On the Renormalized Random Phase Approximation for Dilute Magnetic Alloys. II

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The Anderson model for dilute magnetic alloys is re-examined in the magnetic limit \( U/\mathcal{A} \gg 1 \) (\( U \) and \( \mathcal{A} \) are the Coulomb energy and the level width at the impurity site, respectively) in the renormalized random phase approximation, which has recently been studied by Hamann and the present author. The result is rather different from previous works and proves that the characteristic temperature \( T_o \) in our model is given by \( T_o \sim \mathcal{A} \cdot (\mathcal{A}/U)^{2.683} \), which is too large in comparison with the corresponding Kondo temperature, and that the magnetic susceptibility \( \chi \) does not show the Curie law but \( \chi \sim 1/T \cdot (T/U)^{0.548} \) at high temperature \( (T \gg T_o) \).

§ 1. Introduction

Recently Hamann\(^1\) and the present author\(^2\) have studied the Anderson model\(^3\) for dilute magnetic alloys in the renormalized random phase approximation (RRPA), which was applied to the Wolff model\(^4\) by Suhl and co-workers.\(^5\) The former authors solved the resultant integral equations approximately but analytically to examine the correspondence between the rigorous results obtained by other methods\(^6\) and the latter authors', which was derived by a computer calculation. In this paper the same problem as in previous papers is again examined almost rigorously in the magnetic limit \( U/\mathcal{A} \gg 1 \), where \( U \) and \( \mathcal{A} \) are the Coulomb energy and the level width at the impurity site, respectively.

The essential points of our result are the following: The characteristic temperature \( T_o \) in our model is given by \( T_o \sim \mathcal{A} \cdot (\mathcal{A}/U)^{2.683} \), which is too large in comparison with the corresponding Kondo temperature.\(^4\) The magnetic susceptibility \( \chi \) is given by \( \chi \sim (1/\mathcal{A}) \cdot (U/\mathcal{A})^{0.688} \) at low temperature \((T \ll T_o)\) and \( \chi \sim 1/T \cdot (T/U)^{0.548} \) at high temperature \((T \gg T_o)\). It is also proved that the approximation which has been applied in the previous work\(^1\)-\(^3\) estimates an effect of the spin-fluctuation on the self-energy too small.

As for the introductory talks and for discussions about the model and the formulation for RRPA, the readers should refer to the previous works,\(^1\)-\(^3\) where they are studied rather in detail. Here we only talk about some preparations to solve the resultant integral equations in § 2. The procedure to solve them and its result at absolute zero temperature and those at finite temperature are
§ 2. Self-consistent equations

Similarly to in Ref. 2, here we also apply the temperature-Green's-function technique. Then we shall start our study by writing down a set of resultant integral equations to be solved:

\[ G_d(i\omega_n) = [i\omega_n - \Sigma_d(i\omega_n) + i\Delta \cdot \text{sgn}(\omega_n)]^{-1}, \]  
(2.1)

\[ \Sigma_d(i\omega_n) = \frac{3}{2} U^1 \cdot T \sum_{\varepsilon_m} G_d(i\omega_n - i\varepsilon_m) \cdot \chi_{+-}(i\varepsilon_m), \]  
(2.2)

\[ \chi_{+-}(i\varepsilon_m) = \chi_0(i\varepsilon_m) \cdot [1 - U \cdot \chi_0(i\varepsilon_m)]^{-1}, \]  
(2.3)

\[ \chi_0(i\varepsilon_m) = -T \sum_{\sigma_n} G_d(i\omega_n) \cdot G_d(i\omega_n + i\varepsilon_m) \]  
(2.4)

with

\[ \omega_n = (2n + 1) \cdot \pi \cdot T, \quad (n: \text{integer}) \]
\[ \varepsilon_m = 2m \cdot \pi \cdot T, \quad (m: \text{integer}) \]
\[ \Delta = \pi \cdot \rho_0 \cdot V, \]

where \( \rho_0 \) and \( V \) are the density of states of the s-electron and the interaction-energy of s-d admixture, respectively (c.f. Ref. 2). By the use of the symmetry relations

\[ G_d(i\omega_n) = -i \cdot \text{sgn}(\omega_n) \cdot |G_d(i|\omega_n|)|, \]
\[ \chi_0(i\varepsilon_m) = \chi_0(i|\varepsilon_m|), \]
\[ \chi_{+-}(i\varepsilon_m) = \chi_{+-}(i|\varepsilon_m|), \]  
(2.5)

