tion (35) and $e_0$ is not a function of $y$ in steady flow, equation (36) becomes
\[
\frac{d^2 u}{dy^2} - \frac{\sigma B_0^2}{\mu} u = \frac{1}{\mu} \frac{dP}{dx} + \frac{\sigma}{\mu} e_0 B_0. \tag{37}
\]
Solving equation (37) subject to the no-slip condition $u(\pm a) = 0$ results in
\[
u = \frac{1}{B_0} \left( e_0 + \frac{1}{\sigma B_0} \frac{dP}{dx} \right) \left( 1 - \frac{\cosh My/a}{\cosh M} \right). \tag{38}
\]
where $M = B_0 a (\sigma / \mu)^{1/4}$ and is called the Hartmann number. The equation can be changed to equation (10.4) of reference [5] by a change of coordinates. In order to develop the resistance factor of equation (11), the local velocity $u$ must be found in terms of the average velocity $U$. We express the factor $\left( e_0 + \frac{1}{\sigma B_0} \frac{dP}{dx} \right) \left( 1 - \frac{\tanh M y/a}{\tanh M} \right)$ in terms of $U$ by integrating equation (38) with respect to $y$ over the channel since
\[
U = \frac{1}{a} \int_0^a u \, dy. \tag{39}
\]
The indicated operations yield
\[
- \frac{1}{B_0} \left( e_0 + \frac{1}{\sigma B_0} \frac{dP}{dx} \right) = \frac{U}{1 - \left( \tanh M \right) / M}. \tag{40}
\]
Then equation (38) becomes
\[
\frac{\nu}{U} = \frac{1 - \left( \cosh My/a \right) / \cosh M}{1 - \left( \tanh M \right) / M}. \tag{9}
\]
It should be emphasized that equation (9) is true regardless of the total electric current which flows between the walls of the channel ($y = \pm a$).

The substitution of $e_0$ from equation (40) into Ohm’s law, equation (5), yields
\[
J_y = - \sigma \left( B_0 U \frac{\tanh M}{M - \tanh M} + \frac{1}{\sigma B_0} \frac{dP}{dx} \right). \tag{41}
\]
after $J_y$ is averaged over $y$. If no net current $J_y$ flows then
\[
\frac{dP}{dx} = - \sigma B_0^2 U \frac{\tanh M}{M - \tanh M} = \frac{U \mu}{a^2} M^2 \tanh M, \tag{42}
\]
which clearly demonstrates that the pressure gradient is proportional to the mean velocity for a given Hartmann number. Equation (42) is equivalent to equation (11) when $\partial U / \partial t = 0$ and $J_y = 0$. With regard to equation (11) when $\partial U / \partial t \neq 0$, it must be remembered that $\partial P / \partial x$ is time varying and of course is not equal to the steady-state value indicated by equation (42).

**DISCUSSION**

F. T. Brown

The authors have made a fine contribution, showing specifically how the magnetohydrodynamic effect is precisely the same as a linear resistance to flow. It should be noticed that the resulting equations of motion [equations (18) and (19)] are the same as for the well-documented and fondly used constant L-R-C model of the electrical transmission line.

It may be helpful for the reader to generalize these equations into the form
\[
\frac{d}{dx} \begin{bmatrix} P \\ U \end{bmatrix} = \begin{bmatrix} 0 & Z \\ Y & 0 \end{bmatrix} \begin{bmatrix} P \\ U \end{bmatrix},
\]
which represents any linear symmetrical uniform transmission line with two variables, one undirected ($P$) and one directed ($U$). The solution of this equation, stated in terms of the pairs of variables at two stations of the line, $x = 0$ and $x = L$, is
\[
\begin{bmatrix} P(0) \\ U(0) \end{bmatrix} = \begin{bmatrix} \cosh T & Z_y \sinh T \\ \frac{1}{Z_y} \sinh T & \cosh T \end{bmatrix} \begin{bmatrix} P(L) \\ U(L) \end{bmatrix},
\]
Transmission Matrix

where

- **Propagation Operator**: $T = \sqrt{Z_y L}$
- **Characteristic Impedance**: $Z_y = \sqrt{Z / Y}$

The symbols and terminology in the transmission matrix are apparently and hopefully becoming standard.

For the present example, we have
\[
Z = \frac{1}{\rho} D + R
\]
\[
Y = \frac{1}{K_1} D
\]
in which $D$ is the operator $\partial^2 / \partial t$. For the specific problem discussed by the authors, $P(0)$ is zero and $U(L)$ is a step change. Other boundary conditions also are handled readily.

**Authors’ Closure**

The authors appreciate Dr. Brown’s interest in the paper and wish to thank him for his comments. The operator form of the equations is valuable in many instances, but if one wishes to put the solution in terms of time as an independent variable the inverse Laplace or Fourier transform can be quite nasty. For the present problem the inverse Laplace transform is given in reference [2].

4 Assistant Professor of Mechanical Engineering, Massachusetts Institute of Technology, Cambridge, Mass. Mem. ASME.

3 All the equations in this discussion are in operational form; operational methods greatly simplify the mathematics. For example, the substitution $D \rightarrow iw$ gives directly the frequency response.