light histograms, respectively. Curves denote the photoabsorption cross sections.](https://academic.oup.com/ptp/article-abstract/48/3/840/1871427)
that of the former. This suggests the possibility of the strong coupling between
the spin-flip and the non-spin-flip states, which makes the photoabsorption strength
be equally distributed in these two states. In this argument it is supposed about
the dips at 22.8 MeV and 24.7 MeV in the photoabsorption curve that the ap-
parently observed four peaks in the energy region of 22~25 MeV may not be
related to four different structures, or rather two peaks split into four due to the
existence of many-particle-many-hole states. 10

For Ca 40 (Fig. 3(b)) the observed peaks at 18.5 MeV and 20.5 MeV correspond
to predicted peaks at 19.4 MeV and 22.3 MeV. The former is the dipole state
which is composed of non-spin-flip states and the latter is the superposition of
spin-flip ones. Also here the predicted strength of the latter is fairly smaller
than the experiment. 10

In general, the observed photoabsorption strength of a spin-dipole state is
fairly large as compared to the prediction. This indicates the strong possibility
of the coupling between the spin-dipole and the dipole states.

2.2. The role of the two-body spin-orbit force

The coupling is thought to be induced by noncentral forces and one of these
is the two-body spin-orbit force. This force has the following form:

\[ V_{LS}(r) = \frac{1}{2} V_{L}(r) \left[ \left( \mathbf{r}_1 - \mathbf{r}_2 \right) \times \left( \mathbf{p}_1 - \mathbf{p}_2 \right) \right] \left( \sigma_1 - \sigma_2 \right) \]

\[ = \frac{1}{2} V_{L}(r) \left[ \left\{ \left( \mathbf{r}_1 \times \mathbf{p}_1 \right) \sigma_1 + \left( \mathbf{r}_2 \times \mathbf{p}_2 \right) \sigma_2 \right\} + \left\{ \left( r_1 \times p_1 \right) \sigma_1 + \sigma_1 \left( r_1 \times p_2 \right) \right\} \right. \]

\[ + \left\{ r_1 \left( \sigma_1 \times p_1 \right) - p_1 \left( \sigma_1 \times r_1 \right) + \left( \sigma_1 \times r_2 \right) r_2 - \left( \sigma_1 \times r_1 \right) p_2 \right\} \] \quad (2·8)

From the information of the high energy nucleon-nucleon scattering, we assume
that this force has the short range character and the Serber exchange mixture. 11
The potential depth is determined by assuming that this force is responsible for
about a half of the spin-orbit splitting. 11 The Force 1L in Table II assure the
spin-orbit splittings of 0.95 MeV for the 0p shell, 1.93 MeV for the 0d shell
and 3.07 MeV for the 0f shell.

The main coupling terms should be the last four terms on the right-hand
side of Eq. (2·8), for the coupling between a non-spin-flip particle-hole state and
a spin-flip one is induced by \( r_1 \left( \sigma_1 \times p_1 \right) \) and \( p_1 \left( \sigma_1 \times r_1 \right) \). It should, however, be
noticed that the contributions of these terms are almost completely cancelled out
for each other so long as the harmonic oscillator wave functions are used, e.g.,

\[ \langle V(r) r_1 (\sigma_1 \times p_1) \rangle = \langle V(r) p_1 (\sigma_1 \times r_1) \rangle = 0 \] \quad (2·9)

So the coupling terms had to be the 3rd and the 4th terms in Eq. (2·8), which
are expected to contribute in random phases.

Figures 4a and 4b show distributions of dipole oscillator strengths of O 16
and Ca 40 by using the force 1C + 1L. The dipole and the spin-dipole states
are more or less decoupled with each other. However, this force introduces
only a slight change in the distribution of dipole oscillator strengths as expected
above.
The Coupling of the Spin-Dipole Mode with the Dipole Oscillations

2.3 The role of the tensor force

The tensor force is another well-known noncentral force and has the following form:

\[ V_T(r) = V(r) \left\{ \frac{3(\sigma_1 \cdot r)(\sigma_2 \cdot r)}{r^3} - (\sigma_1 \cdot \sigma_2) \right\}, \tag{2.10} \]

where \( r = r_1 - r_2 \). From the phase shift analysis of the nucleon-nucleon scattering, this is strongly attractive in the triplet even state and weakly repulsive in the triplet odd one. So we assume that such an exchange character is preserved also in a nucleus. The potential depth is assumed to be determined so as to fit the calculated spectrum to the observed one or by considering that this force accounts for about a half of the spin-orbit splitting. In this paper values cited in the paper of Kim and Rasmussen are used.