(2.2) is rewritten as

\[ \Sigma_d(i\omega_n) = G_d(i\omega_n) \cdot \frac{3}{2} U^1 \cdot T \sum_{|\varepsilon_m| < |\varepsilon_n|} \chi_{+-}(i\varepsilon_m) \]
\[ -i \cdot \text{sgn}(\omega_n) \cdot \frac{3}{2} U^1 \cdot T \sum_{|\varepsilon_m| < |\varepsilon_n|} [\{G_d(i|\varepsilon_m| + i\omega_n)| - |G_d(i|\varepsilon_m|)|]\chi_{+-}(i\varepsilon_m) \]
\[ -i \cdot \text{sgn}(\omega_n) \cdot \frac{3}{2} U^1 \cdot T \sum_{|\varepsilon_m| > |\varepsilon_n|} [\{G_d(i\varepsilon_m + i\omega_n)| - |G_d(i\varepsilon_m)|]\chi_{+-}(i\varepsilon_m) \]  
(2.6)

\[ = \alpha(\omega_n) \cdot G_d(i\omega_n) \cdot \frac{3}{2} U^1 \cdot T \sum_{|\varepsilon_m| < |\varepsilon_n|} \chi_{+-}(i\varepsilon_m), \]  
(2.7)

where \( \alpha(\omega_n) \) is defined by (2.7):

\[ \alpha(\omega_n) = 1 + \frac{\text{the last 2 terms in (2.6)}}{\text{1st term in (2.6)}}. \]  
(2.8)

Hereafter it is assumed that \( \alpha(\omega_n) \) is at most of the order of unity, which will be proved consistently as a result. Introducing a new function \( Q(\omega_n) \),
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\[ Q(\omega_n) = T \sum_{|\epsilon_m| < |\epsilon_n|} \chi_{\pm}(i\epsilon_m), \]  

(2.9)

one obtains easily formal solutions for \( \Sigma_d \) and \( G_d \) from (2.1) and (2.7).\(^7\)

\[ \Sigma_d(i\omega_n) = \frac{-i \cdot \text{sgn}(\omega_n) \cdot 3 U^2 \cdot \alpha(\omega_n) \cdot Q(\omega_n)}{|\omega_n| + \Delta + \sqrt{|\omega_n| + \Delta^2 + 6 U^2 \cdot \alpha(\omega_n) \cdot Q(\omega_n)}}, \]  

(2.10)

\[ G_d(i\omega_n) = \frac{-2i \cdot \text{sgn}(\omega_n)}{|\omega_n| + \Delta + \sqrt{|\omega_n| + \Delta^2 + 6 U^2 \cdot \alpha(\omega_n) \cdot Q(\omega_n)}}. \]  

(2.11)

Again applying the symmetry relations (2.5) to (2.4), one obtains

\[ \chi_0(i\epsilon_m) = 2T \sum_{0 < \epsilon_n < |\epsilon_m|} |G_d(i\omega_n)| \cdot |G_d(i\omega_n + i\epsilon_m)| \]

\[ -T \sum_{0 < \epsilon_n < |\epsilon_m|} |G_d(i\omega_n)| \cdot |G_d(i\epsilon_m - i\epsilon_n)|. \]  

(2.12)

It is also rewritten as follows:

\[ \chi_0(i\epsilon_m) = \chi_0(0) - A\chi_0(i\epsilon_m) \]  

(2.13)

with

\[ \chi_0(0) = 2T \sum_{0 < \epsilon_n} |G_d(i\omega_n)|^2, \]  

(2.14)

\[ A\chi_0(i\epsilon_m) = T \sum_{0 < \epsilon_n < |\epsilon_m|} |G_d(i\omega_n)|^2 \]

\[ + T \sum_{0 < \epsilon_n < |\epsilon_m|} |G_d(i\omega_n)| \cdot \left[ |G_d(i\omega_n) - G_d(i\epsilon_m - i\epsilon_n)| - 2|G_d(i\omega_n + i\epsilon_m)| \right] \]

\[ + 2T \sum_{|\epsilon_m| < \epsilon_n} |G_d(i\omega_n)| \cdot \left[ |G_d(i\omega_n) - G_d(i\epsilon_m)| \right] \]  

