**A THEORETICAL STUDY OF GASEOUS HOMEOSTASIS IN THE MAGILL CIRCUIT**

C. M. Conway

**SUMMARY**

Equations have been derived to determine the alveolar gas concentrations which occur when the Magill (Mapleson A) circuit is used with a low fresh gas flow. Alveolar oxygen and carbon dioxide concentrations are determined by the fresh gas flow and composition, carbon dioxide output and oxygen uptake. Gas mixing within the circuit and alterations in the inspired gas concentrations do not affect the final equilibrium. If oxygen uptake and carbon dioxide output are constant, the alveolar gas concentrations are unaffected by alterations in ventilation.

The economical behaviour of the Magill (Mapleson A) circuit during spontaneous ventilation is well recognized. Because of the relative positions of the components, alveolar gas is vented selectively from the circuit when the expiratory valve opens in the latter part of expiration. Mapleson (1954) showed that if the fresh gas flow were equal to, or greater than, the subject’s alveolar ventilation, no rebreathing of alveolar gas would occur, all the alveolar gas being vented from the system. The validity of this analysis has been supported in clinical studies (Kain and Nunn, 1968; Norman, Adams and Sykes, 1968).

Mapleson (1958), analysing the behaviour of this circuit when the fresh gas flow did not exceed the alveolar ventilation, showed that the alveolar carbon dioxide concentration was a function of the fresh gas flow and the carbon dioxide output. He deduced the relationship:

\[ F_{A,CO_2} = V_{CO_2}/V_F \]

A similar relationship was deduced by Nunn and Newman (1964). These authors, like Mapleson, emphasized that under low-flow conditions \( F_{A,CO_2} \) was not influenced by alterations in ventilation.

The present analysis was stimulated by the results of a detailed study of gaseous homeostasis in the Magill circuit with low fresh gas flows (Conway et al., 1976). It was noted that, despite marked alterations of inspired gas concentrations and ventilation, when the fresh gas flow did not exceed the predicted alveolar ventilation, there were only slight changes in the end-tidal oxygen concentrations.

The following discussion assumes that the subject is breathing spontaneously, a steady state is present, and the fresh gas consists of a mixture of oxygen and nitrogen. There are three separate analyses, assuming first that all gas remaining within the circuit is completely mixed, second that no longitudinal gas mixing exists within the circuit, and last that partial and variable degrees of gas mixing occur.

**NOMENCLATURE**

Standard nomenclature has been used throughout, with the addition of the following:

- \( V_F, V_f \) volume of fresh gas flowing into the circuit per unit time and per respiratory cycle.
- \( V_V, V_v \) volume of gas flowing through the expiratory valve per unit time and per respiratory cycle.
- \( V_{AR}, V_{AR} \) volume of alveolar gas retained in the circuit at end-expiration per unit time and per respiratory cycle.
- \( F_{Fx} \) fractional concentration of component \( x \) in fresh gas.

**ASSUMPTIONS**

1. Fresh gas enters the circuit at a constant flow, \( V_F \). In each respiratory cycle a volume of fresh gas, \( V_F \), enters the circuit. This gas will have the composition \( F_{F,O_2} \, F_{F,N_2} \). Fresh gas contains no carbon dioxide.
2. A steady state of respiration exists. Each successive respiratory cycle is identical, and there is no net exchange of inert gas.
3. The circuit has a constant maximum volume. Gas added in excess of this volume will be vented through the expiratory valve.
4. \( V_A \geq V_F \geq (V_{O_2} - V_{CO_2}) \).
5. The subject is breathing spontaneously.
COMPOSITION OF VENTED AND RETAINED GASES

At equilibrium a volume of gas, $V_v$, will be vented from the circuit during each respiratory cycle. This venting will occur in expiration. When the fresh gas flow equals the minute volume the vented gas will initially be alveolar gas, followed by deadspace gas and fresh gas. When the fresh gas flow does not exceed the alveolar ventilation volume, alveolar gas alone will pass through the expiratory valve.