Distributions of dipole oscillator strengths for \( \text{O}^{16} \) and \( \text{Ca}^{40} \) are shown in Figs. 5(a) and 5(b), where the Force 1C+2T is used.

Comparing these figures and those of Figs. 3a and 3b the tensor force is effective in the coupling of the dipole and the spin-dipole states. This force makes the calculated photoabsorption curves similar to that of the observed one and the positions of these states are also shifted to those of the observed peaks.
due to the attractiveness of the diagonal matrix element of this force. It is important that the tensor force works coherently with the central force. This phase relation is preserved also in heavy nuclei, e.g., Pb$^{207}$ and Bi$^{209}$ as will be seen in the next section. (See Table V.)

In the above subsections, it has been studied how to change the positions of the dipole and the spin-dipole states and the ratio of the dipole oscillator strengths of these two states when the central and the noncentral forces are introduced. In order to see these situations more clearly, the energy difference $E_{SD} - E_D$ and the ratio of the oscillator strengths $f(SD)/f(D)$ of these states are shown in Table IV for various forces. It is noticed that the ratio and the energy difference change fairly for various central forces. It seems that the large energy difference gives the small mixture. Indeed the central force with the Serber exchange character gives the largest energy difference and the smallest mixture. The tensor force with any kind of central force usually used works coherently in enhancing the mixture of these states. It is important that the tensor force works coherently with the central force in the coupling of these two states. On the other hand, the two-body spin-orbit force works in the opposite direction, i.e., although slightly, this force diminishes the coupling. The contribution of this force is, however, expected to become smaller as the collectivities of these
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Table IV. The energy difference and the ratio of oscillator strengths of the spin-dipole and the dipole states. Energies are given in MeV.

<table>
<thead>
<tr>
<th>Force</th>
<th>O^{16}</th>
<th>Ca^{40}</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$E_{SD}-E_D$</td>
<td>$f(SD)/f(D)$</td>
</tr>
<tr>
<td>1C</td>
<td>3.1</td>
<td>0.26</td>
</tr>
<tr>
<td>2C</td>
<td>4.0</td>
<td>0.12</td>
</tr>
<tr>
<td>3C</td>
<td>2.5</td>
<td>0.47</td>
</tr>
<tr>
<td>1C+2T</td>
<td>2.6</td>
<td>0.37</td>
</tr>
<tr>
<td>2C+2T</td>
<td>3.5</td>
<td>0.15</td>
</tr>
<tr>
<td>3C+2T</td>
<td>2.2</td>
<td>0.77</td>
</tr>
<tr>
<td>1C+1L</td>
<td>2.9</td>
<td>0.21</td>
</tr>
<tr>
<td>2C+1L</td>
<td>3.9</td>
<td>0.10</td>
</tr>
<tr>
<td>3C+1L</td>
<td>2.2</td>
<td>0.41</td>
</tr>
</tbody>
</table>

states increase. It will be seen in the next section that this force has the vanishing contribution in heavy nuclei.

As photons cannot excite a spin-dipole state, this state is only seen indirectly in the photoabsorption cross section via the coupling with the dipole state, while the direct observation of this state is due to the inelastic electron scattering.\textsuperscript{16} In this experiment, a spin-dipole state is excited by the transverse electric wave and its cross section increases as the transferred momentum becomes large. The transverse electric operator is, however, $\hbar/m_\ell c$ times as small as the longitudinal one and further the dispersion of the spin-dipole strength is fairly large, so it may not be so easy to observe this state. In fact, the spin-dipole state at 21 MeV in Ca\textsuperscript{40} is not seen at any momentum transfer. This is reasonable, for the maximum value of $|F_\pi|^2=3\times10^{-4}$ is expected at $q=0.7$ fm$^{-1}$, while the experimental fluctuation comes up to $|F_\pi|^2=1.5\times10^{-4}$.\textsuperscript{17} This fact seems to indicate that it is very difficult to find the evidence for the coupling from the transverse electric form factor. On the other hand, the longitudinal form factor at any momentum transfer shows only one broad peak at 19~21 MeV, where the dipole state at 19 MeV and the spin-dipole state at 21 MeV are not separately observed. If the longitudinal and the transverse electric form factors of these states will be able to be separately obtained, they will give a more detail information about the coupling.