(2.15)

\[ = \beta(\epsilon_m) \cdot T \sum_{0 < \epsilon_n < |\epsilon_m|} |G_d(i\omega_n)|^2, \]  

(2.16)

where \( \beta(\epsilon_m) \) is defined by (2.16):

\[ \beta(\epsilon_m) = 1 + \frac{\text{the last 2 terms in (2.15)}}{\text{1st term in (2.15)}}. \]  

(2.17)

Hereafter it is assumed that \( \beta(\epsilon_m) \) is at most of the order of unity, which will be proved consistently as a result. By the use of (2.13), (2.3) is rewritten as

\[ \chi_{\pm}(i\epsilon_m) = \frac{\chi_0(0) - A\chi_0(i\epsilon_m)}{K_F + U \cdot A\chi_0(i\epsilon_m)} \]  

(2.18)

with

\[ K_F = 1 - U \cdot \chi_0(0). \]  

(2.19)

Accordingly (2.9) is written explicitly as

\[ Q(\omega_n) = T \sum_{|\epsilon_m| < |\epsilon_n|} \frac{\chi_0(0) - A\chi_0(i\epsilon_m)}{K_F + U \cdot A\chi_0(i\epsilon_m)}. \]  

(2.20)
After all (2·11), (2·15) and (2·20) constitute a set of self-consistent integral equations by the help of (2·19), (2·14), (2·8) and (2·17). Hereafter similarly to in the previous work, we only consider the problem in the magnetic limit \((U/\Delta \gg 1)\). In this case, (2·19) should read as

\[
\begin{align*}
K_\tau &= 0, \\
\zeta(0) &= \frac{1}{U}.
\end{align*}
\]  

\( (2·21) \)

§ 3. Self-consistent solution \((T=0^\circ K)\)

Now we consider the system at absolute zero temperature. In this case a summation over discrete frequency \(\omega_n\) can be replaced by an integration over continuous frequency \(\omega\),

\[
T \sum_{\omega_n} F(\omega_n) \to \int \frac{d\omega}{2\pi} F(\omega).
\]

To solve the self-consistent integral equations, at first we introduce two characteristic frequencies \(\omega_1\) and \(\omega_\infty\) which are defined by the following equations:

\[
\begin{align*}
\omega_1 &= 6U^2 \cdot \alpha(\omega_1) \cdot Q(\omega_1), \\
\omega_\infty &= 6U^2 \cdot \alpha(\omega_\infty) \cdot Q(\omega_\infty).
\end{align*}
\]

\( (3·1) \)

Now we presume that \(\omega_1 \leq \omega_\infty\), which will be proved consistently as a result. Then the whole frequency range is devided roughly into three parts:

(i) low frequency range \((\omega < \omega_1)\), where \(\Delta \gg \omega\), \(6U^2 \cdot \alpha(\omega) \cdot Q(\omega)\),
(ii) intermediate frequency range \((\omega_1 < \omega < \omega_\infty)\), where \(6U^2 \cdot \alpha(\omega) \cdot Q(\omega) > \Delta, \omega\),
(iii) high frequency range \((\omega_\infty < \omega)\), where \(\omega \geq \Delta, 6U^2 \cdot \alpha(\omega) \cdot Q(\omega)\).

In each of them, a solution will be obtained and finally those three solutions will be joined into a self-consistent solution in order to satisfy proper boundary conditions.

A) Low frequency \((\omega \leq \omega_1)\)

Considering the condition \(\Delta \gg \omega, 6U^2 \cdot \alpha(\omega) \cdot Q(\omega)\), one obtains from (2·11)

\[
G_\Delta(i\omega) \approx -i \cdot \text{sgn}(\omega) \cdot \frac{1}{\Delta}
\]

\( (3·2) \)

and accordingly from (2·16)

\[
\Delta \zeta(i\omega) \sim O\left(\frac{\omega}{\Delta^3}\right).
\]

\( (3·3) \)

The substitution of (3·3) into (2·20) yields

\[
Q(\omega) \approx 2 \cdot \left(\frac{\Delta}{U}\right)^3 \cdot \ln \left| \frac{\omega + T_{\infty}}{T_{\omega}} \right|
\]

\( (3·4) \)
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with

\[ T_s = 2\pi \cdot K_s \cdot \frac{A^4}{U}, \]

\[ K_0 = K_T. \quad (T = 0^\circ K) \]  (3.5)