If the inspired and expired alveolar ventilatory volumes were equal, the volume of gas leaving the circuit per breath ($V_V$) would be equal to the fresh gas flow into the circuit during that breath ($V_F$). Because inspired and expired alveolar ventilatory volumes are usually unequal, $V_V$ is better given by:

$$V_V = V_F - V_A + V_A E$$

When $V_F < V_A$, this vented gas will consist entirely of alveolar gas derived from expired alveolar air, having the composition $F_{A_O_2}, F_{A_CO_2}, F_{A_N_2}$. Under these circumstances a fraction of expired alveolar gas of the same composition, and volume $V_A R$, will be retained within the circuit at end-expiration and form part of the succeeding inspirate. Expired alveolar volume is given by the sums of vented and retained gas volumes. Thus $V_A R$ is given by:

$$V_A R = V_A E - V_V$$

$$= V_A E - (V_F - V_A + V_A E)$$

$$= V_A I - V_F$$

EQUILIBRIUM WITH COMPLETE GAS MIXING

If all gas within the circuit at end-expiration is mixed, the composition of the succeeding inspirate will be between that of the fresh and alveolar gases. At equilibrium the gases contributing to the mixture may be considered to be fresh gas, at a flow of $V_F$, and alveolar gas at a flow of $(V_A I - V_F)$. The initial gas volume within the circuit and the contribution of deadspace gas may be ignored, because at equilibrium these components adopt the composition of mixed gas. The fractional inspired concentrations of oxygen and carbon dioxide will be:

$$F_{I_O_2} = \frac{V_F \cdot F_{F_O_2} + (V_A I - V_F) \cdot F_{A_O_2}}{[V_F + (V_A I - V_F)]}$$

$$= F_{A_O_2} + \frac{V_F}{V_A I} (F_{F_O_2} - F_{A_O_2})$$ (1)

At equilibrium there will be no net inert gas exchange, thus:

$$V_A I \cdot F_{I_N_2} = V_A E \cdot F_{A_N_2}$$ (3)

$$V_A E \cdot F_{A_N_2} = V_A E (1 - F_{A_O_2} - F_{A_CO_2})$$ (4)

$$V_A I \cdot F_{I_N_2} = V_A I (1 - F_{I_O_2} - F_{I_CO_2})$$

Substituting (1) and (2), this reduces to:

$$V_A I \cdot F_{I_N_2} = V_A I \left[1 - \frac{V_F}{V_A I} \left(\frac{F_{F_O_2} - F_{A_O_2} - F_{A_CO_2}}{1 - F_{A_O_2} - F_{A_CO_2}}\right)\right]$$ (5)

Substituting (4) and (5) in (3) gives:

$$V_A E = V_A I \left[1 - \frac{V_F}{V_A I} \left(\frac{F_{F_O_2} - F_{A_O_2} - F_{A_CO_2}}{1 - F_{A_O_2} - F_{A_CO_2}}\right)\right]$$ (6)

The magnitude of the oxygen and carbon dioxide exchanges is given by the differences in the mass of these gases between inspired and expired alveolar gas

$$V_O_2 = V_A I \cdot F_{I_O_2} - V_A E \cdot F_{A_O_2}$$

$$V_CO_2 = V_A E \cdot F_{A_CO_2} - V_A I \cdot F_{I_CO_2}$$

Substituting equations (1), (2) and (6) in these expressions gives:

$$V_O_2 = V_F \left(\frac{F_{F_O_2} - F_{A_O_2} - F_{F_O_2} \cdot F_{A_CO_2}}{1 - F_{A_O_2} - F_{A_CO_2}}\right)$$ (7)

$$V_CO_2 = V_F \cdot F_{A_CO_2} \left(\frac{1 - F_{F_O_2}}{1 - F_{A_O_2} - F_{A_CO_2}}\right)$$ (8)