§ 3. E1 radiative widths of IAS's in Pb\textsuperscript{207} and Bi\textsuperscript{209}

In the previous section, the coupling between the dipole and the spin-dipole states is seen to be induced by the tensor force which works additively to the
central force. Now it becomes interesting to see how much change we can expect in the spin-flip $E1$ transition from the IAS of the ground state of $\text{Pb}^{209}$ to the ground state of $\text{Bi}^{209}$, which provides another important information to see this coupling in heavy nuclei. As stated in the Introduction, two different experimental values are reported for this transition, i.e., one is 62 times$^3$ and the other is less than 4 times$^6$ the single particle unit.

Before the calculation of this radiative width, a brief summary of the $E1$ transition from an IAS in connection with the 1st forbidden $\beta$ decay is useful for later discussions.$^18$ These are related as

$$
\langle \psi_f | g_\beta \vec{O}_\beta | \psi_i \rangle = \frac{g_\beta}{e} \sqrt{2T_0 + 2} \langle \psi_f \left| e\vec{O}_I \frac{1}{\sqrt{2T_0 + 2}} T_+ \right| \psi_i \rangle
$$

$$
= \frac{g_\beta}{e} \sqrt{2T_0 + 2} \langle \psi_f | e\vec{O}_I | \text{IAS} \rangle ,
$$

provided that

$$
T_+ | \psi_f \rangle = 0 ,
$$

where $\vec{O}_\beta = \tau_+ r$, $\vec{O}_I = \tau_+ r$ and $T_0$ is the isospin of the final state $| \psi_i \rangle$. The $E1$ transition matrix element $| M_{\text{IAS}} |_{\text{exp}}$ is obtained from the corresponding radiative width:

$$
\Gamma_{\text{IAS}} = \frac{16\pi}{9} \left( \frac{E_\gamma}{\hbar c} \right)^3 e^2 | M_{\text{IAS}} |_{\text{exp}}^2
$$

$$
= 1.05 \times E_\gamma | M_{\text{IAS}} |_{\text{exp}}^2 \quad \text{in units of eV} ,
$$

where $E_\gamma$ is the emitted photon energy measured in MeV. In the case that the initial and the final states can be written as

$$
| \psi_i \rangle = | j_i(n) \rangle
$$

and

$$
| \psi_f \rangle = | j_f(p) \rangle ,
$$

the $| \text{IAS} \rangle$ of the initial state is expressed as

$$
| \text{IAS} \rangle = \frac{1}{\sqrt{2T_0 + 2}} | j_i(p) \rangle - \sum_a \sqrt{\frac{2j_a + 1}{2T_0 + 2}} [ j_a(p), j_a^{-1}(n) ] | j_i(n) \rangle
$$

and $p(n)$ refers to the proton (neutron) state. The $E1$ transition matrix element from the $| \text{IAS} \rangle$ is related with the single particle matrix element such as

$$
M_{\text{IAS}} = \langle j_f(p) | e\vec{O}_\beta | \text{IAS} \rangle
$$

$$
= \frac{1}{\sqrt{2T_0 + 2}} \frac{e}{\sqrt{2j_i + 1}} \langle j_f | rY_i | j_i \rangle
$$

$$
= \frac{1}{\sqrt{2T_0 + 2}} M_{sp}
$$

(3.7)
and the $E1$ transition operator $\hat{O}_{E1}$ is given as

$$
\hat{O}_{E1} = \frac{N}{A} \sum_{\text{proton}} r_i Y_1(i) - \frac{Z}{A} \sum_{\text{neutron}} r_i Y_1(i).
$$

