Meson Photoproduction in the Statistical Approach to Dual Hadrodynamics

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We study the application of the dual theory of hadron electromagnetic currents to meson photoproduction. The “statistical approximation” of Chang, Freund and Nambu to the dual resonance model is used for calculating the Born approximation Feynman graphs. A technique for constructing gauge-invariant amplitudes in the limit of statistical approximation to the dual amplitudes is developed. It is found that the vector-meson photoproduction amplitude in this model has similar properties to those of the vector-meson dominance model based on the photon vector analogy. The model developed is applied to data of the process $\gamma p \rightarrow \rho p$.

The fit to $\rho$ photoproduction differential cross-section allows us to estimate the charge distribution weight function of the dual hadrodynamics.

§ 1. Introduction

Since the inception of the dual resonance model for the strong amplitude, several attempts have been made to incorporate electromagnetic interactions. One class of attempts consists of satisfying the analyticity properties that would be required of a two-current amplitude by Veneziano-like ansätze. Another class of attempts consists of taking the structure of hadrons implied by the factorization of the dual model in terms of harmonic oscillators through a Lagrangian in an internal space and introducing the electromagnetism through a minimal coupling in the Lagrangian. Such a theory can be made formally gauge-invariant by choosing a non-local interaction in the internal space; however, the actual gauge invariance of the result also hinges on a definite prescription for analytic continuation, which we have described in Appendix. When applied to deep inelastic scattering, such theories give scaling form factors for $\nu W$, which agree well with experimental data, although the function $W$, in such theories does not usually scale. These form factors are also Gaussian, typical of harmonic oscillators. The unsatisfactory nature of such form factors has led to off-mass-shell modification of the photon vertex function. However, if such a theory is applied to vector-meson photoproduction (or to mesons of arbitrary spin) no such off-mass-shell ambiguity remains and one can obtain more clear-cut predictions.

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from the model.

The popular model for vector-meson photoproduction is the so-called vector-meson dominance model based on the photon-vector analogy. These models relate vector-meson photoproduction to hadronic elastic scattering and require exchange of meson trajectories in the $t$-channel, but cannot furnish the residue functions of the 12 independent amplitudes. Precise measurements of the differential cross-section and element of density matrices of the photoproduced vector meson exist up to the incident photon energy of 18 GeV. Most of the features suggested by the hypothesis of vector dominance coupled with quark model are borne out by experimental data. A theoretical derivation of such a model in a definite dynamical scheme has so far been elusive. The dual theory of hadrodynamics provides a definite working model for studying vector-meson photoproduction. This model itself is highly restricted in its applications to physically observed processes as it does not incorporate spin and internal quantum numbers. We use this model, however, hoping that the features predicted by it may resemble those which would be based on a more realistic future theory of hadrons.

In § 2 we briefly review the dual theory of hadrodynamics and give the vertex for the coupling of an external meson of spin $j$ to the states of the dual model. We mention there the "statistical approximation" to the dual model of Chang, Freund and Nambu. We use it for the propagators in the calculations of the Feynman graphs. In § 3 we give the calculation of the amplitude for photoproduction of a meson of spin $j$, mass $\mu$, within the framework of the dual model. In § 4 we point out the need of adding a diffractive contribution to the Born approximation graphs calculated in § 3. It is well known that the Pomeranchuk trajectory is excluded from the Born approximation to the dual resonance model. In taking into account the diffraction contribution we use the well-accepted hypothesis of s-channel helicity conservation near the forward direction. We report the results of comparison of our model with experimental data of the process $\gamma p \rightarrow \rho^0 p$ and summarize the conclusions of this investigation. The Appendix is devoted to the study of special problems which we have encountered in applying the statistical approximation to semi-photon processes. In particular, a procedure to construct gauge invariant amplitude in the statistical approximation is given there.

§ 2. Dual hadrodynamics

The spectrum of hadrons in the dual resonance model follows from the following Lagrangian:

$$\mathcal{L} = \frac{1}{4\pi} \left( \frac{\partial x_\mu \partial x_\nu}{\partial \eta \partial \eta} - \frac{\partial x_\mu \partial x_\nu}{\partial \xi \partial \xi} \right).$$

Here the dynamical variables are the Minkowski co-ordinates $x_\mu(\xi, \eta)$ of the
Meson Photoproduction in the Statistical Approach to Dual Hadrodynamics 2285

medium and $\xi, \eta$ are two co-ordinates internal to the hadron. The co-ordinate $\eta$ plays the role of an internal time. This Lagrangian yields the equation of motion

$$\frac{\partial^2 x_{\rho}}{\partial \eta^2} = \frac{\partial^2 x_{\rho}}{\partial \xi^2}. \quad (2.2)$$

Imposing periodic boundary conditions on $x_{\rho}(\xi)$ enables us to write

$$x_{\rho}(\xi, \eta) = x_{\rho,0}(\eta) + 2 \sum_{n=1}^{\infty} x_{\rho,n}(\eta) \cos n\xi. \quad (2.3)$$

In the usual way we may calculate the conjugate variable

$$p_{\rho} = \frac{\delta L}{\delta \left( \partial_\xi x_{\rho} \right)} = \frac{1}{2\pi} \frac{\partial x_{\rho}}{\partial \eta}. \quad (2.4)$$

