Interactions of Baryons in a Relativistic Composite Model

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The interactions of baryons and their resonances are investigated on the basis of a relativistic composite model in terms of the B-S amplitudes. By imposing appropriate conditions, the B-S amplitudes are constructed, which are different from those obtained by a method of SU(6) or \( \bar{U}(12) \) symmetry. This difference may be attributed partially to the effects of interactions and relative motion of urbaryons. From the analysis of various interactions, it is shown that these effects are absorbed in a simplified form of B-S amplitudes as if they were small. The absolute values of magnetic moments, \( NN_\pi \) coupling constant and \( N^*N_\pi \) coupling constant are calculated in good agreement with experiment. The experimental branching ratios of \( \beta \)-decay are also reproduced by our approach.

§ 1. Introduction and summary

Going with increasing experimental information, many attempts have been made at a unified description of elementary particles on the basis of various kinds of composite models, with successful results. For example, an approach by nonrelativistic quark model or \( \bar{U}(12) \)-group theory which rests explicitly or implicitly on a composite hypothesis for elementary particles, predicts the ratio of magnetic moments \( \mu_n/\mu_p = -2/3 \), the \( F-D \) ratio \( F/D = 2/3 \) in \( \beta \)-decay of nucleon and so on, which are in excellent agreement with experiments. On the other hand, a different approach by Regge pole model or Veneziano model seems to reveal the composite aspects of elementary particles in a sense, elucidating the dynamics at the subhadronic level.

In a series of papers, we have investigated various interactions of \( J^P = 0^-, 1^- \) mesons \( (\pi, K, \eta, \eta', \rho, K^*, \omega, \phi)^{\text{(*)}} \) and \( J^P = 2^+, 1^+, 0^+ \) mesons \( (A_2, K_N, f, f'; A_1, K_\Delta; B; \delta)^{\text{(**)}} \) which were assumed respectively, to be s-wave and p-wave bound states of urbaryon and its antiparticle on the basis of a relativistic composite model. In the present paper, we investigate the interactions of baryons and their resonances along the same lines of thought, by making use of a relativistic formulation for the bound state problems developed by Mandelstam, Nishijima and Zimmermann, which enables us to treat the strong, the electromagnetic and the weak interactions, in a unified way.

*) We call Ref. 5)
**) We call Ref. 6)
In the formulation of the problems concerned here, it is a vital point to construct the wave functions or B-S amplitudes describing the composite system of three urbaryons. In the attempts made so far, the B-S amplitudes were constructed by imposing $SU(6)$ or $\bar{U}(12)$ symmetry in some approaches, and were approximated by a direct product of three spinors corresponding to free urbaryons moving with the same momentum of the baryon, by neglecting the effects of interactions and relative motions among urbaryons in the others (see papers of Peking group). As was analyzed in I and II, and was stressed in an analysis of $\pi^\pm \rightarrow 2\rho^\mp$, the effects mentioned above are rather large and are reflected, for example, in the large magnitudes of parameters $\kappa$ corresponding to the masses $m_\sigma, m_\rho$, etc., in the static case.

In § 2, we construct the B-S amplitudes of baryons in a rather unrestricted form compared with those of Ref. 3), taking account of the effects mentioned above as much as possible, and discuss our results comparing with those obtained from other points of view like $SU(6)$ or $\bar{U}(12)$ symmetry. The electromagnetic interactions of baryons are investigated in § 3, in order to normalize B-S amplitudes through the charge form factor of baryons on the one hand and thereby to get the absolute values of magnetic moments of baryons on the other hand. The results for magnetic moments ($\mu_p = 2.72, \mu_n = -1.82, \mu_A = -0.85$ and $\mu_s = 2.51$ in units of $e/2M_p$) agree very well with experimental data ($\mu_p = 2.79, \mu_n = -1.91, \mu_A = -0.7 \pm 0.07$ and $\mu_s = 2.59 \pm 0.46$ in units of $e/2M_p$).

In § 4, we investigate $\beta$-decay of baryons in a similar approximation. As meson states contributing to weak form factors, the existing mesons, i.e., pseudoscalar, vector and axialvector mesons ($\pi^\pm, \rho^\pm$ and $A_1^\pm$ etc.) can be taken. It may be very suggestive for weak interactions, however, that the axialvector form factor cannot be obtained by the effects of these mesons as long as they are assumed to be $s$-wave and $p$-wave bound states. This may be an indication for the existence of "yet to be observed" weak bosons. Assuming the existence of such weak bosons with suitable B-S amplitudes, we analyze the $\beta$-decay of baryons by using $g_A/g_\gamma$ ratio and life time of neutron as input data. The results obtained are in reasonable agreement with experimental data (decay branching ratios and $g_A/g_\gamma$ ratios). Of course, the same results are obtained also in a scheme, where the direct weak interactions of urbaryons coexist with the effects of observed mesons. Strong vertices of baryon and meson are discussed in § 5. The coupling constant $g_{NN^\pi}^{\text{theor}} = 14.5$ is predicted, which should be compared with $g_{NN^\pi}^{\text{exp}} = 13.7$ ($\gamma = 15$). The decay widths of decuplet resonance to a baryon and a pion are also calculated with reasonable results. In the last section (§ 6), some remarks will be added. The explicit forms of baryon vertices are summarized in the Appendix.

§ 2. The B-S amplitudes of baryons

From a point of view of the composite model for elementary particles, it is
supposed that all hadrons are made out of a few number of basic particles (urbaryons). To describe such bound states as baryons and mesons relativistically, it is convenient to use the B-S amplitudes of them. It is, however, impossible to determine the amplitudes without knowledge of subhadronic interactions, and furthermore it will be scarcely possible to get the solutions of three-body B-S equation for baryons, even with a simplified subhadronic interactions. In the analysis made so far, the baryon B-S amplitudes are constructed formally by imposing $SU(6)$ symmetry in some nonrelativistic cases or $\bar{U}(12)$ symmetry and Bargmann-Wigner equations in other relativistic cases. Taking a composite picture of elementary particles explicitly, we can obtain also similar results in terms of a direct product of free urbaryon spinors moving with the same momentum of baryon by neglecting the effects of interaction and relative motion of urbaryons. Such procedures are very profitable for explaining some experimental data. However, the effects of internal interaction and relative motion reveal themselves in the case of mesons (see I and II), and it would be very important to investigate their effects in the baryon cases, and further to find out the reason why $SU(6)$ or $\bar{U}(12)$ symmetry is so successful in a sense.

From such a point of view, we investigate here, the B-S amplitudes of baryons as three-body bound states, taking account of the interactions and relative motions among urbaryons effectively. In order to determine forms of the B-S amplitude, we impose the following conditions:

i) The amplitudes are eigenfunctions with a definite spin in the rest system of baryons.

ii) The amplitudes have an appropriate unitary spin character.

iii) The amplitudes consist of positive energy parts, the meaning of which will be explained in detail, later on.