(3·8)

It is observed in many nuclei that the $|M_{IAS}|_{\text{exp}}$ is smaller than the $|M_{sp}|/\sqrt{2T_0+2}$. This hindrance phenomenon is easily understood if one considers the $\beta$ decay process. The operator $\hat{O} = \tau_{-r}$ generates a collective mode in the daughter nucleus, where the final state corresponds to a low lying unperturbed state. Because of the $(\rho n)$ interaction, a part of the $\beta$ decay strength to the final state is deprived and shifts to upper states which are energetically forbidden. Indeed hindrance factors, which are defined as $|F|^{-1/2} = \sqrt{2T_0+2} |M_{IAS}|_{\text{exp}}/|M_{sp}|$, are 5.3 for the Bi$^{209}$ ($E1: 0i_{13/2} (\text{IAS}) \rightarrow 0h_{9/2}$) and 3.1 for the Pb$^{207}$ ($E1: 2s_{1/2} (\text{IAS}) \rightarrow 2p_{1/2}$), while the transition Bi$^{209}$ ($E1: 1g_{9/2} (\text{IAS}) \rightarrow 0h_{9/2}$) cannot be explained in this way.

Three reasons may be responsible for these hindrance or enhancement phenomena; (1) the coupling of the $T^>$ giant dipole resonance with the $|\text{IAS}\rangle$, (2) the coupling of the charge exchange collective states $|j_p (\rho)n^{-1}(n)\rangle|j_t\rangle$ with the final state as mentioned above and (3) the coupling of the $T^<$ giant dipole resonance with the $|\text{IAS}\rangle$ via the Coulomb interaction. In the lead region, the $T^<$ giant resonance is quite weak and the main contribution comes out from the 2nd reason. These are schematically shown in Fig. 6. These contributions to the $\beta$ decay matrix element are estimated by the 1st order perturbation theory.

The contributions (1) and (2) are formulated as follows: i.e., the initial state is written as

$$
|\psi_i\rangle = |j_i(n)\rangle + \sum_{\rho n} \alpha_{\rho n} [j_p(\rho)n^{-1}(\rho)]j_f(\rho):j_t\rangle
$$

(3·9)

and the final state is

$$
|\psi_f\rangle = |j_f(\rho)\rangle + \sum_{\rho n} \beta_{\rho n} [j_p(\rho)n^{-1}(n)]j_t(n):j_f\rangle,
$$

(3·10)

where $\alpha$ and $\beta$ are the perturbed amplitudes for the mixed states.

The isospin projection procedure is necessary for these mixed states and this is carried out using the following projection operator:

$$
P^T_j \rho \rho^3 = 1 - \frac{1}{2T_0+2} T_+ T_+ + \cdots.
$$

(3·11)

This operator projects a state with the $T_z = [T_0]$ into a subspace of a good isospin $T_q$. In our calculation the series expansion up to the 2nd term is sufficient.
Now, supposing that there are two states \(|i\rangle\) and \(|f\rangle\) and \(\hat{\lambda}\) denotes the eigenstate of the isospin, the two-body and the \(\beta\) matrix elements are given as

\[
\langle f | V | i \rangle = \sqrt{\frac{2T_0 + 2}{2T_0 + 1}} \langle f | V | i \rangle
\]

(3·12)

and

\[
\langle f | \hat{O}_\beta | i \rangle = \sqrt{\frac{2T_0 + 1}{2T_0 + 2}} \langle f | \hat{O}_\beta | i \rangle
\]

(3·13)

provided that either of these states is not the eigenstate of the isospin. Equations (3·12) and (3·13) show that the contribution of these mixed states to the transition strength is the same irrespective of whether the isospin projection procedure is taken into account or not.