The Fourier-decomposed components of the conjugate momentum are related to the corresponding components of the dynamical variable as

$$p_{\rho,n} = \partial_\eta x_{\rho,n} \quad (2.5)$$

and obey the canonical commutation relations

$$[x_{\rho,m}, p_{\rho,n}] = i\delta_{mn}. \quad (2.6)$$

It is also customary to introduce the harmonic oscillator creation and annihilation operators for each mode $n$ of this system as follows:

$$x_{\rho,n} = \sqrt{n + 1} \left( a_{\rho,n} + a_{\rho,n}^\dagger \right), \quad (2.7)$$

$$p_{\rho,n} = -i \sqrt{n + 1} \left( a_{\rho,n} - a_{\rho,n}^\dagger \right), \quad (2.8)$$

for $n \geq 1$ with $[a_{\rho,m}, a_{\rho,n}^\dagger] = \delta_{mn}. \quad (2.9)$

The Hamiltonian of the medium described by the Lagrangian (2.1) is

$$H = \sum_n n a_{\rho,n}^\dagger a_{\rho,n} + p^2, \quad (2.10)$$

where the zero point motion has been absorbed into $p_{\rho,0}$ to define the physical four-momentum $p_{\rho}$. The form (2.10) of the Hamiltonian exhibits its separation into contributions due to the centre-of-mass motion (since the equations of motion are invariant under the translation group $x_{\rho} \rightarrow x_{\rho} + c_{\rho}$) and a relative motion, expressed in terms of normal modes. We now pick those solutions of the equation of motion, Eq. (2.2), which satisfy

$$[H - \alpha(0)] x_{\rho} = [p^2 + M^2] x_{\rho} = 0, \quad (2.11)$$

and interpret the eigenvalue $\alpha(0)$ as the intercept of the leading Regge trajectory. This fixes the particle spectrum as

* We use the Pauli metric with an imaginary time-like component.
A. Hacinliyan and A. Maheshwari

\[ M^2 = \sum_{n, \mu} na_n^+ a_{\mu, n} - \alpha(0), \]  
(2.12)

\( -\alpha(0) \) is the ground-state eigenvalue of the operator \( M^2 \). This, in turn, shows that the particles lie on linear Regge trajectories of unit slope in the units* chosen. The operator expression for the propagator of the Regge trajectory of the dual model for states with momentum \( P \) is given by

\[ A(P^2) = \frac{1}{P^2 + \sum na_n^+ a_{\mu, n} - \alpha(0) - i\varepsilon}. \]  
(2.13)

The statistical approximation of Ref. 14) is based on the observation that "the absorptive part of \( A(P^2) \) is an energy \( \delta \)-function reminiscent of that characteristic of the microcanonical ensemble of statistical mechanics. One can pass to the canonical ensemble by the replacement

\[ A(P^2) = \frac{1}{\sum na_n^+ a_{\mu, n} - (-P^2 + \alpha(0)) + i\varepsilon} \rightarrow \tilde{A}(P^2) = \beta \exp[-\beta \sum na_n^+ a_{\mu, n}] \]  
(2.14)

with

\[ \beta^{-1} = (-P^2 + \alpha(0)) = \alpha(-P^2). \]  
(2.15)

This representation is valid for \( P^2 > 0 \). We give in the Appendix its derivation using the method of steepest descent and discuss an approach for its analytical continuation to the region \( P^2 < 0 \).

It is well known that an interaction vertex of the form \( e^{i\rho \cdot x(0)} \) gives the scalar Veneziano amplitude. Recently, successful attempts have been made to generalize this interaction to handle electromagnetic currents. The starting point for the inclusion of electromagnetic interactions in dual hadrodynamics is the free Lagrangian given in Eq. (2.1). In the \((\xi, \eta)\) space the minimal substitution

\[ p_\mu(\xi) \rightarrow p_\mu(\xi) - e\rho(\xi) A_\mu(x(\xi)) \]  
(2.16)

yields the interaction Lagrangian

\[ \mathcal{L}_{em} = e \int \rho(\xi) \frac{\partial x_\mu}{\partial \eta} A_\mu(x(\xi)) d\xi. \]  
(2.17)

Here \( \rho(\xi) \) is the weight function of the charge distribution over the dynamical variable describing the hadron in this model. The substitution \( A_\mu \rightarrow A_\mu + \partial_\mu A \) reveals that the weight function \( \rho(\xi) \) breaks the gauge invariance of (2.17). The achievement of the approaches\(^b\) is the observation that a non-local minimal substitution reinstates gauge invariance. We shall take the following Lagrangian as the model for the electrodynamics of hadrons:

\(^a\) We use \( \hbar = c = 1 \) and measure energy in GeV. It makes \( \alpha' \), the slope of the Regge trajectory, unity.
Meson Photoproduction in the Statistical Approach to Dual Hadrodynamics 2287

\[ \mathcal{L}_{em} = e \int \rho(\xi) \frac{\partial x_{\mu}}{\partial \eta} A_{\mu}(\bar{x}) d\xi \]  \hspace{1cm} (2.18)

with

\[ \bar{x} = \int \rho(\xi) x(\xi) d\xi = x_0 + 2 \sum_{n=1}^{\infty} \rho_n x_n. \]  \hspace{1cm} (2.19)