The respiratory exchange ratio $R$ is the ratio of carbon dioxide output to oxygen uptake:

$$V_CO_2 = R \cdot V_O_2$$

Substituting (7) and (8) and solving for $F_{A_O_2}$:

$$F_{A_O_2} = F_{F_O_2} - F_{A_CO_2} \left(\frac{1 - F_{F_O_2}}{R}\right)$$ (9)

EQUILIBRIUM WITH NO GAS MIXING

Under circumstances of no gas mixing, the first portion of inspired gas will consist of alveolar gas of
volume \((V_{AI} - V_F)\), and the remaining alveolar inspirate will be fresh gas of volume \(V_{AI} - (V_{AI} - V_F) = V_F\). The mean inspired gas concentrations will be given by:

\[
F_{IO_2} = \frac{(V_{AI} - V_F) \cdot FA_{O_2} + V_F \cdot FF_{O_2}}{V_{AI}}
\]

\[
= FA_{O_2} + \frac{V_F}{V_{AI}} (FF_{O_2} - FA_{O_2})
\]

\[
F_{IC_02} = \frac{(V_{AI} - V_F) \cdot FA_{C_02}}{V_{AI}}
\]

\[
= FA_{C_02} \left(1 - \frac{V_F}{V_{AI}}\right)
\]

As these expressions are identical to (1) and (2) above, the analysis with no gas mixing has the same solution as that with total gas mixing. Thus:

\[
FA_{O_2} = FF_{O_2} - FA_{C_02} \left(FF_{O_2} + \frac{1 - FF_{O_2}}{R}\right)
\]

**EQUILIBRIUM WITH PARTIAL GAS MIXING**

Let a fraction of retained alveolar gas, \(a(V_{AR})\), equal to \(a(V_{AI} - V_F)\), mix with a fraction of fresh gas, \(\beta V_F\), within the system. The inspirate will consist of an initial portion of alveolar gas, a mixed fraction and a final aliquot of fresh gas. Thus:

\[
V_{AI} = (1 - \alpha)(V_{AI} - V_F) + \alpha(V_{AI} - V_F) + \beta V_F
\]

The mean inspired oxygen concentration will be:

\[
F_{IO_2} = \left[(1 - \alpha)(V_{AI} - V_F) \cdot FA_{O_2} + \alpha(V_{AI} - V_F) \cdot FA_{O_2} + \beta V_F \cdot FF_{O_2} + (1 - \beta)V_F \cdot FF_{O_2}\right]/V_{AI}
\]

\[
= FA_{O_2} + \frac{V_F}{V_{AI}} (FF_{O_2} - FA_{O_2})
\]

which is the same as in equation (1). Similarly, the mean inspired carbon dioxide concentration is identical with that deduced in equation (2). Thus, once again, the alveolar oxygen concentration will be given by equation (9).

**DISCUSSION**

The above analyses can be seen to follow the traditional lines of the alveolar air equation. Whilst the concept of "ideal" alveolar gas is, of necessity, simplified and whilst the analysis in this form demands limiting assumptions as to inert gas equilibrium and the uniformity of ventilation, the use of "ideal" alveolar gas concentrations is accepted as being the standard practicable approach to considerations of alveolar gaseous homeostasis.

The other constraints on this analysis are few. The assumption of a constant maximum volume of the circuit requires that the expiratory valve, when open, will pass varying flows of gas without a change in the pressure decrease across it. No assumptions need be made concerning the pressure at which this valve opens. Variations in valve opening pressure will produce variations in the pressure within the circuit and therefore of the circuit's limiting volume, but at equilibrium will not alter the identity or quantity of gas vented from the circuit. That unmixed alveolar gas alone is vented from the circuit at low fresh gas flows is an essential feature of the Magill circuit, and differentiates it from other semi-closed rebreathing systems. The behaviour of the system is identical if gas mixes within the circuit or if no gas mixing occurs. Any intermediate degree of mixing within the circuit will produce the same end-result.