The contribution (3) is expressed as\(^{(3)}\)

\[
M_{\text{IAS}} = \frac{1}{\sqrt{2T_0 + 2}} \frac{1}{\sqrt{2j_f + 1}} \left\{ \langle j_f | [\hat{O}_{li}, T_-] | j_i \rangle + \sum_d \frac{\langle j_f | \hat{O}_{li} | d \rangle \langle d | H_{\text{res}}^{(-)} | j_i \rangle}{E_A - E_d} \right\}
\]

(3·14)

where

\[
H_{\text{res}}^{(-)} = [H_{\text{res}}, T_-]
\]

\[
= \frac{e^2}{4} \sum_{\text{p-hole}} \sum_{\text{p-hole}} \frac{4}{2l + 1} \left( \mathbf{Y}_l \cdot \mathbf{Y}_l \right) \left[ (1 - \tau_s \tau') \tau_s \tau' + (1 - \tau_s \tau') \tau_s \tau' \right]
\]

(3·15)

and the summation \(d\) is taken all the states with the dipole particle-hole excitations. For the Coulomb matrix element \(\langle d | H_{\text{res}}^{(-)} | j_i \rangle\), the exchange term is neglected. The effect of the giant dipole resonance is taken into account by using the transition charge density \(\rho_{\ell}\). The particle-hole energies \(E_d\) are assumed to be degenerate at \(E_d = 13.5\) MeV, which is the center of the giant dipole resonance. Then Eq. (3·14) is written as

\[
M_{\text{IAS}} = \frac{1}{\sqrt{2T_0 + 2}} \frac{1}{\sqrt{2j_f + 1}} \left\{ \langle j_f | [\rho_{\ell} Y_l] | j_i \rangle \left[ 1 + \frac{4\pi}{9} \frac{\langle j_f | f(r) | j_i \rangle}{\langle j_f | r | j_i \rangle} \right] + \sum_{\text{p-hole}} \frac{\langle j_p | \mathbf{Y}_l | j_i \rangle^2}{E_A - E_D} \right\}
\]

(3·16)

and the form factor \(f(r)\) is

\[
f(r) = \int r^\alpha dr' \frac{r^\alpha}{r^\alpha} \rho_{\ell} (r') / \int r^\alpha dr' \rho_{\ell} (r').
\]

(3·17)

Two kinds of forms are assumed for the transition charge density, i.e., the \(\delta\) function at the nuclear surface and the step function up to the surface. The radial integral \(\langle j_f | f(r) | j_i \rangle / \langle j_f | r | j_i \rangle\) is evaluated by the solutions in a Saxon-Woods potential, for the radial wave functions at the nuclear surface should be
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Table V. Various contributions to the 1st forbidden $\beta$ matrix element. Figures are given in units of fm. The underlined figure is thought to be a more preferable one. The calculated result in the 4th column is the summation of the underlined figures wherever they appear.

<table>
<thead>
<tr>
<th>Transition</th>
<th>$\langle r\cdot r \rangle_{exp}$</th>
<th>$\langle r\cdot r \rangle_{fp}$</th>
<th>$\langle r\cdot r \rangle_{theory}$</th>
<th>Central</th>
<th>Tensor</th>
<th>L-S</th>
<th>Coulomb</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pb$^{209}\rightarrow$Bi$^{209}$ $1g_{7/2}^\pi$ 0$^+_h$</td>
<td>2.0±0.2 (&lt;0.53)</td>
<td>-0.250</td>
<td>-0.415</td>
<td>-0.089</td>
<td>-0.024</td>
<td>0.004</td>
<td>-0.025</td>
</tr>
<tr>
<td>Pb$^{208}\rightarrow$Bi$^{209}$ $0h_{11/2}^\pi$</td>
<td>2.0±0.2 (~2.1)</td>
<td>-4.682</td>
<td>-1.735</td>
<td>3.291</td>
<td>0.074</td>
<td>0.062</td>
<td>-0.468</td>
</tr>
<tr>
<td>Tl$^{207}\rightarrow$Pb$^{207}$ $2s_{1/2}^\pi$</td>
<td>1.5±0.2</td>
<td>2.681</td>
<td>1.077</td>
<td>-1.806</td>
<td>-0.022</td>
<td>0.000</td>
<td>0.256</td>
</tr>
</tbody>
</table>

as realistic as possible in the case that the transition charge density is localized at the surface.