The comparison of the action \( S_{em} = \int \mathcal{L}_{em} d\eta \) with its usual form \( \int j_{\mu}(x) A_{\mu}(x) d^4x \), permits us to isolate the current operator

\[ j_{\mu}(x) = \frac{e}{2} \int \rho(\xi) \left\{ \frac{\partial x_{\mu}}{\partial \eta}, \delta^4(x - \bar{x}) \right\} d\xi d\eta. \]  \hspace{1cm} (2.20)

In momentum space it is

\[ j_{\mu}(q) = \frac{e}{2\pi} \left\{ (p_{\mu,0} + \sum_n p_{\mu,n} \rho_n), \exp(-i q \cdot x_0 - i q \cdot \sum_n \rho_n x_n) \right\}. \]  \hspace{1cm} (2.21)

The matrix element of this current between two states labelled by momenta \( P_1, P_2 \) and internal quantum numbers \( 0 \) and \( m \) (Fig. 1) is easily seen to be

\[ \langle P_2, m | j_{\mu}(q) | P_1, 0 \rangle = \frac{e}{2\pi} \left[ (P_1 + P_2)_\mu \langle m | \exp \{ 2iq \cdot \sum_n \rho_n x_n \} | 0 \rangle \right. \\
\left. + \sum_n \rho_n m \left[ (p_{\mu,n} + \sum_n \rho_n x_n) \exp \{ 2iq \cdot \sum_n \rho_n x_n \} \right] | 0 \rangle \right] \hspace{1cm} (2.22)

\[ P_1 = P + q. \hspace{1cm} (2.23) \]

Having defined the electromagnetic vertex, we should like to know the corresponding hadron vertex for the coupling of a neutral meson of spin \( j \) to the states of the dual model. Such a vertex function has been proposed by Gallardo and Galli. \( ^{17} \) The operator expression of this vertex corresponding to Fig. 2 is

\[ \varepsilon_{p_1 p_2 \ldots p_f}(k) M_{p_1 \ldots p_f}(k) = \frac{1}{r_{P_1}} \varepsilon_{p_1 p_2 \ldots p_f} \left[ \prod_{k=1}^{l} \left( \frac{\partial}{\partial \gamma_{p_k}} \right) \right]_{r_{P_1} = 0} \times \left\{ \exp \left\{ \sum a_i \cdot \left( k \sqrt{\frac{2}{l} + \gamma \sqrt{2l}} \right) \right\} \cdot \exp \left\{ \sum a_i \left( k \sqrt{\frac{2}{l} - \gamma \sqrt{2l}} \right) \right\} \right\} \times \exp \{ \gamma (P_1 + P_2) \}. \hspace{1cm} (2.24) \]
\[ \varepsilon_{\mu_{1}...\mu_{j}}(k) \] is the polarization tensor of the external meson. One can notice that this vertex gives the Veneziano form \( e^{i\mathbf{q}\cdot x} \) for the scalar vertex operator.

At this point the apparatus that we have in our hands is sufficient to calculate, at least in principle, photoproduction amplitudes for particles of arbitrary spin. In the next section we shall compute the amplitudes for photoproduction of vector particles from the ground state \( |0\rangle \) of the hadron spectrum.

§3. Calculation of the photoproduction amplitudes

The physical process that we shall study consists of a photon of four-momentum \( q_{\mu} = (iq, q) \) hitting a scalar target of mass \( M \), the ground state of the dual model, such that a meson of spin \( j \), mass \( \mu \), four-momentum \( k_{\mu} = (i\omega, k) \), is photoproduced along with a ground-state hadron. This is not an experimentally observed situation, but it is well known that attempts to write a dual theory of baryons have been plagued by several difficulties.

\[ \varepsilon^{\mu_{1}...\mu_{j}}(q) T_{\mu_{1}...\mu_{j}} e^{*}_{\mu_{1}...\mu_{j}}(k) \]

\[ = \varepsilon^{\mu_{1}...\mu_{j}}(k) \left[ \langle P_{\mu_{1}} | M_{\mu_{2}...\mu_{j}}(-k) | P_{\mu_{1}} \rangle = \exp \left\{ -\beta_{1} \sum n a^{*}_{\mu_{1}} a_{\mu_{1}} \right\} j_{\mu_{1}}(q) \right] | P_{\mu_{1}}, 0\rangle \]

\[ + \langle P_{\mu_{1}} | j_{\mu_{1}}(q) \beta_{2} \exp \left\{ -\beta_{2} \sum n a^{*}_{\mu_{1}} a_{\mu_{1}} \right\} M_{\mu_{2}...\mu_{j}}(-k) | P_{\mu_{1}}, 0\rangle \] \( \varepsilon^{\mu_{1}...\mu_{j}}(q) \),

(3.1)
where
\[ \beta_1 = \frac{1}{-(P_i + q)^2 - M^2} = \frac{1}{S - M^2} \]  \hspace{1cm} (3·2)
and
\[ \beta_2 = \frac{1}{-(P_i - k)^2 - M^2} = \frac{1}{U - M^2}. \]  \hspace{1cm} (3·3)