Let us start with the definition of the B-S amplitude and the adjoint of baryon as a bound state of three urbaryons:

$$\gamma_{a\beta\gamma}(P, x', s, \xi) = (2\pi)^{3/2} \sqrt{\frac{E}{m}} \langle 0 \{ T \{ \psi_{a}^{\alpha}(x_{1}') \psi_{\beta}^{\beta}(x_{2}') \psi_{\gamma}^{\gamma}(x_{3}') \} | P, s, \xi \rangle,$$

(2.1a)

$$\gamma_{a\beta\gamma}^{\dagger}(P, x', s, \xi) = (2\pi)^{3/2} \sqrt{\frac{E}{m}} \langle P, s, \xi \{ T \{ \bar{\psi}_{a}^{\alpha}(x_{1}') \bar{\psi}_{\beta}^{\beta}(x_{2}') \bar{\psi}_{\gamma}^{\gamma}(x_{3}') \} \rangle 0 \rangle,$$

(2.1b)

where $\psi_{a}^{\alpha}(x)$'s are urbaryon fields with spinor indices $\alpha$ and unitary spin indices $a$ in Heisenberg representation, and the ket vector $| P, s, \xi \rangle$ denotes the bound state with momentum $P$, mass $m$, spin $s$ and unitary spin $\xi$. The coordinate $x_{i}$ of the $i$-th urbaryon, the coordinate $x_{i}'$ relative to the c.m. of baryon and the relative coordinate $x_{ij}'$ between the $i$-th and $j$-th urbaryons are related to each other as follows:
In general, the function \( \chi_{a\beta\gamma}(P, x', s) \) with three spinor indices is divided into two parts, a totally symmetric part \( \chi_{[a\beta]}(P, x', s) \) and a mixed symmetric one \( \chi_{\{a\beta\gamma\}}(P, x', s) \) by using sixteen \( r \)-matrices** (symmetric \( (\gamma_{\mu}C)_{a\beta} \), \( (\sigma_{\mu}C)_{a\beta} \) and antisymmetric \( (C)_{a\beta} \), \( (\gamma_{\mu}\gamma_{5}C)_{a\beta} \) with respect to the interchange of \( \alpha \) and \( \beta \):

\[
\chi_{a\beta\gamma}(P, x', s) = \chi_{[a\beta]}(P, x', s) + \chi_{\{a\beta\gamma\}}(P, x', s),
\]

where

\[
\begin{align*}
\chi_{[a\beta]}(P, x', s) &= (\gamma_{\mu}C)_{a\beta} \mathcal{T}_{\mu}(P, x', s) + (\sigma_{\mu}C)_{a\beta} \mathcal{T}^{\mu\nu}(P, x', s), \\
\chi_{\{a\beta\gamma\}}(P, x', s) &= (C)_{a\beta} \mathcal{T}_{\gamma}\mathcal{T}_{\mu}(P, x', s) + (\gamma_{\mu}\gamma_{5}C)_{a\beta} \mathcal{T}_{\gamma}(P, x', s) \\
&\quad + (\gamma_{\mu})_{a\beta} \mathcal{T}_{\gamma}(P, x', s), \\
&\quad - \mathcal{T}_{\gamma}(P, x', s) + \gamma_{\mu}\gamma_{5}\mathcal{T}_{\mu}(P, x', s) + \gamma_{\mu}\gamma_{5}\mathcal{T}_{\mu}(P, x', s) = 0.
\end{align*}
\]

In order to get more explicit forms of \( \mathcal{T}_{\mu}(P, x', s) \) with spinor index \( \gamma \), we expand \( \mathcal{T}_{\mu}(P, x', s) \) in terms of spinor \( \mathcal{U}_{\mu}(P) \) with momentum \( P \) and mass \( m \) of the baryon, and impose on the B-S amplitude the conditions i) and iii),*** the space-reflection invariance and the s-wave condition**** for the composite baryon state. Then, we have the following expressions for \( \chi_{[a\beta]}(P, x', s) \) and \( \chi_{\{a\beta\gamma\}}(P, x', s) \):

\[
\begin{align*}
\chi_{[a\beta]}(P, x', s) &= (\gamma_{\mu}C)_{a\beta} \mathcal{U}_{\mu}(P) \mathcal{F}^{\mu}(P, x', s) + i(\sigma_{\mu}C)_{a\beta} P^{\mu} \mathcal{U}_{\mu}(P) \\
&\quad \times \mathcal{F}^{\tau\mu}(P, x', s) + i(\sigma_{\mu}C)_{a\beta} (\gamma^{\mu}\mathcal{U}^*(P))_{\gamma} \mathcal{F}^{\tau\gamma}(P, x', s)
\end{align*}
\]

with the condition

\[
\begin{align*}
\mathcal{F}^{\tau\mu}(P, x', s) &= \frac{1}{2}(\mathcal{F}^{\mu}(P, x', s) - m\mathcal{F}^{\tau\gamma}(P, x', s) \mathcal{F}^{\tau\mu}(P, x', s)), \\
\chi_{\{a\beta\gamma\}}(P, x', s) &= (\gamma_{\mu}\gamma_{5}C)_{a\beta} \mathcal{U}_{\mu}(P) \mathcal{F}^{\mu}(P, x', s) + (\gamma_{\mu}C)_{a\beta} P^{\mu} \mathcal{U}_{\mu}(P) \\
&\quad \times \mathcal{F}^{\mu1}(P, x', s) + (\gamma_{\mu}\gamma_{5}C)_{a\beta} (\gamma^{\mu}\mathcal{U}^*(P))_{\gamma} \mathcal{F}^{\tau\gamma}(P, x', s)
\end{align*}
\]

For simplicity, we omit here unitary spin indices.

Throughout this paper, we use the following representation for \( r \)-matrices:

\[
\begin{align*}
\sigma_{\mu\nu} &= \frac{1}{2} [T_{\mu\nu}, \tau_{\alpha}], \\
C^{-1} &= C^*, \\
C^{T} &= -C \\
C^{-1} \gamma_{\mu} C &= -\gamma_{\mu}^{*},
\end{align*}
\]

where the asterisk * and \( T \) denote hermitian conjugation and transposition, respectively.

*** In the expansion of the function \( \mathcal{T}_{\mu}(P, x', s) \), we retain only the terms which have a component of positive eigenvalue with respect to \( \gamma \cdot P \). This is the meaning of the condition iii) which corresponds partially to the more restrictive Bargmann-Wigner equation, \( (\gamma \cdot P - m)_{\alpha \beta} x_{\alpha \beta} = 0 \), etc., in usual approaches.

**** This means that the B-S amplitude (2·1) is an even function of \( x' \).
with the condition

\[ F^{4s}(P, x'_{ij}) = \frac{1}{2}[F'^s(P, x'_{ij}) - mF^{4s}(P, x_{ij})], \tag{2.5b} \]

where the \( F^s(P, x'_{ij})'s \) are functions of Lorentz invariants made by \( P \) and \( x'_{ij} \), especially even functions of \( x'_{ij} \), and \( u^s_\tau(P) = f^s(P)u_\tau(P) \) with the polarization vector \( f^s(P) \).

On the other hand, if we impose the Bargmann-Wigner equation on Eqs. (2.3a) and (2.3b), we obtain

\[ \mathcal{T}^{s_s(P, x'_{ij}) = 0}, \quad (\gamma \cdot P - m) \mathcal{T}^{P_s(P, x'_{ij})} = (\gamma \cdot P - m) \mathcal{T}^{f_s(P, x'_{ij})} \]

\[ = (\gamma \cdot P - m) \mathcal{T}^{4s_s(P, x'_{ij}) = 0}, \quad \mathcal{T}^{4s_s(P, x'_{ij})} + \frac{P^s}{m} \mathcal{T}^{f_s(P, x'_{ij})} = 0, \]

\[ \mathcal{T}^{rs_s(P, x'_{ij}) = i \frac{1}{2m} (P^s \mathcal{T}^{f_s(P, x'_{ij})} - P^s \mathcal{T}^{f_s(P, x'_{ij})})}, \]

which mean

\[ \mathcal{T}^{rs_s(P, x'_{ij})} = i \frac{1}{m} P^s u^s_\tau(P) F^s(P, x'_{ij}), \]

\[ \mathcal{T}^{4s_s(P, x'_{ij})} = \frac{1}{m} P^s u_\tau(P) F^s(P, x'_{ij}) \]

\[ F^{4s}(P, x'_{ij}) = 0, \]

or

\[ F^{rs_j}(P, x'_{ij}) = F^{4s}(P, x'_{ij}) \]

if we set \( \mathcal{T}_r^s(P, x'_{ij}) = u^s_\tau(P) \mathcal{T}^f_s(P, x'_{ij}) \) and \( \mathcal{T}_r^f(P, x'_{ij}) = u_\tau(P) \cdot F^f_s(P, x'_{ij}) \) as the solutions of Bargmann-Wigner equations. Comparing Eqs. (2.4a) \sim (2.5b) with Eq. (2.6), we can see that Eqs. (2.4a) \sim (2.5b) contain the terms \( (\gamma \cdot u^s(P))_\tau \) and \( (\gamma^s u(P))_\tau \), which do not appear in the B-S amplitudes with Eq. (2.6), and further that the invariant functions \( F^s(P, x'_{ij}) \) of the former case are less restrictive than those of the latter one.

