$$B_0 = 0.0255 + 1.35(10)^{-4}\frac{1}{r} \qquad [24]$$

where r is taken in feet units.

Extrapolation of Equation [24] to $r = \infty$ (the case of the flat plate) yields $B_0 = 0.0255$. From Table 6 it is seen that Latzko's theoretical value for the flat plate as interpreted by Jakob and Dow is 0.0253. They converted Latzko's equation into the form

$$N_{Nu} = 0.0356(N_{Re})^{0.80}N_{Pr} \qquad [25]$$

noticing that this equation was derived under the assumption that $N_{Pr} = 1$. Then they assumed that the equation may approximately hold for $N_{Pr} = 0.71$ (for air). The result is shown by the dotted line in Fig. 17.

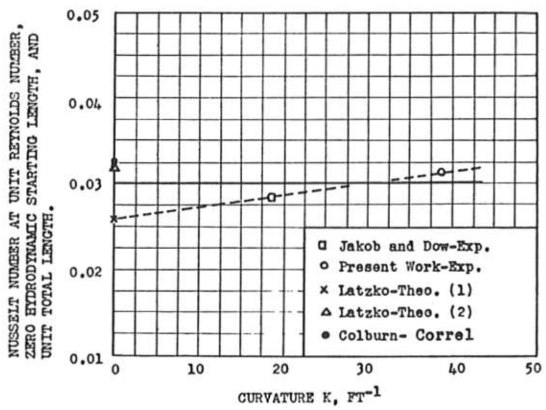

Fig. 17 Influence of Curvature
(1, As interpreted by Jakob and Dow; 2, As interpreted by Tessin and Jakob.)

Another interpretation would be that for $N_{Pr} = 1$, Latzko's equation converts to

$$N_{Nu} = 0.0356(N_{Re})^{0.80} \qquad [26]$$

Now there is evidence from empirical data as shown, for instance in Fig. 20 of Colburn's (5) paper, that

$$N_{Nu} = C_0(N_{Re})^{0.8}(N_{Pr})^m \qquad [27]$$

Colburn correlated data for flat plates with $m = 1/3$. Accepting this, comparison of Equations [26] and [27] yields

$$N_{Nu} = 0.0317(N_{Re})^{0.8}$$

for air streaming parallel to a flat plate. According to this interpretation of Latzko's equation, $B_0 = 0.0317$ for $r = \infty$.

For the time being, the constant $B_0 = 0.03$ independent of curvature, in the range investigated, seems to be the most probable result. This is shown by the full line in Fig. 17.

Therefore it is recommended that the following equation for air in the range $r = 0.3$ in. to $r = \infty$ be used

$$(N_{Nu})_x = 0.03(N_{Re})_L{}^{0.8}\left(\frac{x}{L}\right)^{0.91}$$

and for other fluids

$$(N_{Nu})_x = 0.034(N_{Re})_L{}^{0.8}(N_{Pr})^{1/3}\left(\frac{x}{L}\right)^{0.91}$$

The differences of the values of other observers cannot be explained at this time. It may be that different levels of turbulence in the arrangements of the various observers have caused this discrepancy. Continuation of the experiments with this in mind is planned.

Summary and Conclusions

1 A cylindric heat-transfer element, 0.624 in. diam, and various wooden nosepieces with hemispherical tips to be exposed to air streaming parallel to the axis were constructed. The ratio of thermal length to total length could be varied from 0.141 to 0.983.

2 In the experiments Reynolds numbers from $2.6(10)^4$ to $2.4(10)^6$ were obtained with transition starting at Reynolds number of 50,000 and ending from 300,000 to 600,000.

3 A correlation for Nusselt number based on mean surface coefficients was obtained, which included the effects of starting length.

4 An equation for Nusselt number based on local surface coefficients was derived, which was in satisfactory agreement with the experimental results.

5 Vibration and oblique crossflow studies were made to show that the present results were not influenced by these factors.

6 Comparing the results of a previous investigation by Jakob and Dow with the present results indicates a decrease of the heat transfer by 8.8 per cent if the cylinder diameter is increased from 0.624 in. to 1.3 in. Extrapolation to infinite diameter (flat plate), however, is uncertain and, therefore, it is recommended that an average value for the heat-transfer coefficient be used, independent of the curvature until further evidence of the influence of curvature and turbulence level is available.

BIBLIOGRAPHY

1 "Heat Transfer From a Cylindrical Surface to Air in Parallel Flow With and Without Unheated Starting Sections," by M. Jakob and W. M. Dow, Trans. ASME, vol. 68, 1946, pp. 123–134.