Specializing to vector meson photoproduction, (3·1) reads
\[
\varepsilon^{(\gamma)}(q) T_{\mu \nu} \varepsilon^*_{\rho}(k) = \frac{g_2 \beta_1}{2 \pi} \left( \sum_{n=1}^{\infty} \rho_n \left( a_n^+ + a_n \right) \right) e^{i \beta^1 \cdot \left( -k \sqrt{\frac{2}{l}} + \gamma \sqrt{2l} \right)} + a_i \cdot \left( -k \sqrt{\frac{2}{l}} - \gamma \sqrt{2l} \right) \cdot \exp \left[ \gamma \cdot (2P_i - k) \right] \\
\times \exp \left[ -\beta_1 \sum_n n a_n^+ a_n \right] \left( 2P_i + q \right) \exp \left[ \frac{i q \cdot \sum_n \sqrt{\frac{2}{n}} \rho_n (a_n^+ + a_n) }{2} \right] \\
- \sum_n 2i \rho_n \sqrt{\frac{n}{2}} (a_{\mu,n} - a_{\mu,n}^+) \\
\times \exp \left[ i q \cdot \sum_n \rho_n \sqrt{\frac{2}{n}} (a_n^+ + a_n) \right] |P_i, 0\rangle + \text{crossed graph} \varepsilon^{(\gamma)}(q). \hspace{1cm} (3·4)

The properties of coherent states which are necessary and sufficient for calculating the above amplitude can be found in Ref. 18). The final result appears as follows:
\[
T_{\mu \nu} = \frac{eg \beta_1}{2 \pi} \exp \left[ 2k \cdot q C(\beta_1) \right] \left[ 4P_i P_\nu + 2(P_i q_\nu + P_\nu k_\mu) \right] \\
\times (1 - 2C'(\beta_1)) + k_{\mu q_\nu} (1 - 2C'(\beta_1)) + 2\delta_{\mu \nu} C''(\beta_1) \]  \\
+ \frac{eg \beta_1}{2 \pi} \exp \left[ 2k \cdot q C(\beta_1) \right] \left[ 4P_i P_\nu - 2(P_i q_\nu + P_\nu k_\mu) \right] \\
\times (1 - 2C'(\beta_1)) + k_{\mu q_\nu} (1 - 2C'(\beta_1)) + 2\delta_{\mu \nu} C''(\beta_1), \hspace{1cm} (3·5)
\]
where
\[
C(\beta) = \sum_{n=1}^{\infty} \beta_n e^{-n\beta}, \hspace{1cm} (3·6)
\]
\[
C'(\beta) = \frac{d}{d\beta} C(\beta) \hspace{1cm} (3·7)
\]
and
\[
P = \frac{1}{2} (P_i + P_f). \hspace{1cm} (3·8)
\]
If we want to check the gauge invariance of the amplitude $T_{\mu}$, a word of caution is in order here. The statistical approximation breaks gauge invariance, $q \cdot T_{\mu} = 0$, but the exact amplitude which one would have obtained using the representation (2.13) for the propagator can be shown to be gauge invariant. This is carried out in the Appendix, where a prescription for removing the statistical approximation is found. This then allows us to take the statistical approximation on the photoproduction amplitude after the constraint of gauge invariance has been explicitly checked. It is also seen that a contact term must be added to the graphs of Fig. 3. Amplitude $T_{\mu}$ in this limit is

$$T_{\mu} = C \left[ P_\mu q - P \cdot q \delta_{\mu \nu} - k \cdot q P_\mu P - k \cdot q P_\mu P + k \cdot q P \right] + D [k_\mu q - k \cdot q \delta_{\mu \nu}], \quad (3.9)$$

where

$$C = \frac{e g}{2\pi} \left[ \beta_1 \exp \left[ 2k \cdot q C (\beta_1) \right] (1 - 2C' (\beta_1)) - \beta_1 \exp \left[ 2k \cdot q C (\beta_1) \right] (1 - 2C' (\beta_1)) \right], \quad (3.10)$$

and

$$D = \frac{e g}{2\pi} \left[ \beta_1 \exp \left[ 2k \cdot q C (\beta_1) \right] (1 - 2C' (\beta_1)) + \beta_1 \exp \left[ 2k \cdot q C (\beta_1) \right] (1 - 2C' (\beta_1)) \right]. \quad (3.11)$$

To proceed further we must specify a working model for the charge distribution weight functions $\rho_n$. We shall follow the discussion of Ref. 9, where it is shown that a physical charge distribution must lie between the two extreme limits: i) Point charge distribution—In this case $\rho_n = 1$ and so corresponds to the charge of the hadron being localized at a point. Of course, in this limit the electromagnetic current becomes unphysical. The outstanding feature of this limit is that the photoproduction amplitudes develop a Regge-type asymptotic behaviour. ii) Uniform charge distribution—In this case $\rho_n = 1$, $\rho_n = 0$ for $n \neq 0$ and corresponds to the charge of the hadron being uniformly distributed over the internal space variable $\xi$. In this limit the electromagnetic current is also unphysical and the amplitude develops a fixed pole type asymptotic behaviour.