Finally, for the convenience of later calculations, we summarize the spin and unitary spin character of B-S amplitudes, the latter of which is disregarded so far. As is easily seen from the definition of the B-S amplitude (2.1), the amplitude \( \chi^s_n(P, x'_{ij}, s) \) with the s-wave condition (even function of \( x'_{ij} \)) is symmetric or antisymmetric with respect to the interchange of the pair \( (a, a), (\beta, b), (\gamma, c) \) for the case of Bose or Fermi basic particle, respectively. This means that a single triplet model with basic Fermions of spin 1/2 contradicts the favourable symmetric 56-dimensional representation of \( SU(6) \) for the s-wave composite baryons.

\(^*\) The polarization vector \( f^a(P) \) satisfies the following relations:

\[ \sum_{a=1}^4 f^a_n(P)f^a_j(P) = -\delta_{ij}, \quad \sum_{a=1}^9 f^a_n(P)f^a_j(P) = -\left( g_{\mu\nu} - \frac{P_\mu P_\nu}{m^2} \right), \]

\[ P_\mu f^a_n(P) = 0. \]
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Expecting that such difficulties may be overcome by using the three-triplet model for basic Fermions or a single triplet model of basic particles obeying para-Fermi statistics, we consider the symmetric amplitudes with respect to the interchange of spin and unitary spin indices;

\[ \chi_{a\beta r}^{abc}(P, x_i, s, \xi) = \frac{\sqrt{3}}{2\sqrt{2}} D^{(abc)}(\xi) \chi_{(a\beta r)}(P, x_i, s) \]

\[ + \frac{1}{2\sqrt{6}} [\phi^{ab} N^c_{\rho}(\xi) \chi_{(a\beta r)}(P, x_i, s) + \phi^{bc} N^a_{\rho}(\xi) \chi_{(\beta a r)}(P, x_i, s) \]

\[ + \phi^{cd} N^e_{\rho}(\xi) \chi_{(\beta e a)}(P, x_i, s)], \quad (2\cdot7) \]

where \( D^{(abc)} \) and \( N^e_{\rho} \) represent usual decuplet and octet components of \( SU(3) \) group.

§ 3. The electromagnetic interactions of baryons and normalization of the B-S amplitudes

In this section, we investigate the electromagnetic form factors of baryons with another purpose to normalize the B-S amplitudes. A possibility to obtain the absolute values of the magnetic moment of baryons will be shown also.

The electromagnetic current of our system is given in terms of basic fields \( \phi^a(x) \) in the following form:

\[ j_{\mu}^{em}(X) = T\{\phi(X)\tau_\mu Q\psi(X)\}, \quad (3\cdot1) \]

where the number of basic fields \( \phi^a(x) \) and the charge matrix \( Q \) depend on the choice of our subhadronic models. Here, we consider, for simplicity, the case of quark-ace model favourable for the symmetry properties of baryons and their resonances, setting

\[ Q = \frac{1}{3} \begin{pmatrix} 2 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{pmatrix} \]

The form factors will be obtained by substituting \( j_{\mu}^{em}(X) \) for \( J_{\mu}^{BF}(X) \) in Eq. (A·2) and by taking account of only the vector mesons \( (\rho, \omega, \phi) \) as the possible intermediate states connected with the one-photon state (vector-meson-dominance (VMD) approximation for electromagnetic form factors). The B-S amplitudes of vector mesons corresponding to \( \chi^a \) in Eqs. (A·2) and (A·3) are given in the following form as was discussed in previous papers [see Eq. (3·2) in II]:

\[ \chi_{a\beta r}^{ab}(P, x, s, \xi) = \left( \gamma_\mu + i\frac{\sigma_\mu P^\mu}{\kappa} \right)_{a\beta} f^\nu(P) T(\xi)_{ab} F_0(x^\nu), \]

\[ \chi_{a\beta r}^{ab}(P, x, s, \xi) = - \left( \gamma_\mu - i\frac{\sigma_\mu P^\mu}{\kappa} \right)_{a\beta} f^\nu(P) T(\xi)_{ab} F_0(x^\nu), \quad (3\cdot2) \]
with $\kappa = 1.09$ GeV.

For the later calculations of various vertices, we simplify the baryon B-S amplitudes, Eqs. (2·4a) and (2·4b), as we factorized those of mesons to the product form Eq. (3·2) of $P$-dependent part and $x^2$-dependent one, on the basis of some reasonable considerations. We factorize the B-S amplitudes also in the case of baryons as follows:

$$
F^P(P, x'_i) = a_0(P^2) F(x'_i), \quad F^{A_1}(P, x'_i) = a_0(P^2) F(x'_i),
$$

$$
F^{A_2}(P, x'_i) = a_0(P^2) F(x'_i), \quad F^V(P, x'_i) = a_0(P^2) F(x'_i),
$$

$$
F^{T_1}(P, x'_i) = a_0(P^2) F(x'_i), \quad F^{T_2}(P, x'_i) = a_0(P^2) F(x'_i),
$$

where $a, b, c$ are functions of $P^2$ only and $F$ of $x'_i$, the subscripts $O$ and $D$ mean octet and decuplet, respectively.

The electromagnetic form factors of baryons $F_{i}^{em}(q^2)$ and those of their resonances $H_i^{em}(q^2)$ are defined as follows:

$$
\langle P_\nu, s_\nu, \xi_1 | j_{\mu}^{em}(0) | P_{\nu}, s_{\nu}, \xi_2 \rangle
$$

$$
= \frac{e}{(2\pi)^3} \sqrt{\frac{m^3}{E_1 E_2}} [\bar{u}^{(r)}(P_1) \{ i \sigma_{\mu,\nu} q^{\nu} F_{i}^{em}(q^2) \} u^{(r')} (P_2) ]
$$

$$
- \bar{u}^{(r)}(P_1) \{ i \gamma^\nu \gamma_\mu H_{i}^{em}(q^2) + ig^{\nu\lambda} \sigma_{\mu,\nu} H_{i}^{em}(q^2) \}
$$

$$
+ (q^2 g_{\mu\nu} - q^\nu q^\mu) H_{i}^{em}(q^2) - q^2 \sigma_{\mu,\nu} q^{\nu} H_{i}^{em}(q^2)
$$

$$
- q^2 \sigma_{\mu,\nu} q^{\nu} H_{i}^{em}(q^2) \} u^{(r')} (P_2)
$$

(3·4)

with the normalization $\bar{u}^{(r)}(P) u^{(r')} (P) = \delta_{rr'} (r, r' = 1, 2)$ and $\bar{u}^{(r)}(P) u^{(r')} = -\delta_{rr'} (r, r' = 1, 2, 3, 4)$. Here, $q_{\mu} = (P_2 - P_1)_{\mu}$. Then, using Eqs. (A·2), (A·6) and the B-S amplitudes of baryons and vector mesons, the relevant form factors at $q^2 = 0$ are calculated as follows:

$$
F_{i}^{em}(0) = \sum \frac{q_{\mu} g_{\nu}}{m^3} \cdot F_{i}(0) \frac{1}{12} \left[ 3 \{ - (a_o - M_{bo})^3 + 8c_o (a_o - M_{bo}) \}
$$

$$
- 4c_o^3 \} d_o + 2 \{ - (a_o - M_{bo})^3 + 8c_o (a_o - M_{bo}) - 4c_o^3
$$

$$
+ 6a_o (M_{bo} + c_o) \} f_o + \{ (a_o - M_{bo})^3 - 8c_o (a_o - M_{bo})
$$

$$
+ 4c_o^3 + 12a_o (M_{bo} + c_o) \} c_o \}
$$

(3·5a)

$$
F_{i}^{em}(0) = \sum \frac{q_{\mu} g_{\nu}}{m^3} \cdot F_{i}(0) \frac{1}{12} \left[ 3 \{ (2M_{bo}^3 + 4b_{co}c_o) + \frac{1}{\kappa} (a_o^3 + 2M_{bo}b_o)
$$

$$
+ M_{bo}^3 b_o^3 + 4M_{bo} c_o + 12c_o^3 \} d_o + 2 \{ - (3a_o b_o - 2M_{bo}^3 - 4b_{co}c_o)
$$

$$
+ \frac{1}{\kappa} (a_o^3 + 2M_{bo} b_o + M_{bo}^3 b_o^3 + 4M_{bo} c_o + 18c_o^3) \} f_o
$$

$$
+ \{ - (6a_o b_o + 2M_{bo}^3 + 4b_{co}c_o) + \frac{1}{\kappa} (a_o^3 + 2M_{bo} b_o
$$

$$
+ M_{bo}^3 b_o^3 + 4M_{bo} c_o + 12c_o^3 \} c_o \}
$$

(3·5b)
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\[ H_{\text{sm}}(0) = \sum \frac{g_{\nu}}{m_{\nu}^2} 4F_0(0) \left( \frac{3}{8} \cdot 8a_D(M^*b_D + c_D) + 8a_0b_0 \right) I, \]

where \( M \) and \( M^* \) denote masses of octet and decuplet baryons respectively, and the summation \( \sum_{\nu} \) is taken over the possible vector-meson states. Here, other notations are given by

\[ c_{\nu}' = D(e_{\nu}\xi) T_{a\nu}(v) D(e_{\nu}\xi) (\xi), \]

\[ 4F_0(0) g_{\nu} f_\rho(\xi) = \text{Tr}[\chi(\xi, 0, s_\nu \xi) T_\rho Q], \]

\[ I = -i \int \prod_{i=1}^{3} d^4y_i d^4x_i d^4x_i' 3m_0 \delta^4(\sum_i y_i) \delta^4(\sum_i x_i') \delta^4(x_i + x_i') \]

\[ \text{exp}[i \{ P_i(-y_i + x_i) \prod_{i=1}^{3} F(y_i, f_0, f_0, f_0)\}]. \] (3.6)

First, we note that in Eqs. (3.5a) and (3.5b), the term \( \sum_{\nu} (g_{\nu}/m_{\nu}^2) F_0(0) c_{\nu} \) is simplified in an approximation \( m_{\nu}^2 = m_{\nu}^0 = m_{\nu}^0 = m_{\nu}^0 \) as follows:

\[ \sum_{\nu=m, n, \ldots} \frac{1}{m_{\nu}^2} g_{\nu} c_{\nu} \sim \frac{F_0(0)}{m_{\nu}^2} \text{Tr}(N\bar{N}) \sum_{\nu} \text{Tr}(T^0) \text{Tr}(T^0 Q) = \frac{F_0(0)}{m_{\nu}^2} \text{Tr}(N\bar{N}) \text{Tr}(Q), \]

because of the normalization \( \text{Tr}(T^0 T^0) = \delta_{\nu\nu} \), and therefore that this term does not contribute to form factors in the case of models with traceless charge \( \text{Tr}(Q) = 0 \), i.e., in the case of quark-ace model or three-triplet model. Restricting ourselves to this case, hereafter we neglect this term as approximately very small.

Next we normalize the B-S amplitude by using the definition of the charge of baryons and their resonances. Imposing a condition on Eq. (3.5a) that the charge form factors at \( q^2 = 0 \) of such neutral baryons as \( n, \Lambda, \ldots \) should be zero (or nearly zero in the present approximation), we obtain the following relation:

\[ (a_0 - M b_0)^2 - 4c_0(a_0 - M b_0) + 4c_0^2 = 0 \] (3.7)

or combined with Eqs. (2.5b) and (3.3)

\[ c_0 = 0 \quad \text{and} \quad a_0 = M b_0. \] (3.7')

Then, in the context of VMD together with the approximation \( m_{\nu}^2 = m_{\nu}^0 = m_{\nu}^0 \), the charges (or the charge form factors at \( q^2 = 0 \)) of baryons and their resonances are given from Eqs. (3.5a) and (3.6):

\[ G_{\nu}(0) = F_{\text{sm}}(0) = \frac{1}{m_{\nu}^2} \frac{4F_0(0)}{a_0} I \sum_{\nu} g_{\nu} f_\nu, \]

\[ (3.8a) \]

*Note that the \( d \)-type contribution, Eq. (3.7) in Eq. (3.5a), is very small (the order of \( c_0^2 \)) even if \( c_0 \) is nonzero.
\[ H^{\text{em}}(0) = \frac{1}{m_p^2} A F_0(0) a_D (M^* b_D + c_D) I \sum \gamma_{\nu} c_{\nu}'. \]  

The values of \( \sum \gamma_{\nu} f_{\nu} \) and \( \sum \gamma_{\nu} c_{\nu}' \) for baryons and resonances with the charge +1 are 1 and 1/3, respectively, and therefore from Eqs. (3·8a) and (3·8b) the following relation among \( a_0, a_D, b_D \) and \( c_D \) is obtained:

\[ \frac{1}{m_p^2} A F_0(0) a_0 I = \frac{1}{m_p^2} A F_0(0) a_D (M^* b_D + c_D) I = 1 \]

or

\[ a_0 = a_D (M^* b_D + c_D). \]

It is worthwhile to remark here that the effects of interactions and relative motions of urbaryons are well absorbed in a relatively simple forms of B-S amplitudes inferred from \( SU(6) \) or \( \bar{U}(12) \) symmetry, in contrast with the case of vector and pseudoscalar mesons. Although the \( c_D \) term can be estimated to be small by the decay rate of \( N^* \to N + \gamma \), we do not enter into this problem because of the theoretical and experimental complexity.

Finally, we calculate the magnetic moments \( G_N(0) \) of baryons by using Eqs. (3·5a) and (3·5b) together with Eq. (3·7') as follows:

\[ G_N(0) = F_1^{\text{em}}(0) + 2MF_1^{\text{em}}(0) \]

\[ = \frac{4F_0(0) a_0 I}{m_p^2} \frac{1}{3} (1 + \frac{2M}{\kappa}) \sum \gamma_{\nu} (2f_{\nu} + 3d_{\nu}), \]

which turns in virtue of the charge normalization (3·9) to

\[ G_N(0) = \frac{1}{3} \left( 1 + \frac{2M}{\kappa} \right) \sum \gamma_{\nu} (2f_{\nu} + 3d_{\nu}). \]

From Eq. (3·11), the magnetic moment of proton, for example, is predicted to be \( \mu_p^{\text{theor}} = 2.72 \) in units of \( e/2M_p \), which should be compared with the experimental value \( \mu_p^{\text{exp}} = 2.79 \). Magnetic moments of other baryons are shown in Table I, which are in good agreement with experiment. It should be remarked that the value of \( \kappa (= 1.09 \text{ GeV} \neq m_\pi) \) which was determined in I plays an essential role in reproducing the magnitude of the experimental magnetic moments.