The order of magnitude of \( \omega_i \) is estimated by the substitution of (3.4) into (3.1) as

\[ \omega_i \sim T_s. \]  (3.6)

B) Intermediate frequency (\( \omega_i < \omega < \omega_n \))

At first we assume that \( \alpha(\omega) \sim \alpha, \quad \beta(\omega) \sim \beta(\alpha \text{ and } \beta \text{ are independent of } \omega) \)

and \( \Delta \chi_0(i\omega) \ll \chi_0(0) \) in this range, which will be justified consistently as a result. Then (2.20) is rewritten as

\[ Q(\omega) = \int_{\sigma_1}^{|\omega|} \frac{d\varepsilon}{\pi U} \cdot \frac{1}{K_0 + U \cdot \Delta \chi_0(i\varepsilon)} + Q_i \]  (3.7)

with

\[ Q_i = Q(\omega_i). \]

By the use of the condition \( 6 \cdot U^3 \cdot \alpha \cdot Q(\omega) \gg A, \omega_i \), (2.16) is also rewritten with the most dominant terms as

\[ \Delta \chi_0(i\omega) \sim \frac{\beta}{3\pi \cdot \alpha \cdot U^3} \cdot \int_{\sigma_1}^{|\omega|} \frac{d\varepsilon}{Q(\varepsilon)} + \Delta \chi_1, \]  (3.8)

where

\[ \Delta \chi_1 = \Delta \chi_0(\omega_i). \]

If another function \( t(\omega) = \frac{\beta}{3\pi \alpha U} \cdot \int_{\sigma_1}^{|\omega|} d\varepsilon \cdot (1/Q(\varepsilon)) \) is introduced, it is straightforward to see that integral equations (3.7) and (3.8) are reduced to a single differential equation as

\[ -\frac{\beta}{3\alpha} \cdot \frac{t''(\omega)}{[t'(\omega)]^2} = \frac{1}{K_0 + U \cdot \Delta \chi_1 + t(\omega)}. \]

The differential equation (3.9) can easily be integrated to give a solution as

\[ t(\omega) = (a \cdot \omega + b)^{\beta/(3\alpha + \beta)} - K_0 - U \cdot \Delta \chi_1, \]

where \( a \) and \( b \) are integration constants which should be determined under the boundary conditions, \( t(\omega_i) = 0 \) and \( Q(\omega_i) = Q_i \). Finally \( Q(\omega) \) is given by

\[ Q(\omega) = A \cdot \left( \frac{\omega}{\pi U} + B \right)^\gamma \]  (3.9)

with

\[ \gamma = \frac{3\alpha}{3\alpha + \beta}, \]  (3.10)
Hereafter for the consistency of the approximation we only take the most dominant terms in every case, discarding trivial factors of the order of unity.

Making use of the above result, one can easily find the following expressions:

\[ G_s(i\omega) \simeq -i \cdot \text{sgn}(\omega) \cdot \frac{1}{U \cdot \sqrt{A}} \cdot \left(\frac{\omega}{U}\right)^{-r/2}, \]  
\[ \chi_s(i\omega) \simeq \frac{1}{A \cdot U} \left(\frac{\omega}{U}\right)^{r-1}, \]  
\[ \chi_{-+}(i\omega) \simeq \frac{A}{U} \left(\frac{\omega}{U}\right)^{r-1}. \]

If these are substituted into (2.8) and (2.17), one obtains the following equations to determine \( \alpha \) and \( \beta \) self-consistently:

\[
\alpha = 1 - \frac{1}{2} \cdot \frac{\sum_{n=1}^{\infty} \left(1 - \left(-1\right)^n \cdot \frac{1}{n - \gamma/2} \cdot \frac{n - 1 + \gamma/2 \cdot \left(n - 2 + \gamma/2 \cdot \cdots \cdot \left(\gamma/2\right)\right)}{n - 1 + \gamma/2} \right)}{n!},
\]

\[
\beta = 2 - \frac{1 - \gamma}{1 - \gamma/2} + (1 - \gamma) \cdot \frac{\sum_{n=1}^{\infty} \left(1 - \left(-1\right)^n \cdot \frac{2 \cdot \left(-1\right)^n}{n + 1 - \gamma/2} \cdot \frac{2}{n - 1 + \gamma/2} \cdot \left(n - 2 + \gamma/2 \cdot \cdots \cdot \left(\gamma/2\right)\right)}{n!},
\]

while \( \gamma \) is given by (3.10). These self-consistent equations can be easily solved numerically to give

\[
\alpha = 0.813,
\]

\[
\beta = 2.049,
\]

\[
\gamma = 0.543.
\]

