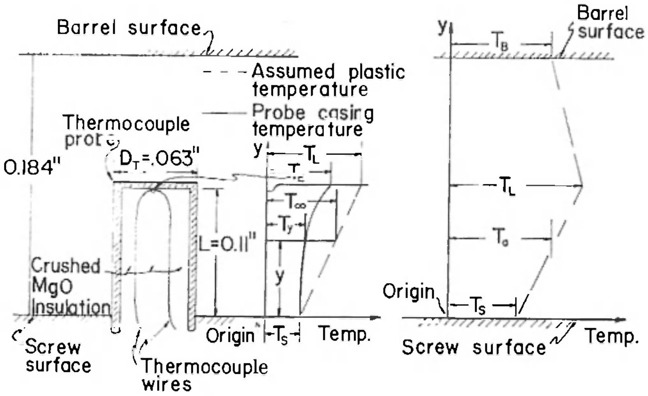

Fig. 5 Quantities and distributions involved in the determination of average plastic temperatures

assumption of a symmetric temperature distribution across the channel height will not introduce appreciable error into the determination of the average temperature, T_a. Pilot tests tended to support this assumption. It should also be noted that errors in the measurement and evaluation of average temperature are not really important directly so long as the same error is present in each measurement, since only the gross temperature difference along the screw is desired. However, since the viscosity coefficients a and b are computed at T_a, then the temperature determination should be accurate enough to yield reasonably accurate values of these quantities.

3 Heat transfer away from the thermocouple junction is due primarily to the probe casing of stainless steel. Estimates support this assumption. Referring to Fig. 5, let:

L = distance of thermocouple juncture $y = L$ from screw surface $y = 0$
T_y = probe temperature at a distance y from the screw surface
T_E = temperature at thermocouple junction sensed by the thermocouple (at $y = L$)
T_S = screw surface temperature
T_∞ = plastic temperature at a distance y from screw surface
T_L = plastic temperature at $y = L$
T_a = average temperature of plastic in which the thermocouple probe is embedded

For heat transfer along the y axis of the probe,

$$\frac{d^2T_y}{dy^2} = m^2(T_y - T_\infty) \qquad \text{(B-1)}$$

where $$m^2 = \frac{\pi D_T h_c}{K A_T} \quad \text{and} \quad \pi = 3.1416$$

D_T = outside diameter of probe casing
h_c = convective heat transfer coefficient between probe casing surface and plastic. (In BTU per hour-square foot-degree Fahrenheit.)
K = thermal conductivity of probe. (In BTU per hour-foot-degrees Fahrenheit.)
A_T = cross-sectional area of probe casing

The general solution to equation (B-1) may be written:

$$T_y = B_1 e^{my} + B_2 e^{-my} + T_s + (T_L - T_S)\frac{y}{L} \qquad \text{(B-2)}$$

Applying the boundary conditions:

a) $T_y = T_S$ at $y = 0$

b) $-KA_T \left.\frac{dT_y}{dy}\right|_{y=L} = \frac{\pi D_T^2 h_c}{4}(T_E - T_L)$

c) $T_y = T_E$ at $y = L$

there results:

$$T_E = \frac{T_L + \left[T_s - T_L\left(1 - \frac{\pi D_T^2 h_c L}{4A_T K}\right)\right]\frac{\tanh(mL)}{mL}}{1 + \frac{\pi D_T^2 h_c L}{4A_T K}\,\frac{\tanh(mL)}{mL}} \qquad \text{(B-3)}$$

For the present case, $K \sim 10$ and $h_c \approx 200$ BTU per hour square foot-degree Fahrenheit. By selecting

D_T = 0.0053 ft (1/16 in.)
L = 0.009 ft (0.11 in.)
A_T = 0.000012 sq ft (for wall thickness of 0.010 in.)

equation (B-3) reduces to:

$$T_E = \frac{T_s + T_L}{2} \qquad \text{(B-4)}$$

But for a linear temperature distribution in the plastic,

$$T_a = \frac{T_s + T_L}{2} \qquad \text{(B-5)}$$

Thus, $T_E = T_a$. At several locations along the barrel (Table 1) the inner barrel surface had hemispherical cavities into which the plastic flowed. Experiments with colored plastics showed that the plastic flowed smoothly up into the cavities and then returned to the screw channel. Thermocouple probes were projected from the barrel into these cavities. These probes were also designed to measure average plastic temperature.

DISCUSSION

Zeëv Rotem[3]

The authors are to be congratulated on their efforts to extend the reliability of prediction of non-Newtonian extruder pump performance to the case of nonisothermal flow. The following remarks may help to put their paper in perspective.