Whilst, in this analysis, the respired mixture is assumed to consist of oxygen, carbon dioxide and nitrogen alone, it can be extended to include the presence of anaesthetic gases. At equilibrium there will be no net exchange of anaesthetic, thus the fractional concentration of anaesthetic gases may be considered as part of the nitrogen fraction.

If there is no rebreathing of alveolar gas, and thus fresh gas alone forms the inspired mixture, equation (9) reduces to the standard form of the alveolar air equation:

\[
FA_{O_2} = FF_{O_2} - FA_{C_02} \left(FF_{O_2} + \frac{1 - FF_{O_2}}{R}\right)
\]

In the presence of rebreathing of alveolar gas, the alveolar oxygen concentration will be, from equation (9), a function of \(FA_{C_02}\) and be independent of \(FI_{O_2}\), \(FI_{C_02}\) or alveolar ventilation. However, \(FA_{C_02}\) itself will be determined by the fresh gas flow and ventilation. At equilibrium, carbon dioxide output will be represented by the mass of carbon dioxide in gas vented from the system. Thus:

\[
\dot{V}_{C_02} = \dot{V}_F \cdot FA_{C_02}
\]

\[
= (\dot{V}_F - \dot{V}_{AI} + \dot{V}_{AE}) \cdot FA_{C_02}
\]

Assuming \(\dot{V}_{C_02}\) and \((\dot{V}_{AI} - \dot{V}_{AE})\) to be constant, \(FA_{C_02}\) will be determined completely by \(\dot{V}_F\). Similarly, at equilibrium \(FA_{O_2}\) must be a function of the quality and quantity of gas vented from the circuit, and these are independent of ventilation or of the composition of the inspired gas.
The failure of mixing of gases within the circuit to affect the alveolar gas concentrations may be considered an unexpected outcome of this analysis. A more general approach to the system under consideration is to regard it as consisting of three regions—a fresh gas supply which is fixed and under control, an intermediate subject–circuit complex, and an external environment to which is added gas vented from the circuit. The composition and mass of vented gas are determined by the fresh gas flow and, because of the characteristics of this particular circuit, are unaffected by perturbations within the circuit. Thus as the final region of the system reaches an equilibrium state which is independent of the behaviour of the intermediate region, it may be expected that the equilibrium state of the subject will also be determined by the fresh gas supply and be independent of changes occurring within the circuit.

Because the final equilibrium state of the Magill circuit is independent of gas mixing within the circuit or of changes in the subject’s ventilation, the entire foregoing analysis can be undertaken by considerations of the fresh gas flow and vented gas alone.

In the absence of inert gas exchange, the masses of nitrogen added to and leaving the system must be equal. Thus:

\[ V_F \cdot F_{F_{N_2}} = (V_F - V_{AI} + V_{AE}) \cdot F_{A_{N_2}}, \]

or

\[ V_F(1 - F_{F_{O_2}}) = (V_F - V_{AI} + V_{AE})(1 - F_{A_{O_2}} - F_{A_{CO_2}}) \]

From this equation an expression for \( V_{AE} \) can be determined. Similarly oxygen uptake and carbon dioxide output are given by:

\[ V_{O_2} = V_F \cdot F_{F_{O_2}} - (V_F - V_{AI} + V_{AE}) \cdot F_{A_{O_2}} \]
\[ V_{CO_2} = (V_F - V_{AI} + V_{AE}) \cdot F_{A_{CO_2}} \]

Substitution of these equations to derive the respiratory exchange ratio leads to the same solution for \( F_{A_{O_2}} \) as in equation (9).

These expressions apply to the Magill circuit at a low fresh gas flow because the gas vented from the system is always alveolar gas, the mass of which is independent of the behaviour of the system.