Three kinds of perturbation effects mentioned above are summarized in Table V. The figures in the 2nd column are the $\beta$ matrix elements deduced from the observed $E1$ radiative widths. These are reported by the electron linear accelerator group at Tohoku University.\(^5\) The figures in brackets are those of the Stony Brook.\(^5\) The calculated results are shown in the 4th column whose contents are shown in the 5th to the 8th columns. The 5th column shows the contribution of the central force with the potential depth of $-50$ MeV. Three figures correspond to those with three different exchange characters listed in Table II. The contribution of the tensor force is shown in the 6th column where three figures correspond to those in Table II. A figure in the 7th column is the contribution of the two-body spin-orbit force (Force $1L$) with the range parameter $\lambda=0.4$. In these calculations, the same particle and hole energies as those in the paper of Gillet et al.\(^22\) are used. The 8th column gives the contribution (3) when the transition charge density is assumed to be the $\delta$ function or the step function. Underlined figures are thought to be more preferable ones.

For the non-spin-flip transitions, i.e., Pb$^{209}(0_{i1/2}^\pi)\rightarrow$Bi$^{209}(0h_{9/2}^\pi)$ and Tl$^{207}(2s_{1/2}^\pi)\rightarrow$Pb$^{207}(2p_{1/2}^\pi)$, hindrance phenomena are beautifully explained in our calculation and the hindrance factor $|F|$ is about 4. For the spin-flip transition Pb$^{209}(1g_{9/2}^\pi)\rightarrow$Bi$^{209}(0h_{9/2}^\pi)$, however, the enhancement occurs and the predicted width comes up to 4 times the single particle unit. The central force plays the most important role for these hindrance or enhancement phenomena. It should be noted that the tensor force contributes coherently to the central force. For the spin-flip transition, its contribution comes up to 30~50% of that of the central force, while for the non-spin-flip transitions, the former is only 10% of the latter. As stated in the previous section, the contribution of the two-body spin-orbit force is very
small due to the cancellation of the main coupling terms. Although the central and the tensor forces work coherently to increase the spin-flip transition, i.e., \( \text{Pb}^{209}(1g_{9/2}) \rightarrow \text{Bi}^{209}(0h_{11/2}) \), the width of only 4 times the single particle unit is expected and this is much smaller than the experimental value obtained at Tohoku University.

The enhancement phenomenon, as will be discussed in the Appendix, is expected to come out only for a spin-flip transition. Moreover the following two conditions should be satisfied for its appearance: (1) The \( (\tau \cdot \tau)(\sigma \cdot \sigma) \) part of the central force is responsible for its coming out. (2) The particle-hole state whose spins are up in the mixed configurations is responsible for the enhancement, while the partner state whose spins are down is responsible for the hindrance and the latter contribution cancels out that of the former. So the enhancement phenomenon appears in a nucleus where the former kind of state exists and the latter kind of state is not in the mixed configurations. This condition is satisfied only for nuclei near some doubly closed shells. Indeed, in the \( \text{Pb}^{209}(1g_{9/2}) \rightarrow \text{Bi}^{209}(0h_{11/2}) \) transition, the mixed configuration with \( 1g_{9/2}(n)0h_{11/2}(\bar{p}) \) for the initial state and the \( 0i_{13/2}(p)0i_{13/2}(n) \) configuration for the final state are responsible for almost all the enhancement.

§ 4. Summaries

Vibrational states in nuclei have been extensively studied together with the progress of the \( \gamma \) ray spectroscopy. These states are, however, what can be observed by the \( \gamma \) ray, while there are other vibrational modes which are neither excited nor disintegrated with the absorption or the emission of \( \gamma \) ray. One of these is a spin mode of oscillation generated by the operator \( (2 \cdot 2) \) whose collective state should be observed by the inelastic electron scattering or by the muon capture process for the charge exchange collective mode. The strength of a spin mode is generally scattered in many excited states due to the existence of the spin-orbit interaction in the single particle field. The spin-dipole mode with \( (\lambda^* = 1^-, \tau = 1) \) is, however, found to be fairly collective.