2 "Der Waermeuebergang an einen turbulenten Fluessigkeits- oder Gasstrom," by H. Latzko, *Zeitschrift für angewandte Mathematik und Mechanik*, vol. 1, 1921, pp. 268–290 (translated in US NACA Tech. Memo. No. 1068, Washington, D. C., 1944).

3 "Der Waermeuebergang an einer ebenen Wand," by W. Juerges, *Beihefte zum Gesundheits-Ingenieur*, Reihe 1, Beiheft 19, Munich and Berlin, Germany, 1924.

4 "Die Waermeuebertragung einer geheizten Platte an stroemende Luft," by F. Élias, *Abhandlungen aus dem Aerodynamischen Institut an der Technischen Hochschule Aachen*, No. 9, 1930, pp. 10–39; (translated in US NACA Tech. Memo. No. 614, Washington, D. C., April, 1931).

5 "A Method of Correlating Forced-Convection Heat Transfer Data and a Comparison With Fluid Friction," by A. P. Colburn, Trans. American Institute of Chemical Engineers, vol. 29, 1933, pp. 174–209.

6 "An Analytical Investigation of Convective Heat Transfer From a Flat Plate Having a Stepwise Discontinuous Surface Temperature," by M. W. Rubesin, advance copy ASME Paper No. 48—A-43.

7 "Evaporation of Liquids Into Turbulent Gas Streams," by D. S. Maisel and T. K. Sherwood, *Chemical Engineering Progress*, vol. 46, 1950.

8 "The Effect of an Arbitrary Surface-Temperature Variation Along a Flat Plate on the Convective Heat Transfer in an Incompressible Turbulent Boundary Layer," by M. W. Rubesin, US NACA Tech. Note No. 2345, Washington, D. C., April, 1951.

Discussion

D. S. Maisel.[8] This paper represents an extension of previous work carried out at the Illinois Institute of Technology on the transfer characteristics from a surface, where conditions for momentum transfer differ from those for mass or heat transfer. These have been concerned with the influence of a "starting section" over which momentum transfer alone occurs on the

[8] Esso Laboratories, Development Division, Standard Oil Development Company, Linden, N. J.

heat and mass transfer rates from the downstream transfer surface.

The authors conclude that the well-founded correlation between ratio of the length of the heat-transfer surface to the sum of the lengths of the starting section plus the transfer section means that the "correlation is independent of the absolute lengths of the thermal or starting portions and it seems to indicate that the thermal boundary layer reaches the thickness of the hydrodynamic layer almost immediately." This is very interesting since it would seem appropriate to believe that the two lengths should be independently important. The point of conversion from a laminar to a turbulent zone either in the starting length or heat-transfer surface, for example, should seem to affect the heat-transfer rates.

The various configuration functions shown by the authors in Table 2 do not seem to differ by any great degree. Considering errors inherent in measurement, each, with the possible exception of the Jakob and Dow function for small values of (X/L) should correlate data equally well. Therefore there may be some preference to use the form suggested by Rubesin, which is based on a theoretical analysis of the transfer dynamics.

Authors' Closure

We agree with Dr. Maisel's statement that the rates of heat transfer should be different according to whether heating starts at a Reynolds number below or above the critical point. We might have mentioned that all points from which Equation [9] was obtained, were taken from experiments in which s was larger than the critical distance s_{cr} at which turbulence started. Our equations of correlation do not cover the transient region where heating may have started below or above the critical Reynolds number; in this region the points for Y do not fall in a unique line, as can be seen from Fig. 16 at $(N_{Re})_L \leqq 500{,}000$. In addition to the ratio x/L a ratio $(s_{cr} + x)/L$ or any other length ratio, including s_{cr}, might enter the correlation. It would be difficult to determine s_{cr} with reasonable accuracy. Anyway, we did not find any influence of s_{cr}.

The first part of the passage, verbally quoted in Maisel's discussion, was included in the preprint of our paper, but was omitted as self-explanatory in the final draft; obviously, the right side of Equation [9] is independent of the absolute value of x or L-x, but dependent on their relative values x/L or $(L$-$x)/L$.

The second part of that quotation refers to the high exponent (0.91) of x/L. If the exponent were 1.0, then h_m would be independent of x. This means that the thermal boundary layer over the heating length would have the same mean thermal resistance, at whatever place of the hydrodynamic boundary layer heating were started. If one could assume that the resistance of the buffer layer were negligible compared with that of the laminar sublayer and the thickness of the sublayer did not change with x, then the mean thermal resistance would be the same for any value $s \gtreqless s_{cr}$; however, since the exponent is not 1, but only 0.91, the resistance need not be exactly the same. This assumption may be closer to reality than that made in our paper and quoted by Dr. Maisel.