It is perhaps regrettable that the authors seem to be unaware of previous efforts in the field, and that they positively state that no previous paper predicts both the pressure-rise and the axial and/or radial temperature-rise in a molten plastic material when pumped through an extruder. In this context one may draw attention to the numerical results of Colwell [33],[4] the theoretical results of Rotem and Shinnar [34], and the experimental measurements of Marshall, et al. [35] (where further references are given).

Moreover, reference [6] of the author's paper seems to have been rather misunderstood. The equations given in that research (c.f. equation (24*a*)) give the complete integration of the equations of flow for a "general" (Reiner-Rivlin) fluid (devoid of cross-viscosity and elastic effects) for the isothermal case, while equations (8) and (9) of reference [34] do the same for the case of nonisothermal flow for viscosity pumps, using Colwell's "steady-state" model of flow. The variation of the rheological constants for two widely used models of temperature dependency is properly taken into account in [34]. The case of fluid behavior first stipulated by Rabinowitsch is obtained as a *special example* in both the isothermal derivation [6] and the nonisothermal [34]. Likewise, the pseudoplastic isothermal model is given in paragraph III of [6] complete with numerical results, while [34] gives

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[4] Numbers in brackets designate Additional References at end of Discussion.

the same for nonisothermal flow (without computational results) as well as the flow for a fluid following the Andrade model. Finally, in [36] the isothermal flow of a Ree-Eyring fluid is derived, and in [37] this is done for a "simple" fluid with three rheological constants.

Turning now to the author's theory: first, the assumptions stated after equation (1), specifically $\partial q_z/\partial z = 0$, imply not only that "the shear-stress gradients along the channel are comparatively small" but also stipulate that the flow profile at the inlet to the zone where the material is fully molten is also fully developed. This contention may be acceptable for viscosity pumps, but in the authors' case not only is the channel inlet tapered (their paragraph "Test Equipment") but there is also a rapid variation of rheological properties due to rapid temperature variation at the inlet. Therefore, the writer believes the assumptions call for more explanation.

Further, in equation (18), presumably d/dt is the substantive differential operator ("comoving derivative"?). But then, is there not a supplementary assumption involved in passing to equation (19) where only *one* partial derivative of the temperature is retained? For while q_y (authors' notation) may be small, as assumed after equation (1), the term

$$q_y \frac{\partial T}{\partial y}$$

in their equations is implicitly assumed to be possibly of the same order of magnitude as the term

$$q_z \frac{\partial T}{\partial z}$$

through the equation following (19) and their statement after equation (10). This result may be derived by using the equation of continuity,

$$\nabla_i q_i = 0 \tag{24}$$

and the assumption stated in the section "Theoretical Considerations,"

$$(w/h)_{\min} \sim 10. \tag{25}$$

The authors state that β is very small, in the text before equation (20) and in Appendix C. Then we obtain from equations (20) and (22) the following order-of-magnitude result,

$$\frac{\partial T}{\partial y} \sim O\left|\frac{\rho c_p q_z}{k} \cdot y \frac{\partial T}{\partial z}\right| \tag{26}$$

or, in dimensionless form, choosing the channel height (say) as reference length,

$$\frac{\partial T}{\partial \hat{y}} \sim O\left|\text{Pe}\; \hat{y}\frac{\partial T}{\partial \hat{z}}\right| \tag{27}$$

where Pe is the Péclet number, $\text{Pe} = q_z h/\kappa$; κ is the local thermal diffusivity, $\hat{y} = y/h$ and $\hat{z} = z/h$. Therefore, combining (27) and (25), we conclude that the term $\rho c_p q_y \partial T/\partial y$ must be retained on the left-hand side of equation (19) for values of Pe certainly not larger than of order 100. Now, the writer believes that for molten plastics Pe exceeds 100 considerably. From the authors' Table 2, Fig. 3 and Appendix C, taking as typical average values of the variables T = 450 deg F, M = 2 lb/min, p_a = 4000 psi we calculate for Pe a value of 2635. It is not surprising, therefore, to find that the computed values for the temperature-rise are occasionally found to be negative (authors' Table 6). Choosing again from Table 2 some average value for the rotational speed of 0.8 rps and calculating the rheological coefficients a and b from Appendix C one obtains

$$\xi^2 b \sim 0|0.57|$$

which leaves one to wonder whether the model of behavior under shear of the plastic chosen by the authors, which is characterized by 2 rheological coefficients, is truly adequate. What is the scatter of values found experimentally around the correlation assumed?