Finally the order of magnitude of \( \omega_u \) is estimated by the substitution of (3.9) into (3.1) as

\[ \omega_u \sim U \cdot A^{1/(3 - \gamma)}. \]  

If \( \omega_u \) is determined by the condition \( \chi_s(i\omega_u) \sim \chi_s(0) = 1/U \), one obtains another expression for \( \omega_u \):

\[ \omega_u \sim U \cdot A^{1/(3 - \gamma)}, \]

which is proved to be the same as (3.15), because it is verified as a result that \( A \) must be of the order of unity.

C) High frequency \((\omega \gg \omega_u)\)

Considering the condition \( \omega \gg A, 6U^2 \cdot \alpha(\omega) \cdot Q(\omega) \), one obtains from (2.11)
\[ G_{\varepsilon}(i\omega) \simeq -i \cdot \frac{\text{sgn}(\omega)}{|\omega|} \cdot \frac{1}{|\omega|} \]  

(3·16)

and accordingly from (2·12)

\[
\chi_0(i\omega) \simeq \int_{-\infty}^{\infty} d\varepsilon \cdot \frac{1}{\pi |\omega|} |G_{\varepsilon}(i\varepsilon)| + \int_{|\omega|}^{\infty} d\varepsilon \cdot \frac{1}{\pi \cdot \varepsilon \cdot (|\omega| + \varepsilon)} \\
- \int_{-\infty}^{\infty} d\varepsilon \cdot \frac{1}{\pi |\omega|} |G_{\varepsilon}(i\varepsilon)| + \int_{|\omega|}^{\infty} d\varepsilon \cdot \frac{1}{2\pi \varepsilon (|\omega| - \varepsilon)} \\
- \int_{|\omega|}^{\infty} d\varepsilon \cdot \frac{1}{2\pi \varepsilon} |G_{\varepsilon}(i|\omega| - i\varepsilon)|. 
\]  

(3·17)

The most dominant terms of \( O(1/\omega) \) in (3·17) are cancelled out with each other only to leave terms of \( O(\omega_u/\omega^2) \). The substitution of (3·17) into (2·20) yields

\[ Q(\omega) \simeq Q_u + O(\frac{\omega_u}{\omega}) \]  

(3·18)

with

\[ Q_u = Q(\omega_u). \]

D) Self-consistent condition

By the use of the foregoing results (3·2), (3·12) and (3·16), (2·14) can be easily calculated to give

\[ \chi_0(0) = \chi_0^{(1)} + \chi_0^{(2)} + \chi_0^{(3)}, \]  

(3·19)

where

\[ \chi_0^{(1)} = \int_{0}^{\infty} d\omega \cdot \frac{1}{\pi} |G_\varepsilon(i\omega)|^2 \sim \frac{\omega_1}{A^2}, \]

\[ \chi_0^{(2)} = \int_{\omega_1}^{\infty} d\omega \cdot \frac{1}{\pi} |G_\varepsilon(i\omega)|^2 \sim \frac{1}{A} \cdot \frac{\omega_u}{U^2}, \]

\[ \chi_0^{(3)} = \int_{\omega_u}^{\omega_1} d\omega \cdot \frac{1}{\pi} |G_\varepsilon(i\omega)|^2 \sim \frac{1}{\omega_u}. \]

Substituting (3·19) into (2·19) and taking into account the expressions for \( \omega_1 \) and \( \omega_u \) (3·6) and (3·15), one obtains a resultant self-consistent condition

\[ 1 - [O(K_0) + O(A^{-(1-\tau)/(3-\tau)}) + O(A^{-1/(3-\tau)})] = K_0, \]  

(3·20)

while \( A \) is defined by (3·11). Therefore, considering that \( \Delta \chi_i \) and \( Q_i \) are estimated roughly by (3·3) and (3·4) to be respectively of \( O(K_0/U) \) and of \( O(\Delta'/U^2) \) and accordingly \( A \sim O((\Delta/U)^{(1-\tau)} \cdot K_0^{-\tau}) \), one further obtains the final solution from (3·20),

\[ K_0 \sim \left( \frac{\Delta}{U} \right)^{2(1-\tau)/\tau} = \left( \frac{\Delta}{U} \right)^{1.480} \]  