Matsuda and Manassah have suggested a model for charge distribution

$$\rho_n = e^{-n^3}, \quad (3.12)$$

which interpolates the distribution between the two extreme limits. The case of the point charge distribution is given by $\delta \to 0$ and the case of uniform charge distribution is given by $\delta \gg 1$.

We shall see in §5 that application of this model to $\rho^8$ photoproduction favours a Regge-type behaviour of the amplitude. The quark models based on the assumption of vector dominance also require explicit exchanges of tensor-meson trajectories in the $t$-channel.\(^{15}\) We shall therefore discuss the case i) in detail.
Using the distribution \( (3 \cdot 12) \) in the definition of the function \( C(\beta) \) given by Eq. \((3 \cdot 6)\) we get

\[
C(\beta) = \sum_{n=1}^{\infty} \frac{e^{-n\beta}e^{-n\beta}}{n} = -\ln(1-e^{-\beta}). \tag{3.13}
\]

Recall from Eqs. \((3 \cdot 2)\) and \((3 \cdot 3)\),

\[
\beta_1 = \frac{1}{S-M^2}
\]

and

\[
\beta_2 = \frac{1}{U-M^2},
\]

and in the high-energy limit where the statistical approximation is valid both \( \beta_1 \) and \( \beta_2 \) go to zero through the positive and negative values, respectively. Operationally the limit \( \delta \to 0 \) at high energy is to be taken in two stages; \( \beta, \delta \ll 1 \) and then \( \delta \to 0, \beta \to 0 \) in that order. The quantities \( C \) and \( D \) defined in Eqs. \((3 \cdot 10)\) and \((3 \cdot 11)\) are then evaluated in this limit. The calculations are elementary; we give the result

\[
C \to \frac{2eg}{\pi} (S-M^2)^{3/2} (1-e^{i\delta+k\cdot q}) \tag{3.14}
\]

and

\[
D \to \frac{2eg}{\pi} (S-M^2)^{3/2}e^{i1}(1-e^{i\delta+k\cdot q}). \tag{3.15}
\]

The differential cross-section \( d\sigma/dt \) is proportional to the spin summed square of the amplitude \( T_{\mu \nu} \), i.e.,

\[
\frac{d\sigma}{dt} \propto |T_{\mu \nu}T_{\nu \nu}^*| \left( \beta_{\gamma} + \frac{k_{\gamma}k_{\mu}}{\mu^2} \right). \tag{3.16}
\]

From \((3 \cdot 9)\) and \((3 \cdot 16)\) we find

\[
\frac{d\sigma}{dt} \propto |C|^2 \left[ 2(P\cdot q)^2 + \frac{(k\cdot q)^2}{(P\cdot q)^2} P^4 - 2(k\cdot q) P^3 - \mu^2 P^3 \right]
\]

\[
+ |D|^2 [2(k\cdot q)^2] + 2|CD| \left[ -\mu^2 P\cdot q + P^4 (k\cdot q)^2 / P\cdot q \right], \tag{3.17}
\]

where

\[
-2P\cdot q = -2P\cdot k = S-M^2 + k\cdot q = -(U-M^2 + k\cdot q), \tag{3.18}
\]

\[
2k\cdot q = t - \mu^2 \tag{3.19}
\]

and
Using (3.14) and (3.15) in (3.17), the differential cross-section in the limit $S \to \infty$ and forward direction $|t| \sim 0$ is

$$\frac{d\sigma}{dt} \propto |S^{2k-2}_{-1}(1 + e^{i\pi(2k-2)})|^2. \quad (3.21)$$

In the vector dominance model for photoproduction, the non-diffractive contribution to the forward direction differential cross-section is due to an even signature tensor trajectory $\alpha(t)$ and is given by

$$\left(\frac{d\sigma}{dt}\right)_{\text{non-diffractive}} \propto |S^{\alpha(t)}(1 + e^{i\pi\alpha(t)})|^2. \quad (3.22)$$

Comparing (3.21) and (3.22) we find that our dual model for the photoproduction of vector mesons gives an output trajectory

$$\alpha(t) = 2k \cdot q + 1$$

$$= t - \mu^2 + 1. \quad (3.23)$$

We note

$$\alpha(\mu^2) = 1$$

and

$$\alpha(0) = 1 - \mu^2 \sim 0.5.$$
Meson Photoproduction in the Statistical Approach to Dual Hadrodynamics

The photon to be the z-axis, x-axis in the production plane and y-axis normal to it.\textsuperscript{a)} The various helicity amplitudes can be obtained from

\[ T_{\lambda_\mu,\lambda_\tau} = \varepsilon_{\mu}^{(\tau)}(\lambda_\tau) T_{\rho_\sigma} \varepsilon_{\sigma}^{(\rho)}(\lambda_\rho). \]  

(4.1)

Using (3·9) and (4·1) the following expressions for the helicity amplitudes are obtained:

\[ T_{++} = C \left[ \frac{|k| |q|}{2} \sin^2 \theta \left( 1 + \frac{1}{4} k \cdot q - P \cdot q \cos^2 \theta \right) \right. \]
\[ - D \left[ \frac{|k| |q|}{2} \sin^2 \theta + k \cdot q \cos^2 \theta \right], \]  