In this analysis changes in oxygen uptake and carbon dioxide output have not been considered. As a result of inspired carbon dioxide concentrations being greater than zero, ventilation will be stimulated. Because of increased muscle work increased ventilation will tend to increase the oxygen uptake and the carbon dioxide output. It is well known that, whilst the oxygen cost of quiet breathing may be as little as 0.25 ml oxygen per litre ventilation (Milic-Emili and Petit, 1960), increased ventilation considerably increases this oxygen cost, and this increase will be reflected in an increased carbon dioxide production.

At constant values of \( V_F \) and of \( R \), as \( V_{CO_2} \) increases, so will \( (V_{AI} - V_{AE}) \). Thus two factors cause \( F_{A_{CO_2}} \) to increase and \( F_{A_{O_2}} \) to decrease. Conway and colleagues (1976) have shown gradual progressive increases in \( F_{A_{CO_2}} \) using a Magill circuit in spontaneously breathing volunteers at a fresh gas flow held constant at a value equal to or just less than predicted \( V_A \). These changes were associated with increasing ventilation and could be ascribed to an increased work of ventilation and, therefore, an increase in carbon dioxide output.

Gas mixing within the circuit, whilst having no effect on the state of equilibrium, may affect the time pattern of alveolar gas composition, and the consequent alterations in oscillating gas tensions within arterial blood might affect the level of ventilatory response to rebreathing. With no gas mixing, inspired gas will have the composition of alveolar gas at the start of inspiration, decreasing suddenly to equal that of fresh gas. Changes in the carbon dioxide concentration will be greater here than if inspired gas is mixed and has a constant composition. Such “tube-breathing” with marked oscillations in carbon dioxide concentration has been shown to enhance the ventilatory response to carbon dioxide (Fenner, Jansson and Avery, 1968). However, Cunningham (1974) pointed out that these effects are often associated with mismatching of \( P_{A_{O_2}} \) and are of significance only if the subject breathes a hypoxic mixture. In the presence of adequate \( P_{A_{O_2}} \) values the ventilatory response, and thus the carbon dioxide output at equilibrium, should not be greatly affected by the degree of gas mixing within the circuit.

The treatment of the Magill circuit in this study applies only to a spontaneously breathing subject. Under conditions of controlled ventilation, gas loss from the circuit occurs mainly during inspiration, and the gas vented is not specifically alveolar gas. Replacement of the usual simple expiratory valve of the Magill circuit by a valve which does not permit gas venting during inspiration would allow this simple analysis to be extended to include the passively ventilated subject.

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REFERENCES


ÉTUDE THEORIQUE DE L’HOMEOEASE GAZEUSE DANS LE CIRCUIT DE MAGILL

RESUME
On a établi des équations pour déterminer les concentrations de gaz alvéolaire qui se produisent lorsque l’on utilise le circuit de Magill (Mapleson A) avec un faible débit de gaz frais. Les concentrations d’oxygène alvéolaire et de gaz carbonique sont déterminées par le débit et la composition du gaz frais, par la production de gaz carbonique et par la consommation d’oxygène. Le mélange de gaz à l’intérieur du circuit et les changements dans les concentrations de gaz inspiré n’affectent pas l’équilibre final. Tant que la consommation d’oxygène et la production de gaz carbonique restent constantes, les concentrations de gaz alvéolaire ne sont pas affectées par les variations de ventilation.

UN ESTUDIO TEORICO DE HOMEOSTASIA GASEOSA EN EL CIRCUITO MAGILL

SUMARIO
Se han derivado ecuaciones para determinar las concentraciones de gas alveolar producidas cuando se usa el circuito Magill (Mapleson A) con un flujo bajo de gas fresco. Las concentraciones de oxígeno alveolar y anhídrido carbónico se determinan a partir del flujo y composición del gas fresco, producción de anhídrido carbónico y captación de oxígeno. La mezcla de gas dentro del circuito y las alteraciones en las concentraciones del gas inspirado no afectan el equilibrio final. Si sean constantes la captación de oxígeno y la producción de anhídrido carbónico, las concentraciones de gas alveolar no se ven afectadas por alteraciones en la ventilación.