It is not clear to the writer from the section, "Materials and Properties," and from Appendix C, which polystyrene plastic was being used and how the rheological coefficients a and b were obtained using "the Couette method of capillary rheometry." The importance of channel taper is unfortunately not discussed though it is said to be of great importance [38, 39], nor that of channel curvature and aspect ratio discussed in [40–42], or the influence of "end effects" on temperature distribution [43]. Thus, it seems that the authors' basic assumptions would need more than just cursory justification.

The leakage between the screw landings and the barrel, not considered by the authors, may be calculated from [44], in which the solution is also given for both boundaries of the gap in movement.

Mention is made of "normal stresses" in the authors' section, "Theoretical Considerations," and in Appendix A. The writer thinks that the fact that these may have possibly nothing to do with the elastic properties of the material (see Serrin [45]) or, on the other hand, may be a function of the strain tensor, not taken into account in the paper in the 1st place, is not well appreciated by the authors. The writer understands that the paper was not meant to discuss the fundamentals of Rheology: then, in view of the controversy surrounding this point [46] the title of Appendix A ("Elastic Stresses") might be considered inappropriate. Appendix D gives the heat transfer analysis through a simple one-dimensional fin, available from any heat transfer textbook. The subject matter of this Appendix is thoroughly discussed in [35].

By the authors' own statement, the measured temperatures are liable to be in error "by 10 deg F to 15 deg F"; i.e., by about 30 percent of the total temperature-rise measured. In view of the strong dependence of the rheological coefficients a and b upon the temperature (for b indirectly, through the pressure-rise), one is led to wonder whether the close agreements between theory and test results claimed through Table 6, Figs. 3 and 4 and in the text are not purely fortuitous.

The writer believes that his comments draw attention to the danger of neglecting terms in the basic equations intuitively, without proper order-of-magnitude analysis. He hopes sincerely that the authors will be able to show that his remarks are misplaced and looks forward to their reply.

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James L. Throne[5]

It is obvious that the authors have approached a most complex process, that of pumping of macromolecular materials, from the fundamental viewpoint of continuum mechanics. Application of the continuum mechanical methodology to other complex processes such as calandering and mixing are beginning to yield valuable information as to the behavior of polymeric materials under nonisothermal, variable shear loads; information that is needed if accurate design and prediction methods are to be developed. This paper, then, represents a significant step in this direction.

There are, however, two assumptions that I feel the authors have not discussed in sufficient detail in their paper.

1 There is no discussion in the paper to indicate whether the polystyrene test material was (*a*) pumped in a molten form into the test apparatus, or (*b*) was fed as solid pellets into the screw, plasticated, melted and pumped. For the purpose of the remaining discussion, I shall assume the latter. In analysis of the melt metering section of the pump only, it is necessary to specify the initial conditions of the melt, e.g., the temperature and velocity profiles at the time the melt enters the test section. Owing to the complex nature of the plasticating process (see authors' reference [7] for visual details), prediction of these conditions seems impossible at the moment. The question then is twofold. First, how did the authors account for the presence of the feed and plasticating sections of the extruder and assure themselves that they had excluded evidence of these sections from their melt pumping analysis. Second, since polystyrene, while "nonNewtonian" in the power-law sense, is fairly nonviscous, was any consideration given to the possibility that, for materials that exhibit large amounts of viscous dissipation, velocity and temperature conditions at the $z = 0$ conditions could be significantly different than those assumed (or measured with a single-point thermocouple), and in fact consideration of velocity and/or temperature profiles might significantly affect the pressure per formance?

2 While the authors indicate agreement between theory and experiment for polystyrene material, I feel that the constitutive model chosen is probably too limited for general use. In particular, I feel that exclusion of normal stress effects on extruder performance is valid only for those materials, such as polystyrene which are relatively nonelastic. For elastic (or memory) materials such as ABS, energy storage is considerably enhanced, as is viscous dissipation and compressibility. While I am not convinced that the observed increases in power input for memory fluids are consequences of effects such as normal stresses (one cannot omit backflow the presence of screwflights in the rectangular channel, possible recoil energy losses upon fluid egress from the screw flights, the effect of channel curvature, etc.), the continuum mechanical approach seems to provide the clearest analysis. I would appreciate the authors' comments on the foregoing, particularly with regard to the suitability of their analytical approach to design prediction of extruder performance when the material is an elastic liquid.

[5] Supervisor, Heat Transfer, American Standard Research and Development Center, New Brunswick, N. J.