(3·21)
and accordingly from (3·5)

$$T_t \propto A \cdot (A/U)^{3-\gamma/T} = A \cdot \left(\frac{A}{U}\right)^{3.685}.$$  \hspace{1cm} (3·22)

If it is assumed that a static magnetic susceptibility $\chi$ is described mainly by $\chi_{\tau-}(0)^9$, then it is given by (2·18) as

$$\chi \approx \chi_{\tau-}(0) \approx \left(\frac{U}{A}\right)^{3-\gamma/T} \cdot \frac{1}{A} = \left(\frac{U}{A}\right)^{3.685} \cdot \frac{1}{A}.$$  \hspace{1cm} (3·23)

§ 4. Self-consistent solution (finite temperature)

Here we study a system at finite temperature $T(0 \leq T < A)$. At first it should be noticed that both in the intermediate frequency range ($\omega \ll \omega_n \ll \omega_{in}$) and in the high frequency range ($\omega_{in} \ll \omega_n$), the problem can be manipulated with the most dominant terms exactly in the same manner as in the previous section, because a replacement of a summation over discrete frequencies ($\omega_n$) by an integration over continuous frequencies ($\omega$) only leads to a relative error of the order of $T/A$ or $T/U$ at most in each case. Therefore, it is sufficient for us in this section to confine ourselves to the study of the low frequency range and to know the magnitude of $\omega_n$ or $Q_n$ at finite temperature as is inferred from the manipulation of the problem in § 3.

Considering that $\omega_n \ll A$ in our concerned frequency range and introducing new notations

$$Q_n = Q((2n+1)\pi T), \quad \Delta \chi_n = \Delta \chi((2n+1)\pi T),$$

$$\alpha_n = \alpha((2n+1)\pi T), \quad \beta_n = \beta((2n+1)\pi T),$$

one obtains iteratively from (2·11), (2·16) and (2·20)

$$\Delta \chi_0 = 0, \quad Q_0 = \frac{T}{K_T \cdot U},$$

$$\Delta \chi_1 = \frac{4\beta \cdot T}{(A + \sqrt{A^4 + 6U^4 \cdot \alpha_0 \cdot Q_0})^3},$$

$$Q_1 = Q_0 + \frac{2T}{K_T \cdot U} \cdot \left(1 + \frac{\Delta \chi_1}{K_T \cdot U}\right)^{-1},$$

and generally

$$\Delta \chi_n = \Delta \chi_{n-1} + \frac{4\beta_n \cdot T}{(A + \sqrt{A^4 + 6U^4 \cdot \alpha_{n-1} \cdot Q_{n-1}})^3},$$

$$Q_n = Q_{n-1} + \frac{2T}{K_T \cdot U} \left(1 + \frac{\Delta \chi_n}{K_T \cdot U}\right)^{-1}. \hspace{1cm} (4·1)$$
Now to advance our study, we shall divide the whole temperature range roughly into two; the low temperature range \((T \ll T_s)\), where \(A^a \gg 6U^a \alpha_s Q_s\) and the high temperature range \((T_s \ll T \ll D)\), where \(6U^3 \alpha_s Q_s \gg D\).

(i) Low temperature \((T \ll T_s)\)

Making use of the condition \(A^a \gg 6U^a \alpha_s Q_s\), one obtains from (2.11)

\[
G_a(i\omega_n) \sim -i \cdot \text{sgn}(\omega_n) \cdot \frac{1}{A}
\]

and from (4.1)

\[
Q(\omega_n) = \frac{1}{\beta} \cdot \left(\frac{A}{U}\right)^2 \left[\frac{\pi T}{T_s} + \psi\left(\frac{\omega_n}{2\pi T} + \frac{1}{2} + \frac{T_s}{2\pi T}\right) - \psi\left(1 + \frac{T_s}{2\pi T}\right)\right]
\]

with \(T_s = 2\pi K_r \cdot (A^a / U)\), where it has been assumed that \(\beta_n = \beta\) (independent of \(n\)) and \(\psi(x)\) is a digamma function.