(4.2)

\[ T_{--} = C \left[ - \frac{|k| |q|}{2} \sin^2 \theta \left( 1 + \frac{1}{4} k \cdot q - P \cdot q \sin^2 \theta \right) \right. \]
\[ + D \left[ \frac{|k| |q|}{2} \sin^2 \theta - k \cdot q \sin^2 \theta \right], \]  

(4.3)

and

\[ T_{0+} = C \frac{\sin \theta}{\sqrt{2}} \left[ - \frac{|k|^2 |q|}{2} + |k| |q| (\omega_k \cos \theta \left( 1 + \frac{1}{4} k \cdot q \right) \right. \]
\[ + P \cdot q \omega_k + \frac{|k|^2}{2 P \cdot q} \left( P \omega_k + P \omega_0 + \omega_k \right) \]
\[ + D \frac{\sin \theta}{\sqrt{2}} \left( |q| |k|^2 + k \cdot q \omega_k - |q| |k| \omega_k \cos \theta \right]. \]  

(4.4)

The other amplitudes are related to these given above by parity transformation

\[ T_{-+} = T_{++}, \]  

(4.5)

\[ T_{+-} = T_{-+}, \]  

(4.6)

and

\[ T_{0-} = - T_{0+}. \]  

(4.7)

We have remarked in the introduction that the Pomeranchuk trajectory is excluded from the Born approximation to the dual resonance model. It is a well known experimental fact that the diffractive effect is important in $\rho^0$ photoproduction. We should have liked to study charged $\rho^+$ photoproduction where the diffractive effect

\[ \text{Fig. 4. Illustration of the kinematics used in the definition of helicity amplitudes.} \]

\textsuperscript{a)} The helicity polarization vectors of the photon and the vector meson are $\varepsilon_\mu^{(\rho)}(\pm) = \mp (0, 1, \pm i, 0)/\sqrt{2}$; $\varepsilon_\mu^{(\rho)}(\pm) = \mp (0, \cos \theta, \pm i, - \sin \theta)/\sqrt{2}$ and $\varepsilon_\mu^{(\rho)}(0) = (i |k|, \omega_k \sin \theta, 0, \omega_k \cos \theta)/\mu.$
would be absent but that is beyond the limit of application of the dual model at our disposal. We use here the well accepted hypothesis that s-channel helicity conservation holds near the forward direction in diffractive processes. This strong assumption allows us to take the unitarity correction to the Born term by the addition of the following amplitude to $T_{+,+}$:

$$
(T_{+,+})_{\text{diff}} = -\cos^2 \frac{\theta}{2} \alpha_P(t) \beta_P(t) \left(1 + \exp \left[-i\pi \alpha_P(t)\right]\right) \frac{\sin \pi \alpha_P(t)}{S^{p(t)}},
$$

where

$$
\alpha_P(t) = 1 + \alpha_P' t.
$$

At this stage our two unknown parameters are the Pomeron residue $\beta_P(t)$ and the charge distribution interpolation variable $\tilde{t}$. We use our model for $\rho^0$ photoproduction to find the value of $\tilde{t}$ and the relative contribution of the dual amplitude and the diffraction term. Although we have neglected the spin of the target, this must not hinder us from applying our model to fit target spin-averaged quantities which are measured.

The experiments with polarized photon beam can measure 9 elements of the following density matrix parameters:

$$
\rho^0_{ik} = \frac{1}{A} \sum_{k'} T_{ik} T_{ik'}^*,
$$

$$
\rho^1_{ik} = \frac{1}{A} \sum_{k'} T_{ik} T_{ik'}^*,
$$

$$
\rho^2_{ik} = \frac{1}{A} \sum_{k'} T_{ik} T_{ik'}^*,
$$

with

$$
A = \sum_{k,k'} T_{ik} T_{ik'}^*.
$$

The differential cross-section in the normalizations used is given by

$$
\frac{d\sigma}{dt} = \frac{1}{4\pi S|q|^2} \times \frac{1}{2} A,
$$

where we have averaged over the incident photon’s helicities.

§ 5. Comparison with the experimental data and conclusions

Many experimental groups have studied $\rho^0$ photoproduction with incident photon beam energies up to 18 GeV. The values of $d\sigma/dt$ in the forward direction for the various groups vary as much as 40%. This discrepancy is largely due

*) We take the slope of the Pomeron to be 0.6 GeV$^{-2}$ in the light of high-energy elastic scattering data.
Fig. 5. Comparison of the differential cross-section for the photoproduction of \( \rho^0 \) following from our model with the experimental results of Ref. 23) at incident photon energies 6.5, 11.5, 13.0, 14.5, 16.0 and 17.8 GeV. The parameters determined from a \( \chi^2 \) minimization are \( 0 = 0.021, \beta_p = 1.95 \) and each amplitude is multiplied by a normalization factor of 5.0.

to the different methods of analysis used. However, the values of the density matrices, which do not depend on the overall normalization, tend to agree among the results of different groups. For the differential cross-section we have therefore used the results of the group of Anderson et al. 23) who have measured the differential cross-section for the reaction \( \gamma p \to \rho^0 p \) for photon energies between 6.5 and 17.8 GeV. The group of Ballam et al. 24) has made precise measurements of the photoproduced \( \rho^0 \)'s density matrix elements at photon energies of 2.8 and 4.7 GeV.