<table>
<thead>
<tr>
<th>( \mu^{\text{theor}}(e/2M_p) )</th>
<th>( \mu^{\text{exp}}(e/2M_p) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.72</td>
<td>2.79</td>
</tr>
<tr>
<td>-1.82</td>
<td>-1.91</td>
</tr>
<tr>
<td>-0.85</td>
<td>-0.70±0.07</td>
</tr>
<tr>
<td>2.51</td>
<td>2.59±0.46</td>
</tr>
<tr>
<td>0.84</td>
<td>0.84</td>
</tr>
<tr>
<td>-0.84</td>
<td>-1.62</td>
</tr>
<tr>
<td>-1.62</td>
<td>-0.81</td>
</tr>
</tbody>
</table>

In this Table, we use experimental data in Ref. 9) for \( \mu^{\exp} \).

\[ \S 4. \text{ Leptonic decays of baryons} \]

In this section, we investigate weak interactions of baryons, especially the
Interactions of Baryons in a Relativistic Composite Model

$\beta$-decay of them. In our analysis of the $\beta$-interaction of baryons, we assume, as is usually accepted, the following form for the weak interaction Hamiltonian at the subhadronic level:

$$H_\pi = (G/\sqrt{2}) \int d^4X J^{\pi}_\mu W(X)J^{\pi*\mu}(X)$$

with currents of the form

$$J^{\pi}_\mu(X) = j^{\pi\in\pi}(X) + j^{\pi\text{had}}(X),$$

the hadronic part of which is expressed in terms of basic fields $\phi_a^u(x)$ as follows:

$$j^{\pi\text{had}}(X) = T\{\bar{\psi}(X)\gamma_\mu(1-\gamma_5) W(\theta) \psi(X)\}, \quad (4 \cdot 1)$$

where the weak coupling constant $G = (1/m^2) \times 1.01 \times 10^{-8}$ and the weak matrix $W(\theta)$ is written by Cabibbo angle $\theta$,

$$W(\theta) = \begin{pmatrix}
0 & 0 & 0 \\
\cos \theta & 0 & 0 \\
\sin \theta & 0 & 0
\end{pmatrix}.$$

For the calculation of the $\beta$-decay rate of baryons, it is necessary to get the matrix elements of the weak current $j^{\pi\text{had}}(X)$ between baryons, which are formally given in Eq. (4 \cdot 2). As the main possible states mediating the weak processes in Eq. (A \cdot 2), we can take account of only so far observed particles such as pseudoscalar mesons ($\pi, K$, etc.), vector mesons ($\rho, K^*$, etc.) and axialvector mesons ($A_1$- and $B$-families), just in the same way as we took only $\rho, \omega$ and $\phi$ vector mesons as the possible mediating states for the calculation of the electromagnetic form factors in § 3. From the point of view we developed so far, these intermediary mesons are considered as s- and p-wave bound state of urbaryon and antiurbaryon, and the explicit forms of their B-S amplitudes were given in II. The ps-meson contributes to the induced pseudoscalar part of weak form factors of baryons, the vector-meson to the vector and tensor parts. As for the axialvector meson which is considered as p-wave bound state, the B-S amplitudes are given as follows:

$$(\gamma^s + \frac{i}{\kappa} \sigma^{\mu\nu} P_\sigma) \left\{ \frac{1}{2} (f^{(d)} f^{(d*)} - f^{(u)} f^{(u*)}) \right\} n^u(x) F_1(x^2) \quad \text{for } A_1\text{-family},$$

$$(1 + \frac{1}{\kappa} \vec{\gamma} \cdot \vec{P}) \gamma^s f^{(d)} \cdot n(x) F_1(x^2) \quad \text{for } B\text{-family}. \quad (4 \cdot 2)$$

The axialvector mesons with the above B-S amplitudes* do not contribute to the axialvector part of weak form factors against apparent expectation, and rather give the vector, tensor or induced pseudoscalar parts which can be neglected for

* Note that the value of the amplitudes (4 \cdot 2) at $x=0$ is not necessarily zero because $n^u(x) = x^u/\sqrt{x^2}$. 
the calculation of decay widths because of the small momentum transfer. Thus, we cannot obtain the axialvector part of weak form factors as contributions of relatively low lying mesons predicted by composite models. This situation should be compared with that of the electromagnetic interactions, in which the various form factors at low energies are rather successfully described in terms of the vector-meson-dominance model. This may indicate the existence of the fundamental weak boson yet to be discovered, the structure of which is somewhat different from those predicted by composite models. We here assume that the charged weak bosons exist and mediate weak leptonic processes. Although there is not any convincing clue to get an explicit form for the B-S amplitude of weak bosons at present, we tentatively assume the following B-S amplitudes for them except for the charge character of them:

\[ \chi_w \sim (\gamma^\nu + i\frac{e}{\hbar c}A^\nu \gamma^\mu \gamma^5 P_\mu) f_s^{(4)}(P) (F_{\gamma \mu}^w(x^2) + \gamma_4 F_{\gamma}^w(x^2)). \]

(4.3)

The form (4.3) assumed may be considered as a characteristic of weak boson which cannot be inferred from the usual composite model.

The weak form factors \( F_{\gamma}^w(q^2) \) and \( G_{\gamma}^w(q^2) \) of baryons defined by

\[ \langle P, s_1, \xi_1 | J_{\gamma}^w(0) | P, s_2, \xi_2 \rangle = \frac{G}{\sqrt{2}} \frac{1}{(2\pi)^3} \int \frac{d^3p_1 p_2}{E_1 E_2} \{ \gamma_\mu F_{\gamma}^w(q^2) + i \sigma_{\mu\nu} q^\nu F_{\gamma}^w(q^2) \}
- \gamma_\mu \gamma_5 G_{\gamma}^w(q^2) - q_\mu \gamma_5 G_{\gamma}^w(q^2) \} u(\nu) (P_1), \]

(4.4)

are calculated from Eq. (A.2) by using the B-S amplitudes (4.3), leaving the following results:

\[ F_{\gamma}^w(0) = \frac{1}{m_w^2} 4 F_{\gamma}^w(0) a_\gamma^0 I_{\gamma} g_w(\theta) f_w, \]

(4.5)

\[ G_{\gamma}^w(0) = \frac{1}{m_w^2} 4 F_{\gamma}^w(0) a_\gamma^0 I_{\gamma} \frac{1}{3} g_w(\theta) (2f_w + 3d_w), \]

where \( m_w \) denotes the mass of weak boson, \( I_{\gamma} \) and \( I_{A} \) are obtained from Eq. (3.6) by inserting \( F_{\gamma}^w \) and \( F_{A}^w \) instead of \( F_{\gamma} \), and \( g_w(\theta) \), \( f_w \) and \( d_w \) are defined by

\[ g_w(\theta) = Tr(T_{\gamma} W(\theta)), \quad f_w = Tr(N[T_{\gamma}(N), N]), \quad d_w = Tr(N[T_{A}(N), N]) \]

(4.6)

with the charge matrix \( T_{\gamma}(N) \) of weak boson.

In order to compare our theoretical results with experiments, now we have to fix the values of some parts contained in our theoretical formulae, referring to relatively exact experimental information, for example, the \( (g_A/g_{\gamma}) \)-ratio and the decay rate of neutron:

\[ (g_A/g_{\gamma})_{\exp} = 1.23, \quad R_n = (1.084 \pm 0.016) \times 10^{-4} \text{ sec}^{-1}. \]

(4.7)

Taking the \( \beta \)-decay data of neutron (4.7) as an input, we can predict the branching
ratios for the $\beta$-decay of other baryons, the results of which are tabulated in Table II. As is seen from Table II, agreement between theory and experiment is reasonable.

