Beta-Rays from Polarized Nuclei and Pseudo-Tensor Current

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The general expression for the angular distribution of the electron and neutrino is given in the case of the allowed beta decay of polarized nuclei under the assumption of the G-parity nonconservation. Explicit formulas are derived for the neutron beta decay and for the pure Gamow-Teller transition. It is pointed out that the magnitude of the pseudo-tensor coupling constant can, in principle, be obtained by measuring the angular distribution of the beta particles with respect to the initial nuclear polarization axis.

§ 1. Introduction

The β decay of the neutron is generally described by the matrix element of the current:  

$$\langle p|J|n\rangle = \frac{i}{\sqrt{2}} \bar{u}_p \gamma_\mu (G_V + G_A \gamma_\mu) \gamma_5 + \delta_{\mu\nu} (G_T + G_{PT} \gamma_5) + i q_\mu (G_S - G_{PS}) u_n, \tag{1}$$

where $q = p_n - p_p$. It is known that the $G_V$ and $G_A$ terms make the dominant contribution to β decay and that the constants $G_T$ and $G_{PS}$ are $\sim 1/M_N$ times $G_V$ and $G_A$, respectively. The other two constants $G_S$ and $G_{PT}$, that is, the so-called terms of the second kind are expected to be small compared with the main terms. Actually, $G_S$ and $G_{PT}$ should be zero if these two terms are required to have the same G-transformation properties as those of the $G_T$ and $G_A$ terms, respectively. On the other hand, we can make a simple model, which leads to these two terms as well as others, under the assumption of strong meson dominance. In this note we consider method of determination of the magnitude of $G_{PT}$.**

As is well known, the $ft$-ratio in the β decays of the mirror nuclei shows a deviation from unity if $G_{PT}$ is not equal to zero.

$$\frac{(ft)^+}{(ft)^-} \approx 1 + \frac{4}{3} \left( W^+ + W^- \right) \frac{\text{Re}(G_A G_{PT}^*)}{|G_A|^2}, \tag{2}$$

where $W$ is the maximum energy of the β particle and ± denote the positron and electron emission, respectively. This deviation is, however, not established

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** For the determination of $G_S$, see Ref. 2).
The measurement of $G_{\text{PT}}$ by Eq. (2) has to deal not only with nuclear effects but also with the intrinsic difficulty that the $ft$-value depends upon the main term $|G_A|^2$ which screens the small effect of $G_{\text{PT}}$. It may not be without interest, then, to look for other quantities more sensitive to $G_{\text{PT}}$ than the $ft$-value.

§ 2. Beta-rays from polarized nuclei

We calculate the angular distribution of electron and neutrino in the allowed $\beta$ transition from polarized nuclei under the assumption of the possible $G$-noninvariance in the nonrelativistic approximation. The general expression is given by $^{(9,8)}$

\[
\omega \langle J \rangle |E_e, \Omega_e, \Omega_\nu \rangle dE_e d\Omega_e d\Omega_\nu = \frac{1}{(2\pi)^3} \frac{p_e E_e (W - E_e) dE_e d\Omega_e d\Omega_\nu}{\frac{1}{J(J+1)} - \frac{3\langle J \cdot f \rangle}{J(2J-1)}}
\]

\[
\times \xi \left\{ 1 + a \frac{P_e \cdot P_\nu}{E_e P_\nu} + c \frac{J(J+1) - 3\langle J \cdot f \rangle}{J(2J-1)} \right\}
\]

\[+ \frac{\langle J \rangle}{J} \left[ A \frac{P_e}{E_e} + B P_\nu + D \frac{P_e \times P_\nu}{E_e P_\nu} \right]. \tag{3}
\]

Here $p$, $E$ and $\Omega$ are the momentum, the energy and the solid angle of the particle shown by suffix, respectively, $J$ is the angular momentum of the initial nuclei with the expectation value $\langle J \rangle$, $j$ is the unit vector in the direction of $\langle J \rangle$, and $\xi$, $a$, $c$, $A$, $B$ and $D$ are defined by

\[
\xi = \left| M_{\text{PT}} \right|^2 \left\{ |G_{\text{PT}}|^2 + m_e^2 |G_s|^2 - 2m_e^2 \text{Re}(G_{\text{PT}}G_s^*) \right\}
\]

\[+ \left| M_{\text{GPT}} \right|^2 \left\{ |G_A - W G_{\text{PT}}|^2 \right\}
\]

\[+ |G_{\text{PT}}|^2 \frac{1}{3} \left( 2E_e^2 - 2WE_e + W^2 - m_e^2 \right)
\]

\[+ |G_{\text{PT}}|^2 \frac{2}{3} \left( 3E_e^2 - 3WE_e + W^2 - 2m_e^2 + \frac{WM_e}{E_e} \right)
\]