We should like to report at the outset that we have found that a pure diffraction term of the type \( (4t) \) alone cannot explain \( d\sigma/dt \) in the forward direction \( (|t| < 0.4 \text{GeV}^2) \) over the whole range 6.5 to 17.8 GeV of the incident pho-
ton energies. This motivated us to study the effect of the dual contribution from the $\rho^0$ photoproduction data.

We have found that data requires $\delta \approx 0$, which throws light on the success of Regge-type models based on vector dominance in explaining $\rho^0$ photoproduction. The fit of our model to the $d\sigma/dt$ data is shown in Fig. 5, and the calculated density matrix elements in Fig. 6. The contribution of the diffraction term to the differential cross-section at $E_\gamma = 11.5$ GeV and $t = -0.20$ GeV$^2$ is $\sim 75\%$. The agreement of the model with the density matrix elements becomes poor at momentum transfers $|t| > 0.4$ GeV$^2$. The hypothesis of $s$-channel helicity conservation is expected to hold only near the forward direction and in the region of large momentum transfer the contribution of the helicity-flip amplitudes may become comparable to that of the helicity conserving amplitude. In the absence of a theory for diffractive contribution the effect of these terms is difficult to take into account.

We also tried to use this model for studying $\omega$ photoproduction on a proton. We have found that the contribution of one-pion exchange at the energies 2.8 and 4.7 GeV, where the precise measurements of density matrix elements of Bal-
Meson Photoproduction in the Statistical Approach to Dual Hadrodynamics 2297

lam et al. exist, is important and the effect of the dual term in the added presence of a diffractive term cannot be extracted in a convincing manner.

We should like to conclude by noting that the dual model has provided us with the justification of the use of vector dominance models based on t-channel Regge exchanges which have been used in the past to explain \( \rho \) photoproduction. We have learnt from our study the residue functions of the tensor trajectory exchange to the different helicity amplitudes. For a more convincing study of photoproduction processes within the framework of dual theory of hadrons one would have to construct a dual theory which takes into account baryons and the internal quantum numbers.

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Appendix

The statistical approach to Veneziano model, gauge invariance and analytic continuation

In this Appendix we first briefly review the statistical approach to the Veneziano model. We then demonstrate a procedure for establishing the gauge invariance of the photoproduction amplitude with particular emphasis on an assumption concerning analytical continuation, which in the literature has been implicitly made but not explicitly stated. Finally, we write the vector photoproduction amplitude in the form (3·9).

A typical 4-point Veneziano amplitude is given by the beta function

\[
V(s, t) = \int_0^1 x^{-\alpha(t)}(1-x)^{-\alpha(t)} dx. \tag{A·1}
\]

One method of obtaining the Regge limit is to set \( x = e^{-\beta} \) in (A·1), getting

\[
V(s, t) = \int_0^\infty e^{\beta \alpha(t)}(1-e^{-\beta})^{-(1+\alpha(t))} d\beta. \tag{A·2}
\]

To obtain the Regge limit as \( \alpha(s) \to -\infty \), a steepest-descent estimate of the integral in Eq. (A·2) is made. The condition produces \( \beta \alpha(s) \to O(1) \) and the integral can be approximated to

\[
\beta (1-e^{-\beta})^{-(1+\alpha(t))} \approx \beta^{-\alpha(t)} = \alpha(s)^{\alpha(t)} \tag{A·3}
\]

in the Regge limit \( (s \to -\infty) \). This result may now be compared with the result which one would have obtained by contracting the "statistical propagator"
between the scalar vertices. This comparison shows that \( \beta \) of the propagator can be identified with \( \beta \) of the integral (A·2) with \( \beta \alpha(s) \sim O(1) \).

The restriction \( \alpha(s) \to -\infty \) is usually removed in the following way:

\[
V(s, t) = \int_0^1 x^{-1-a(s)} (1-x)^{-1-a(t)} \sin \pi(\alpha(s) + \alpha(t)) \sin \pi \alpha(s)
\]

Now putting \( x = e^{-\beta} \), we get

\[
V(s, t) = \frac{\sin \pi(\alpha(s) + \alpha(t))}{\sin \pi \alpha(s)} \int_0^\infty e^{-\beta a(s)} e^{-\beta(a(t)-1)} (1-e^{-\beta})^{-a(t)-1} d\beta,
\]

and the limit \( \alpha(s) \to \infty \) can readily be evaluated. This is the customary analytical continuation.

We next study gauge invariance. First, we pass from the statistical approximation to the exact dual form. It is obvious from the preceding discussion that a term of the form \( \beta_1 F(\beta_1) \) is an approximation of \( \int e^{\beta(u-M)} F(\beta) d\beta \), and similarly a term of the form \( \beta_2 F(\beta_2) \) is an approximation of \( \int e^{\beta(u-M)} F(\beta) d\beta \). Keeping this in mind, we first establish the condition of gauge invariance of the amplitude of scalar photoproduction. From (3·1) it follows that the amplitude for scalar photoproduction in the statistical approximation is

\[
\epsilon_{\mu}^{(r)}(q) T_{\mu} = \frac{e\gamma}{2\pi} \left[ \beta e^{2k-qO(\beta)} \{2P_{\mu} + k_{\mu}(1-2C' (\beta_1)) \}
\right.