Provided that there are two collective states with the same spin, parity and isospin, these are necessarily coupled with each other. Indeed there is a considerable coupling which is induced by the tensor force working coherently with the central force between the dipole and the spin-dipole states. This coupling phenomenon is also seen in heavy nuclei, e.g., in the spin-flip transition of \( \text{Bi}^{209}(E1: 1g_{9/2}(\text{IAS}) \rightarrow 0h_{11/2}) \), where the tensor and the central forces work additively to enhance this transition and the radiative width of 4 times the single particle unit is expected. The role of the two-body spin-orbit force for the coupling is very small, for the main coupling terms of this force are cancelled out with each other so long as the harmonic oscillator wave functions are used.
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Appendix

The conditions for the enhancement phenomena to emerge are discussed by using the schematic model. The contribution of the final state configuration mixing to the 1st forbidden $\beta$ matrix element is given as

$$M_\beta = \frac{1}{\sqrt{2j_f+1}} \langle j_f | \tau \cdot r | j_i \rangle \left[ 1 - \sum_{\alpha} \frac{\langle j_f | V_{ij} [j_p,j_h -1 \rangle : j_f \rangle \langle j_i | [j_p,j_h -1 \rangle : j_f \rangle | \tau \cdot r | j_i \rangle}{(E_{ph} - E_0) \langle j_f | \tau \cdot r | j_i \rangle} \right].$$

(A.1)

We assume in the following that the residual interaction is central and has the separable form

$$V = \frac{G_1}{2} \sum_{\alpha} (\tau_i \cdot \tau_j) (r_i \cdot r_j) + \frac{G_2}{2} \sum_{\alpha} (\tau_i \cdot \tau_j) (\sigma_i \cdot \sigma_j) (r_i \cdot r_j)$$

(A.2)

and only the direct term of a matrix element is taken into account. All the radial integrals are further assumed to be a constant and also all the particle-hole energies are degenerate, i.e., $E_{ph} = E_0$. Then Eq. (A.1) becomes

$$M_\beta = \sqrt{\frac{3}{2j_f+1}} h_{dp}(j_f,j_i) \left[ 1 - \frac{G_1}{E - E_0} h_{dp}(j_f,j_i) \sum_{\alpha} h_{dp}(j_p,j_h) h_{dp}(j_p,j_h) \right. - \frac{G_2}{E - E_0} h_{sp}(j_f,j_i) \sum_{\alpha} h_{sp}(j_p,j_h) h_{sp}(j_p,j_h) \left. \right],$$

(A.3)

where

$$h_{dp}(j_f,j_i) = \frac{1}{\sqrt{3}} \langle j_f | C_i | j_i \rangle,$$

(A.4)

$$h_{sp}(j_f,j_i) = \frac{1}{\sqrt{3}} \langle j_f | (\sigma \times C_i) | j_i \rangle$$

(A.5)

and $C_i$ is the unnormalized spherical harmonics. Provided that the interaction (A.2) has the Serber exchange character, i.e., $G_1 = G_2 > 0$, the contributions of these mixed configurations are then easily calculated.
For a non-spin-flip transition, i.e., $(j_1 i_1) = (l_1 l_1, l_1 l_1)$ or $(l_1 l_1, l_1 l_1)$ where the direction of the spin angular momentum relative to the orbital one is indicated by an arrow, the dominant contribution comes from the 2nd term in the bracket of Eq. (A·3) and as this term is always positive, the hindrance occurs. For a spin-flip transition, i.e., $(j_2 i_2) = (l_2 l_2, l_2 l_2)$ or $(l_2 l_2, l_2 l_2)$, the 3rd term has the dominant contribution. In this case, this term has the negative or the positive sign according to the spins of a mixed particle-hole state are up or down. The contribution of the particle-hole state with spins $(l, l, l, l)$ and that of the partner state with $(l, l, l, l)$ in the 3rd term cancels out with each other and the resultant contribution of these two states is weakly negative and the same order of magnitude of the 2nd term. So the enhancement appears only in nuclei where a particle-hole state whose spins are up exists, while the partner state whose spins are down is not in the mixed configurations. Also the $(\tau \cdot \tau)(\sigma \cdot \sigma)$ part of the central force is responsible for its appearance.

References

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