\[+ \text{Re}[(G_A - W G_{\text{PT}})G_{\text{PT}}^*] \frac{4}{3} \left( 2E_e - W - \frac{m_e^2}{E_e} \right)
\]

\[+ \text{Re}[(G_A - W G_{\text{PT}})G_{\text{PT}}^*] \frac{2}{3} \left( W - \frac{m_e^2}{E_e} \right) \},
\]

\[a \xi = \left| M_{\text{PT}} \right|^2 \left\{ |G_{\text{PT}}|^2 + m_e^2 |G_s|^2 \right\}
\]

\[+ \left| M_{\text{GPT}} \right|^2 \left\{ - \frac{3}{2} |G_A - W G_{\text{PT}}|^2 \right\}
\]

*) We adopt the units $\hbar = c = 1$. The following formulas are obtained for the electron emission.
\[ c^T = |M_{\text{GR}}|^3 A_{TJ} \]

\[ \times \left\{ -|G_A - W G_{PR}|^2 \left[ \frac{1}{3} \frac{p_e \cdot p_s}{E_e p_s} - \frac{(j \cdot p_e) (j \cdot p_s)}{E_e p_s} \right] \right. \]

\[ + |G_{T}|^2 \left[ \frac{p_e^2 p_s^2 - (p_e \cdot p_s)^2}{3 E_e p_s} - \frac{(j \cdot p_e + p_s)^3}{E_e p_s} \right] \]

\[ + |G_{PR}|^2 \left( \frac{1}{2} \left( E_e p_s + p_s \right) \left[ \frac{(p_e + p_s)^3}{3} - (j \cdot p_e + p_s)^3 \right] \right. \]

\[ - |G_{PR}|^2 \left( \frac{1}{2} \left( E_e p_s + p_s \right) \left[ \frac{(p_e + p_s)^3}{3} - (j \cdot p_e + p_s)^3 \right] \right. \]

\[ + \text{Re}[(G_A - W G_{PR}) G^*_{PR}] \times \left[ \frac{1}{3} \left( \frac{p_e + p_s}{p_s} \right) \cdot (p_e + p_s) - \left( j \cdot p_e \frac{p_s}{p_s} \right) (j \cdot p_e + p_s) \right] \]

\[ - \text{Re}[(G_A - W G_{PR}) G^*_{PR}] \times \left[ \frac{1}{3} \left( \frac{p_e + p_s}{p_s} \right) \cdot (p_e + p_s) - \left( j \cdot p_e \frac{p_s}{p_s} \right) (j \cdot p_e + p_s) \right] \]

\[ + \text{Re}(G_T^* G^*_{PR}) \times \left( j \cdot p_e \frac{p_e + p_s}{p_s} \right) \left[ (j \cdot p_s) (p_e \cdot p_s + p_s) - (j \cdot p_s) (p_e \cdot p_s + p_s) \right] \}

\[ A^T = |M_{\text{GR}}|^3 \lambda_{TJ} \]

\[ \times \left\{ -|G_A - W G_{PR}|^2 \right. \]

\[ + |G_T|^2 \left[ 2 E_e^2 - W E_e - m_e^2 - (2 E_e^2 - W E_e) \frac{(p_e \cdot p_s)}{E_e p_s} \right] \]

\[ - \text{Re}[(G_A - W G_{PR}) G^*_{PR}] \left[ 3 E_e - W - E_e \frac{(p_e \cdot p_s)}{E_e p_s} \right] \]

\[ + \text{Re}[(G_A - W G_{PR}) G^*_{PR}] \left[ E_e - W - E_e \frac{(p_e \cdot p_s)}{E_e p_s} \right] \]

\[ + \text{Re}(G_T^* G^*_{PR}) \left[ 2 E_e^2 - 3 W E_e + W^2 - (2 E_e^2 - E_e W) \frac{(p_e \cdot p_s)}{E_e p_s} \right] \]
\begin{align*}
+ 2\delta_J \sqrt{\frac{J}{J+1}}
\times & \text{Re} \left\{ M_T M_{\delta T} \left[ G_T (G_A^* - W G_{\delta T}^*) - G_\delta G_{\delta T}^* m_e^2 \right. \\
& \left. + G_T (E_e - W - E_e \frac{p_e \cdot p_T}{E_e p_T}) \right. \\
& \left. + G_T G_{\delta T}^* E_e \left(1 + \frac{p_e \cdot p_T}{E_e p_T}\right) \right\}, \\
B^2 = |M_{\delta T}\rangle^2 \lambda_J \langle J & \langle G_A - W G_{\delta T} \rangle^2 \\
+ |G_T|^2 (E_e - W) \left[ 2E_e - W - m_e \frac{E_e}{E_e} - (2E_e - W) \frac{p_e \cdot p_T}{E_e p_T} \right] \\
+ \text{Re} \left[ (G_A - W G_{\delta T}) G_T^* \left[ 3E_e - 2W - m_e \frac{E_e}{E_e} - (E_e - W) \frac{p_e \cdot p_T}{E_e p_T} \right] \right] \\
+ \text{Re} \left[ (G_A - W G_{\delta T}) G_{\delta T}^* \left[ E_e - m_e \frac{E_e}{E_e} - (E_e - W) \frac{p_e \cdot p_T}{E_e p_T} \right] \right] \\
+ \text{Re} (G_T G_{\delta T}^*) \left[ 2E_e - W E_e - 2m_e + \frac{W m_e^2}{E_e} - (2E_e - 3E_e W + W^2) \frac{p_e \cdot p_T}{E_e p_T} \right] \right]\}
\end{align*}