<table>
<thead>
<tr>
<th>Decay mode</th>
<th>Branching ratio</th>
<th>$g_A/g_V$ or $g_V/g_A$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Experimental</td>
<td>Theoretical</td>
</tr>
<tr>
<td>$A^+\to p + e^- + \nu$</td>
<td>$(0.80\pm0.06) \times 10^{-3}$</td>
<td>$0.870 \times 10^{-3}$</td>
</tr>
<tr>
<td>$\Sigma^+\to n + e^- + \nu$</td>
<td>$(1.09\pm0.05) \times 10^{-3}$</td>
<td>$0.854 \times 10^{-3}$</td>
</tr>
<tr>
<td>$\Sigma^0\to n + e^- + \nu$</td>
<td>$0.60\pm0.06 \times 10^{-4}$</td>
<td>$0.563 \times 10^{-4}$</td>
</tr>
<tr>
<td>$\Lambda^0\to p + e^- + \nu$</td>
<td>$2.02\pm0.47 \times 10^{-5}$</td>
<td>$1.84 \times 10^{-5}$</td>
</tr>
<tr>
<td>$B^-\to A^+ + e^- + \nu$</td>
<td>$(1.07\pm0.23) \times 10^{-3}$</td>
<td>$0.544 \times 10^{-3}$</td>
</tr>
<tr>
<td>$s^-\to s + e^- + \nu$</td>
<td>$&lt;0.5 \times 10^{-3}$</td>
<td>$0.096 \times 10^{-3}$</td>
</tr>
<tr>
<td>$s^-\to u + e^- + \nu$</td>
<td>$&lt;1.5 \times 10^{-3}$</td>
<td>$0.31 \times 10^{-3}$</td>
</tr>
</tbody>
</table>

As experimental values of the branching ratios and $g_A/g_V$ or $g_V/g_A$, we use those of Ref. 9) except for $g_A/g_V$ data a) of the decay: $\Sigma^0\to n + e^- + \nu$, for which we use the average of new and old data (see 14th International Conference on High-Energy Physics, Vienna, 1968, p. 290). For Cabibbo angle $\theta$, $\sin \theta=0.25$ is used.

In the analysis made so far, we have considered the $\beta$-decay of baryons as a process mediated by weak bosons and have used a degree of freedom of inequality

$$F_A^w(0)I_A/F_V^w(0)I_V \neq 1,$$

referring to the relations $(g_A/g_V)_{\beta}= (5/3)(F_A^w(0)I_A/F_V^w(0)I_V)$ and $(g_A/g_V)_{\beta}^{\text{exp}}=1.23$. It would be worthwhile, however, to remark that if we consider the $\beta$-process as a direct $\beta$-process of urbaryons contained in the baryon, we obtain the following result:

$$F_1^w(0) = a_0 I_A f(\theta),$$
$$G_1^w(0) = \frac{1}{2} a_0 I_A (2f(\theta) + 3d(\theta)), \quad (4.8)$$

with

$$I_A = \prod_{i=1}^{8} d^4 y_i' d^4 z_i' 3m_0 \delta^4 \left( \sum y_i' \right) \delta^4 \left( \sum z_i' \right) \delta^4 \left( y_{12}' - z_{12}' \right) \delta^4 \left( y_{13}' - z_{13}' \right) \times \exp \{ i (P y_i' - P z_i') \} F(\gamma_i') F(z_i'),$$

$$f(\theta) = \text{Tr} \left( N \{ W(\theta), N \} \right), \quad d(\theta) = \text{Tr} \left( N \{ W(\theta), N \} \right),$$

and therefore we are lead to a result, $(g_A/g_V)_{\beta}=5/3$, without a degree of freedom to adjust $(g_A/g_V)_{\beta}$ to the experimental value.
Of course, as mentioned in § 1, the results obtained in the weak-boson scheme can be obtained also in a scheme where the effects of direct weak interactions of urbaryons coexist with the effects of observed mesons.

§ 5. Strong vertices of baryons and mesons

In this section, we investigate the strong vertices of baryons and mesons, especially the \( \pi \cdot N \) and \( \pi \cdot B^* \) interactions.

First we estimate the \( NN\pi \) vertex which is given by Eq. (A·3) and is approximated like a form (A·6) as shown in Figs. 1 and 2 of the Appendix. As the B-S amplitudes to be inserted into these formulae, we use those of Eq. (2·4b) with Eq. (3·3) and those of ps-mesons given in Eq. (5·12) of I or in Eq. (3·2) of II for nucleon and pion, respectively. Then, the \( \pi \cdot N \) vertex is shown to be

\[
K_{N\pi}(P_1, q, P_2) = i(2\pi)^{\delta^4}(P_1 + q - P_2) \frac{1}{12} \left( 1 + \frac{2M}{\kappa} \right) \left( 4 - \frac{q^2}{M^2} \right)
\times 5a_0^3 \Gamma_{\pi, N}(P_1) \gamma_{\pi N}(P_2),
\]

(5·1)

where the numerical factor 5 in Eq. (5·1) comes from the \( SU(3) \) Clebsch-Gordan coefficient. The \( NN\pi \) coupling constant \( g_{NN\pi} \) defined through the interaction Hamiltonian density \( \mathcal{H}(x) \)

\[
\mathcal{H}(x) = g_{NN\pi} \bar{N}(x) \gamma_{\pi N}(x) \cdot \pi(x)
\]

(5·2)

is given in terms of parameters specifying meson and baryon as follows:

\[
g_{NN\pi} = \frac{1}{\sqrt{2}} \frac{1}{12} \left( 1 + \frac{2M}{\kappa} \right) \left( 4 - \frac{m^2}{M^2} \right) 5a_0^3 \Gamma.
\]

(5·3)

Using the normalization \( 4F(0)a_0^3 I/m_{\pi} = 1 \) (see Eq. (3·9)), a relation \( Tr[\chi^{(0)}(P; 0)\gamma_{\pi}Q] = (4F_0(0)/\sqrt{2})f_{\pi}^{(0)}(P) = (m_{\pi}^2/g_\pi f_{\pi}^{(0)}(P) \) connected with the \( \rho \to \mu^+\mu^- \) decay constant \( g_{\rho}^4/4\pi = 2.7 \) [see Eq. (5·13) in I] and \( M = 1.115 \) GeV as the mean mass value of baryon, we obtain a theoretical value \( g_{NN\pi}^{\text{theor}} = 14.5 \) which should be compared with the experimental value \( g_{NN\pi}^{\text{exp}} = 13.7 \) \( (g_{NN\pi}^{\text{exp}})^2/4\pi \approx 15 \). Agreement between theory and experiment is fairly good.