\left. + \beta e^{2k-qO(\beta)} \{2P_{\mu} - k_{\mu}(1-2C' (\beta_2)) \} \right]
\]

We now know that (A·6) is an approximation to

\[
\epsilon_{\mu}^{(r)}(q) T_{\mu} = \frac{e\gamma}{2\pi} \left[ \int_0^\infty d\beta e^{\beta(u-M)} e^{2k-qO(\beta)} \{2P_{\mu} + k_{\mu}(1-2C' (\beta)) \}
\right.

\left. + \int_0^\infty d\beta e^{\beta(u-M)} e^{2k-qO(\beta)} \{2P_{\mu} - k_{\mu}(1-2C' (\beta)) \} \right]
\]

The gauge invariance requires that if we change the polarization vector \( \epsilon_{\mu}^{(r)}(q) \)
by \( q^* \) in (A.7) the result should identically be zero, i.e.,

\[
q^*_T = \frac{eg}{2\pi} \left[ \int_0^\infty d\beta e^{i(S-M^1)} e^{iK+q(1-2C'(\beta))}\right] \\
+ \left[ \int_0^\infty d\beta e^{i(U-M^1)} e^{iK+q(1-2C'(\beta))}\right] = 0.
\]  

(A.8)

We use the kinematic relations (3.18) in (A.8) to get

\[
q^*_T = -\frac{eg}{2\pi} \left[ \int_0^\infty d\beta e^{i(S-M^1)} e^{iK+qC'()}\right] \cdot \\
\left[ S-M^1+2k\cdot qC'() \right] - \left[ \int_0^\infty d\beta e^{i(U-M^1)} e^{iK+qC'()}\right] \cdot \\
\left[ U-M^1+2k\cdot qC'() \right].
\]  

(A.9)

The integrations can be simply performed and one obtains

\[
q^*_T = \frac{eg}{2\pi} \left[ e^{i(U-M^1)} e^{iK+qC()} - e^{i(S-M^1)} e^{iK+qC()}\right] \beta = 0.
\]  

(A.10)

This expression vanishes at the lower limit and one of the two terms is seen to blow up in the limit \( \beta \to \infty \) in the physical region of the scattering \((S>0 \text{ or } U>0)\). Therefore, the requirement of gauge invariance can only be satisfied if the limit \( \beta \to \infty \) in the right-hand side of (A.10) is taken in the unphysical region \((S<0, U<0)\) and then the result of zero is analytically continued to the physical region \((S>0)\). The other alternative is to perform the analytical continuation (A.5), which means that different propagators be used for the \( s \)-channel and the \( u \)-channel graphs of Fig. 3. We prefer the former alternative.

The amplitude \( T^*_\alpha \) defined by Eq. (A.8) has the tensor structure

\[
T^*_\alpha = AP^*_\alpha + Bk^*_\alpha.
\]  

(A.11)

The constraint of gauge invariance, which has been established in the preceding paragraph, allows one to eliminate one of the amplitudes such that

\[
T^*_\alpha = B \left( k^*_\alpha - \frac{k\cdot q}{P\cdot q} P^*_\alpha \right),
\]  

(A.12)

where

\[
B = \frac{eg}{2\pi} \left[ \int_0^\infty d\beta e^{i(S-M^1)} e^{iK+q(1-2C'(\beta))}\right] \\
- \left[ \int_0^\infty d\beta e^{i(U-M^1)} e^{iK+q(1-2C'(\beta))}\right].
\]  

(A.13)

which in the “statistical approximation” is

\[
B = \frac{eg}{2\pi} \left[ \beta e^{iK+q(\beta)} (1-2C'(\beta)) - \beta e^{iK+q(\beta)} (1-2C'(\beta)) \right].
\]  

(A.14)
together with \((A \cdot 14)\) give the proper approximation to our amplitude.

A corresponding discussion of the case of vector photoproduction can be sketched parallel to that of the scalar case which has led us from the form \((A \cdot 7)\) of the amplitude to \((A \cdot 12)\). We remark here only that, unlike the scalar case where the right-hand side of \((A \cdot 10)\) vanished identically in the limit \(\beta \to 0\), one has to add now a contact term. Starting from Eq. \((3 \cdot 5)\) and following the above discussion to the end, we find that the proper approximation to the dual vector photoproduction amplitude is

\[
T_{\mu} = C \left[ P_{\mu q_\nu} - P \cdot q_\mu \delta_{\mu \nu} - \frac{k \cdot q}{P \cdot q} P_\mu P_\nu + k_\mu P_\nu \right] + D \left[ k_{\mu q_\nu} - k \cdot q \delta_{\mu \nu} \right],
\]

where \(C\) and \(D\) are defined in Eqs. \((3 \cdot 10)\) and \((3 \cdot 11)\) respectively.

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