\begin{align*}
+ 2\delta_J \sqrt{\frac{J}{J+1}}
\times & \text{Re} \left\{ M_T M_{\delta T} \left[ G_T (G_A^* - W G_{\delta T}^*) - G_\delta (G_A^* - E_e G_{\delta T}^*) \right. \\
& \left. + G_T (E_e - m_e \frac{E_e}{E_e} - (E_e - W) \frac{p_e \cdot p_T}{E_e p_T}) \right. \\
& \left. - G_T G_{\delta T}^* (E_e - W) \left(1 + \frac{p_e \cdot p_T}{E_e p_T}\right) \right\}
\end{align*}

and

\begin{align*}
D^2 = |M_{\delta T}\rangle^2 \lambda_J \langle J & \langle J (J + 1) - 3 \langle J \cdot J \rangle^2 \rangle \frac{J (J + 1)}{J (2J - 1)} \\
\times \left\{ \text{Im} \left[ (G_A - W G_{\delta T}) G_T^* \right] (j \cdot p_e - p_T) \\
+ \text{Im} \left[ (G_A - W G_{\delta T}) G_{\delta T}^* \right] (j \cdot p_e + p_T) \\
+ \text{Im} (G_T G_{\delta T}^*) (2E_e - W) (j \cdot p_e + p_T) \right\}
\end{align*}
\[ + |M_{oT}|^2 \lambda_{J',J} \]
\[ \times \left\{ \text{Im}\left[ (G_A - W G_{PR}) G_T^* \right] W \right. \]
\[ - \text{Im}\left[ (G_A - W G_{PR}) G_T^* \right] (2E_e - W) \]
\[ - \text{Im}(G_T G_{PR}^*) \left[ 2E_e^3 - 2WE_e + W^2 - m_e^2 \right] \]
\[ - 2(E_e^3 - W E_e) \left( \frac{p_e \cdot p_s}{E_e} \right) \right\} \]
\[ + 2\delta_{J',J} \sqrt{\frac{J}{J+1}} \]
\[ \times \text{Im}\left\{ M_F M_{oT}^* [G_T (G_A^* - W G_{PR}) - m_e^2 G_s G_T^* + G_T G_T^* (2E_e - W)] \right\}, \]

where \( M_F \) and \( M_{oT} \) are the conventional Fermi and Gamow-Teller matrix elements, respectively, the product \( M_F M_{oT}^* \) is defined by
\[ (\Psi(J', M'), \Psi(J, M), \sigma_J \Psi(J', M')) \]
\[ = \delta_{M M'} \delta_{JJ'} M_F M_{oT}^* \]

with the initial nuclear wave function \( \Psi(J, M) \) and the final one \( \Psi(J', M') \), and
\[ \lambda_{J,J'} = \begin{cases} 1, & (J' = J+1) \\ 1/(J+1), & (J' = J) \\ -J/(J+1), & (J' = J-1) \end{cases} \]
\[ \lambda_{J',J} = \begin{cases} 1, & (J' = J-1) \\ 1/(J+1), & (J' = J) \\ -J/(J+1), & (J' = J+1) \end{cases} \]