Next, we consider the strong pion-decay processes \( B^* \to B + \pi \). By using Eqs. (A·4), (A·6) and (A·7), the matrix element of these processes is given as follows:

\[
\langle P_1, \frac{1}{2}, B; P_b, \pi; \text{out} | P_b, \frac{3}{2}, B^* \rangle = -i(2\pi)^{\delta^4}(P_1 - P_2 - P_3) \frac{1}{M^*} \sqrt{M \cdot M^*} \left( \frac{1}{2E_x E_{x*} g_{B^* B} \bar{u}}(P_1) P_1 \bar{u}(P_2) \right)
\]

(5·4)

with

\[
g_{B^* B} = a_D \left[ \frac{1}{M} b_D + \frac{1}{\kappa} \left( a_D + \frac{M^*}{M} b_D + \frac{M^*}{M} c_D \right) \right] \epsilon_{\alpha \beta \gamma \delta_{x \gamma_{\pi x \pi}} T^{a_{\alpha \beta \gamma \delta}}(x_{\alpha \beta \gamma \delta}).
\]

(5·5)
If we use a relation $c_D = \frac{1}{3}(a_D - M^* b_D)$ (see Eq. (2.5) together with the parametrization Eq. (3.3)) and the normalization (3.9), the above coupling constant $g_{BB*}$ is re-expressed as follows:

$$g_{BB*} = a_0 \frac{1}{\sqrt{M}} \left[ a_D + a_0 + \frac{M^*}{a_D} \right] \frac{N_{\epsilon_i}^a T_{a_i}^{a_i} D_{\epsilon_i}^{a_i}}{a_D},$$

(5.6)

by which the decay width $\Gamma$ of the process $B^*(M^*) \rightarrow B(M) + \pi(m_\pi)$ is given as follows:

$$\Gamma = \frac{g_{BB*}^4}{4\pi} \frac{1}{6} \left[ (M + M^*)^3 - m_\pi^3 \right]$$

(5.7)

with

$$p^3 = \left[ \frac{M^3 + M^2 - m_\pi^2}{2M^*} \right]^3 M^2.$$

Using mean mass values of octet baryons and of decuplet baryon resonances, $M = 1.115$ GeV and $M^* = 1.38$ GeV respectively, for the coupling constant $g_{BB*}$, we calculated the decay widths (5.7),* the results of which are shown in Table III as a function of the ratio $a_D/a_0$ together with experimental data. As is seen from Table III, agreement between theory and experiment is obtained by taking $a_D/a_0 = 1$. From Eq. (3.9) this suggests a situation $c_D = 0$, which means that the effects of interactions and relative motions among urbaryons in decuplet baryon resonances are small as well as in octet baryons ($c_0 = 0$). This fact may be reflected on some successful aspects of $SU(6)$- or $\tilde{U}(12)$-symmetry version.

<table>
<thead>
<tr>
<th>Decay mode</th>
<th>Experimental width (MeV)</th>
<th>Predicted width (MeV)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A \rightarrow N + \pi$</td>
<td>109 ± 121</td>
<td>77.4</td>
</tr>
<tr>
<td>$Y_1^* \rightarrow A + \pi$</td>
<td>32 ± 4</td>
<td>29.6</td>
</tr>
<tr>
<td>$Y_1^* \rightarrow \Sigma + \pi$</td>
<td>3.6 ± 1.4</td>
<td>4.45</td>
</tr>
<tr>
<td>$S^* \rightarrow S + \pi$</td>
<td>7.3 ± 1.7</td>
<td>12.3</td>
</tr>
</tbody>
</table>

As experimental widths, we use those of Ref. 9).

§ 6. Concluding remarks

As was stressed in § 1 and was discussed in §§ 3 and 4, it was one of our purposes to investigate the effects of interactions and relative motions among urbaryons by estimating the magnitudes of $c_0$ and $c_D$, which are not contained in

* $M$ and $M^*$ appearing kinematically in Eq. (5.7) should be the physical (not mean) ones.
the B-S amplitudes inferred from usual approaches. It has been shown by analysing electromagnetic form factors of baryons and strong $\Lambda^*\Lambda\eta$ vertex, etc., that the magnitudes of $c_0$ and $c_D$ are small even if such terms exist in the baryon B-S amplitudes and therefore the effect mentioned above are well absorbed in the simplified forms of B-S amplitudes derivable from $SU(6)$ or $\bar{U}(12)$ approaches, in contrast with the case of $0^-$ and $1^-$ mesons.

Next, we should remark that the absolute, not relative, values for the magnetic moments of baryons predicted by our theory are in good agreement with experimental ones. Similar results were obtained also by Salam, Delbourgo and Strathdee. It would be stressed, however, that our electromagnetic vertex, for example, was calculated as a dynamical result of interacting urbaryons, compared with rather formal and artificial definition of their magnetic current. That is, although the expression (3.11) for the magnetic moment is almost the same as that of Salam et al. in its form and magnitude [see Ref. 3]

\[
\mu_p = (1 + 2x), \quad \mu_n = -\frac{1}{2}(1 + 2x), \quad x = M/\langle \mu \rangle
\]

with $\langle \mu \rangle \approx 1000$ MeV (mean mass value of vector mesons), our final expression does not contain the vector-meson mass which is eliminated by the normalization of charge form factor (3.9), and $\kappa$ corresponding to $\langle \mu \rangle$ is just a parameter of B-S amplitudes of $0^-$ and $1^-$ mesons which was determined to be 1.09 GeV as the effects of urbaryon interactions by experimental information.

Final remark is related to the scheme of weak interactions with weak bosons $W_\mu^\pm$, in which the B-S amplitudes of weak bosons cannot be inferred from ordinary composite models. This fact may be considered as a peculiarity of weak interactions or a characteristic of "yet to be discovered" weak bosons.

Acknowledgements

The authors would like to express their sincere thanks to Prof. Y. Katayama for his kind interest in this work. We are also indebted to the members of Theoretical Physics Group of Department of Nuclear Engineering in Kyoto University for valuable conversations.

Appendix

Explicit forms of baryon vertices

In this Appendix we derive matrix elements between baryons of such currents as electromagnetic or weak ones, and of the strong decay of baryon resonances. For the purpose, we apply the same method as that of I and II to the case of baryons and their resonances.

The current $J_{\mu}^R(X)$ of our system considered here is expressed in terms of urbaryon field $\psi^a(x)$ as follows:
Interactions of Baryons in a Relativistic Composite Model

\[ J^{RI}(X) = T \{ \bar{\psi}(X) \Gamma^R \bar{T}^I \psi(X) \}, \quad \text{(A.1)} \]

where \( \Gamma^R \) denotes one of Dirac's \( \gamma \)-matrices and \( T^I \) one of unitary spin matrices. The matrix element of the current between baryons is given in the form:

\[
\int d^4X \exp[iq \cdot X] \langle P_1, s_1, \xi_1 \vert J^{RI}(X) \vert P_2, s_2, \xi_2 \rangle
\]

\[
= -i \sum_{\xi_1} \frac{1}{(2\pi)^{4}N_s} \sqrt{\frac{m_1 m_2}{E_1 E_2}} \frac{1}{q^2 - m_3^2 + i\varepsilon} \text{Tr}[\chi_M(q, 0, s_1, \xi_1) \Gamma^R T^I]
\]

\[
\times K_{s_1, s_2, i_1, i_2}(P_1, q, P_2), \quad \text{(A.2)}
\]

where \( m_1 \) and \( m_2 \) denote the final and initial baryon masses, and \( \chi_M \) is the B-S amplitude of the particle with mass \( m_3 \) mediating the processes concerned here. The function \( K_{s_1, s_2, i_1, i_2}(P_1, q, P_2) \) is expressed in terms of B-S amplitudes and of the irreducible diagram \( \mathcal{M} \) as is shown in Fig. 1:

\[
K_{s_1, s_2, i_1, i_2}(P_1, q, P_2) = \int d^4y_1 d^4y_2 d^4y_3 d^4x_1 d^4x_2 d^4x_3 d^4\varepsilon_1 d^4\varepsilon_2
\]

\[
\times \exp[i(P_1 Y + q X' - P_2 Z)] \chi_{y_1 y_2 y_3}(P_1, y_{1j}, s_1, \xi_1) \chi_{x_1 x_2 x_3}(q, x', s_2, \xi_2)
\]

\[
\times \chi_{\varepsilon_1 \varepsilon_2}(P_2, z_{1}, s_2, \xi_2) \mathcal{M}(y_{1j}, s_1; x_{1j}, s_2; z_1, z_2, z_3), \quad \text{(A.3)}
\]

where

\[
y_{1j} = y_{1j}' - y_j, \quad y_j' = y_j - \frac{1}{2} \sum_j y_j, \quad x_j' = x_j - x_j',
\]

\[
z_{1j} = z_{1j}' - z_j, \quad z_j' = z_j - \frac{1}{2} \sum_j z_j.
\]