By the use of the above result and (3.1), \(\omega_l\) is estimated as

\[
\omega_l \sim T_s,
\]

when

\[
Q_l = Q(\omega_l) \sim \left(\frac{A}{U}\right)^2, \\
\Delta \chi_l = \Delta \chi(\omega_l) \sim \frac{T_s}{A}
\]

After all one finds that \(K_r = K_q\) and accordingly obtains exactly the same result as in the previous section, (3.21) and (3.23), with respect to the most dominant terms.

(ii) High temperature \((T_s < T \ll D)\)

By the use of the condition \(6U^3 \alpha_s Q_s \gg D\), it is obtained from (2.11) that

\[
G_a(i\omega_n) \sim -i \cdot \text{sgn}(\omega_n) \cdot \frac{1}{U \cdot \sqrt{Q(\omega_n)}}
\]

In this case, the low frequency range is missing and the characteristic frequency \(\omega_l\) should be supposed of the order of the temperature

\[
\omega_l \sim T,
\]

when

\[
Q_l = Q(\omega_l) \sim Q_s = \frac{T}{K_r \cdot U}, \\
\Delta \chi_l = \Delta \chi(\omega_l) \sim \frac{K_r}{U}
\]
By the use of these results the self-consistent condition (2.21) can be evaluated analogously as in § 3(D) to give

\[ K_T \sim \left( \frac{T}{U} \right)^{1-\gamma} = \left( \frac{T}{U} \right)^{0.437} \quad (4.9) \]

Correspondingly a magnetic susceptibility \( \chi \) is given by

\[ \chi \sim \chi_+(0) = \frac{1}{K_T \cdot U} \sim \frac{1}{T} \left( \frac{T}{U} \right)^{1-\gamma} = \frac{1}{T} \left( \frac{T}{U} \right)^{0.543} \quad (4.10) \]

§ 5. Concluding remarks

In the previous sections we have obtained an almost rigorous solution in RRPA except trivial factors of \( O(1) \). In comparison with our results one can easily find that a fundamental mistake was made in previous works.1-7 In the latter it was assumed a priori that the spin-fluctuation propagator \( \chi_+(i\omega_n) \) (2.18) must be given by a Lorenzian form \( \zeta/(\omega_n + T_s) \), when \( \zeta \) and \( T_s \) were frequency-independent parameters given by

\[
\begin{align*}
\zeta &= U^{-1} \left. \left( \frac{d\chi_+(\omega_n)}{d\omega_n} \right) \right|_{\omega_n = 0}^{-1}, \\
T_s &= K_T \cdot U^{-1} \left. \left( \frac{d\chi_+(\omega_n)}{d\omega_n} \right) \right|_{\omega_n = 0}^{-1},
\end{align*}
\quad (5.1)
\]

which had been due to an approximation for \( \Delta^2 \chi_+(i\omega_n) \),

\[ \Delta^2 \chi_+(i\omega_n) \approx \omega_n \cdot \left. \left( \frac{d\chi_+(\omega_n)}{d\omega_n} \right) \right|_{\omega_n = 0}^{-1} \quad (\text{for } 0 \leq \omega_n \leq U) \quad (5.2) \]

However it is clear that the approximation (5.2) is not consistent in our model (RRPA), because the resultant expression for \( \Delta^2 \chi_+(i\omega_n)/\omega_n \) is highly frequency-dependent as is seen in (3.13), and that it also underestimates the effect of the spin-fluctuation on the self-energy by an order of magnitude.

We note in passing that such a comment affects also some other work, where it has been assumed that a spin-fluctuation propagator must be given by such a Lorenzian form.

Finally we discuss the meaning of our own results. The facts that the temperature \( (T_n) \) characteristic of the spin-fluctuation is too large in comparison with its corresponding Kondo temperature \( (T_0) \) and that at high temperature \( (T \gg T_0) \) the magnetic susceptibility is given by (4.10) but not by a Curie law8 may be thought to mean that the RRPA can only partially describe an effect of the localized electron spin. The correspondence between the numerical results of Suhl and co-workers9 and ours can be interpreted in the same manner as in the previous work8 since the former does not seem to depend upon the critical behaviour of the system in the magnetic limit \( (U/\pi d) \gg 1 \).
On the Renormalized Random Phase Approximation

To describe an effect of the localized electron spin more completely, it is necessary in our model further to include a renormalized vertex-function instead of a bare one, as is pointed out in several papers. Such a problem will be discussed elsewhere.

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References

7) The similar type of a solution is led originally by T. Kitamura in Prog. Theor. Phys. 47 (1972), 765.