In order to see the effect of the smaller terms more clearly, we give explicit formulas for the angular distribution of the emitted electron about the initial polarization axis in the cases of the Gamow-Teller transition and the \( \beta \) decay of free neutrons. By integrating Eq. (3) over \( d\Omega_e \) and over the azimuthal angle of the electron and also by neglecting the products of the smaller terms, we have
\[ \omega \langle \langle J' \rangle | E_e, \cos \theta_e \rangle dE_e \frac{d \cos \theta_e}{2} \]
\[ = \frac{1}{2\pi^2} p_e E_e (W - E_e) dE_e \frac{d \cos \theta_e}{2} \]
\[ \times |M_{oT}|^2 \left\{ A_{oT} + \lambda_{J,J'} \frac{\langle J' \rangle}{J} \frac{p_e \cdot B_{oT}}{E_e} \right\} \]
\[ \text{(4)} \]
with
\[ A_{\sigma T} = |G_d|^3 
+ \frac{4}{3} \text{Re}(G_d G_T^*) \left[ 2E_e - W - \frac{m_e^2}{E_e} \right] 
- \frac{2}{3} \text{Re}(G_d G_T^*) \left[ 2W + \frac{m_e^2}{E_e} \right] + A_{\sigma f} \frac{J(J+1) - 3\langle J J \rangle}{J(2J-1)} \]
\[ \times \left[ \text{Re}(G_d G_T^*) - \text{Re}(G_d G_T^{* T}) \right] \]
\[ \times \left[ \frac{1}{3} - \frac{(J\cdot p_e)^3}{p_e^4} \right] \left[ E_e - \frac{m_e^2}{E_e} \right] \]
and
\[ B_{\sigma T} = -|G_d|^3 - \frac{4}{3} \text{Re}(G_d G_T^*) (5E_e - 2W) + \frac{2}{3} \text{Re}(G_d G_T^{* T}) (E_e + 2W) \]
for the Gamow-Teller transition and
\[ \omega \langle \sigma \rangle |E_e, \cos \theta_e \rangle dE_e \frac{d \cos \theta_e}{2} \]
\[ = \frac{1}{2\pi^2} \rho_e E_e (W - E_e)^4 dE_e \frac{d \cos \theta_e}{2} \left\{ A_n + \langle \sigma \rangle \frac{p_e}{E_e} B_n \right\} \]
(5)
with
\[ A_n = |G_T|^3 + 3|G_d|^3 - 2 \frac{m_e^2}{E_e} \text{Re}(G_T G_d^*) \]
\[ + 4 \text{Re}(G_d G_T^*) \left( 2E_e - W - \frac{m_e^2}{E_e} \right) - 2 \text{Re}(G_d G_T^{* T}) \left( 2W + \frac{m_e^2}{E_e} \right) \]
and
\[ B_n = -2|G_d|^3 + 2 \text{Re}(G_T G_d^*) \]
\[ - \frac{4}{3} \text{Re}(G_d G_T^*) (5E_e - 2W) \]
\[ + \frac{4}{3} \text{Re}(G_T G_T^*) (E_e + 2W) \]
\[ + \frac{4}{3} \text{Re}(G_T G_T^* + G_T G_T^{* T}) (E_e - W) \]
for the \( \beta \) decay of free neutrons in which \( J = \sigma/2 \) and \( 3 \langle J J \rangle = J(J+1) \) hold.

§ 3. Discussion

The expression (3) is derived from the general current (1) under the non-relativistic approximation. The isospin of nuclei is not taken into account expli-
citely, so it is a simple generalization of the result for $\beta$ transition within an isomultiplet by Holstein and Treiman.\textsuperscript{5,6)}

In the spectral shape factor $A_{\sigma\tau}$, it should be noted that the term $\text{Re}(G_A G_T^*)$ gives almost no energy dependence in contrast to the strong energy dependence of $\text{Re}(G_T G_T^*)$ and that the latter property can hardly be destroyed by the former even if $G_T$ is of the order of $G_T$.\textsuperscript{6)} The expression $B_{\sigma\tau}$ gives another method for the determination of $G_T$, which is independent of the $f_i$-value method making use of the integrated form of $A_{\sigma\tau}$ as is shown in Eq. (2). However, we cannot expect too much from $B_{\sigma\tau}$ measurement, because it is also dominated by the main term $|G_A|^2$. The main terms $|G_A|^2$ and $\text{Re}(G_T G_A^*)$ in $B_a$ cancel with each other, which meet our initial purpose. But the appearance of extra terms $\text{Re}(G_Y G_T^*)$ and $\text{Re}(G_T G_T^*)$ will make it difficult to fix experimentally the magnitude of $G_T$ from $B_a$ for the time being.

The existence of $G_T$ in nuclear $\beta$ decay does not necessarily imply the existence of it in the primary weak interactions, that is, the $\beta$ decay of the free neutrons and $\Sigma^+ \rightarrow \Lambda e^+ \nu(\bar{\nu})$ decays since there is a possibility that it has its origin in the complexity of the nucleus. We propose, for the determination of $G_T$, to study accurately the distribution of the allowed $\beta$-rays from the polarized nuclei and those from the free polarized neutrons.

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\textbf{References}


\textsuperscript{*)} The result of this paper is a special case of ours and it is obtained from our formulas by setting $J=J'$ and by making all coefficients in Eq. (1) real.