![Fig. 1.](https://academic.oup.com/ptp/article-abstract/48/6/2358/1857761)

On the other hand, the matrix element of the decay process \( B^*(P_3, s_3, \xi_3) \)

\[
\rightarrow B(P_1, s_1, \xi_1) + M(P_2, s_2, \xi_2)
\]

is expressed by the function \( K \) as is seen from Fig. 1:
\[ \langle P_{s_t} \xi_1; P_{s_t} \xi_1; \text{out}|P_{s_t} \xi_1 \rangle = -i \sqrt{\frac{m_{s_t} m_{s_t}}{2E_{s_t} E_{s_t}} \frac{1}{(2\pi)^3}} \times K_{s_t s_t s_t s_t} (P_{s_t}, P_{s_t}, P_{s_t}) , \]  

(A-4)

where \( B^*(P_{s_t}, \xi_1) \), \( B(P_{s_t}, s_t, \xi_1) \) and \( M(P_{s_t}, s_t, \xi_1) \) denote baryon resonance, baryon and meson with momentum \( P_{s_t} \), spin \( s_t \) and unitary spin \( \xi_1 \), respectively.

Thus, the matrix elements of currents and of strong decay processes are determined completely in terms of B-S amplitudes and the irreducible diagram \( \mathcal{M} \). For the function \( \mathcal{M} \), we assume the same approximation as that of I and II, that is,

(i) the function \( \mathcal{M} \) can be approximated to the lowest-order irreducible diagram consistent with Okubo ansatz or Iizuka rule including the ordinary spin suffixes,

(ii) the momentum of the urbaryons in the function \( \mathcal{M} \) can be neglected in comparison with the mass \( m_0 \) of urbaryons.

These assumptions correspond in a sense to a partial neglect of the interactions and relative motions of urbaryon in the vertex part. However, taking account of the fact that the mass \( m_0 \) and the momentum \( P_{s_t} \) of baryons participating in a type of vertices (strong, electromagnetic or weak type) are nearly equal, we may expect that the effects neglected will give roughly equal contributions for various processes of a vertex type and therefore can be factorized to a simplified \( \mathcal{M} \). Under the above assumptions, the function \( \mathcal{M} \) is approximated as follows (see Fig. 2):

\[ \mathcal{M}_{a_t b_t c_t d_t e_t f_t} (y_1, y_2, y_3; x_1', x_2'; z_1, x_3, z_2) \]

\[ \cong m_0 [\delta_{a_1 a_2} \delta_{b_1 b_2} \delta_{c_1 c_2} \delta_{d_1 d_2} \delta_{e_1 e_2} \delta_{f_1 f_2}, \delta_{r_1 r_2} \delta_{s_1 s_2} \delta_{t_1 t_2} \delta_{u_1 u_2} \delta_{v_1 v_2} \delta_{w_1 w_2} \delta_{x_1' x_2'} \delta_{y_1' y_2' z_1' z_2'} \delta_{\xi_1' \xi_2'} \delta_{\eta_1' \eta_2'} \delta_{\zeta_1' \zeta_2'} \delta_{\upsilon_1' \upsilon_2'} \delta_{\omega_1' \omega_2'} \delta_{\phi_1' \phi_2'} \delta_{\theta_1' \theta_2'} \delta_{\psi_1' \psi_2'} \delta_{\chi_1' \chi_2'} \delta_{\pi_1' \pi_2'}] . \]

(A-5)

Substituting Eq. (A-5) into Eq. (A-3) and taking account of the fact that

Fig. 2.
the B-S amplitudes of baryons are even functions of internal coordinates $y’_{ij}$ and $z’_{ij}$, and further the fact that amplitudes of baryons are totally symmetric with respect to the interchange of spin and unitary spin index pair, we can show that three types of diagrams in Fig. 2 give the same result. Therefore, the function $K$ in Eq. (A·3) is simplified as follows:

$$K_{i_1 i_2 i_3 i_4 i_5 i_6} (P_1, q, P_2) = (2\pi)^6 \delta^4 (P_1 + q - P_2) \cdot 3m_0 \int \prod_{i=1}^{12} d^4 y_i d^4 z_i d^4 x_i d^4 x_i'$$

$$\times \delta^4 \left( \sum_i y_i' \right) \delta^4 \left( \sum_i z_i' \right) \delta^4 \left( x_i' + x_i \right) \delta^4 \left( y_{12} - z_{12} - x_i' + x_i \right)$$

$$\times \delta^4 \left( y'_{12} - z_{12} - x_i' + x_i \right) \exp \left[ i \{ P_1 \left( -y_i' + x_i \right) - P_2 \left( -z_i' + x_i' \right) \} \right]$$

$$\times \tilde{\tau}_{i_1 i_2 i_3} \left( P_1, y'_{ij}, s_{12}, s_i \right) \tilde{\tau}_{j_1 j_2 j_3} \left( q, x', s_{ij}, s_i \right) \tilde{\tau}_{k_1 k_2 k_3} \left( P_2, z'_{ij}, s_{ij}, s_i \right). \quad (A·6)$$

The product of three amplitudes in Eq. (A·6) is expressed by using Eq. (2·7) and its adjoint amplitude in the following way:

$$\lambda_{i_1 i_2 i_3} a_1 a_2 a_3 \lambda_{j_1 j_2 j_3} a_1 a_2 a_3 \lambda_{k_1 k_2 k_3} a_1 a_2 a_3$$

$$= \frac{8}{5} \tilde{D}^{(a_1 a_2 a_3)}_{(a_1 a_2 a_3)} T_{a_1 a_2 a_3} D_{(a_1 a_2 a_3)}^{(a_1 a_2 a_3)}$$

$$\times \frac{1}{2} \tilde{D}^{(a_1 a_2 a_3)}_{(a_1 a_2 a_3)} T_{a_1 a_2 a_3} D_{(a_1 a_2 a_3)}^{(a_1 a_2 a_3)} N_{a_1 a_2 a_3}^{(a_1 a_2 a_3)}$$

$$+ \frac{1}{3} \tilde{D}^{(a_1 a_2 a_3)}_{(a_1 a_2 a_3)} T_{a_1 a_2 a_3} D_{(a_1 a_2 a_3)}^{(a_1 a_2 a_3)} N_{a_1 a_2 a_3}^{(a_1 a_2 a_3)}$$

$$- \frac{1}{3} \tilde{D}^{(a_1 a_2 a_3)}_{(a_1 a_2 a_3)} T_{a_1 a_2 a_3} D_{(a_1 a_2 a_3)}^{(a_1 a_2 a_3)} N_{a_1 a_2 a_3}^{(a_1 a_2 a_3)}$$

$$+ \frac{1}{3} \tilde{D}^{(a_1 a_2 a_3)}_{(a_1 a_2 a_3)} T_{a_1 a_2 a_3} D_{(a_1 a_2 a_3)}^{(a_1 a_2 a_3)} N_{a_1 a_2 a_3}^{(a_1 a_2 a_3)}, \quad (A·7)$$

where $f_T = \text{Tr} (\bar{N} \{ T, N \} )$, $d_T = \text{Tr} (\bar{N} \{ T \} N )$, $C_T = \text{Tr} (\bar{N} N \text{Tr} (T) )$. 

References

M. Y. Han and Y. Nambu, Phys. Rev. 139 (1965), B1006.
2) R. H. Dalitz, Proceedings of the XIIIth International Conference on High-Energy Physics, 1966, p. 215. An extensive list of references is found at the end of this paper.
G. Morpurgo, Physics 2 (1965), 95.
Research group of the theory of elementary particles, Peking: Papers of the 1966 Summer Physics Colloquium of the Peking Symposium.
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