J. G. Williams[6] and R. T. Fenner[6]

There is little doubt that screw extruders are the most important pieces of equipment used in the plastics industry. Almost all plastic products pass through an extruder at some stage of their manufacture, and there is therefore a considerable interest in the industry in the efficient design and prediction of performance of these machines. A great deal of effort has been put into the field and it is now possible to isolate the main problems involved fairly accurately.

Perhaps, the greatest of these is the problem of the melting process in the feed and compression sections of the screws, but little can be done at this stage to give reliable predictions. The metering section, which normally contains only molten material is, however, more amenable to theoretical analysis. The problem is to solve for a complex three-dimensional, non-Newtonian flow of a very viscous, temperature sensitive fluid to quite a high level of accuracy. The practically important results such as pressures, flow rates, and temperatures are quite sensitive to most of the parameters involved, and so one is faced with an extremely difficult mathematical problem.

To include all the variables in a complete analysis is impractical because of the amount of labor involved, even with the aid of a computer. In many practical cases, however, some of the variables can be neglected as being unimportant for the particular set of conditions under consideration. This means that it is possible to make some assumptions and derive a solution which is practically useful under certain operating conditions. There is a danger, however, that having established the validity of the solution from one set of conditions, it is then applied in circumstances where it is not valid.

Having made these general comments one can now turn to the paper under discussion The method of analysis described consists of making a well-defined set of assumptions, and solving the resulting conservation equations for one particular form of constitutive equation relating stress and rate of deformation. This particular set of assumptions have not been used previously, so far as we are aware, so that the paper is an interesting contribution. Nevertheless we consider the evaluation of viscosity at a mean temperature at a particular cross section to be rather unrealistic for many practical applications. In general, it is necessary to allow for temperature variations between the screw and barrel by including the temperature dependence of viscosity in the equation of motion, and solving it simultaneously with the energy equation. For the experimental trials reported, however, the simplification was reasonable.

The method of solution employed in the paper depends entirely on the particular constitutive equation being applicable. While the empirical Rabinowitsch "law" is very convenient to use, it is not always a good approximation to melt flow behavior. For example, many workers have preferred a power-law representation, which would not be amenable to the analysis described.

In the experiments reported, the authors were fortunate in being able to measure both screw temperature profiles and the pressure at the starting point for the analysis. Both of these pieces of information were required as boundary conditions. In typical design and performance analysis problems, however, these data are not available. Perhaps, the authors would care to comment on how they propose to overcome this difficulty.

Authors' Closure

The authors wish to acknowledge the discussion by Professor Z. Rotem. Unfortunately, a number of his remarks appear to be misplaced, perhaps due in part to lack of clarity on the part of the authors.

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The author's statement at the top left of page 478 was apparently misinterpreted by Professor Rotem. That statement referred to the problem of simultaneous prediction of pressure and temperature rise of melted plastic along the channel of a screw pump. The emphasis was on prediction of axial changes, as well as radial gradients.

With reference to previous theoretical efforts in the field, the following remarks apply. Reference [34] by Rotem and Shinnar completely neglects axial heat convection and cross-channel flow. Basically, the problem solved in [34] is that solved by Colwell and Nickolls [3] and Colwell [33]. Numerous solutions have considered radial heat conduction, including references [3], [4], [7], [8], and [34]. The authors of the present paper only claim that their solution includes *axial heat convection*, radial heat conduction, and cross-channel flow.

The discussion of the convection term $q_y \partial T/\partial y$ is unnecessary when q_y is taken to be zero. Fortunately, q_y is not known to exist in the screw channel, except at the walls where turn-under occurs. Pending valid data on the existence and magnitude of q_y in the bulk of the screw channel, it would appear that discussion of $q_y \partial T/\partial y$ is both conjectural and inappropriate here. In any case, q_y is a wall effect, being positive at one wall and necessarily equal (upon integration) and negative at the other wall. Thus, the term $\rho c_p q_y \partial T/\partial y$ would tend to disappear upon integration over x. In the authors' paper, one computed temperature rise was negative (Table 6), apparently not because of the Peclet number, but rather because the measured initial melt temperature or the measured barrel temperature were slightly in error. The solution then predicted a slight melt cooling effect.

With regard to the discusser's questioning of the basic assumptions of the paper, we note that they are clearly stated as being assumptions or approximations. Many of these same assumptions, either implicitly or explicitly, are made in references [1] through [8], and [36]. In order to solve the problem in the present paper, including the cross channel flow and axial temperature gradients, all of the assumptions made were considered to be reasonable approximations. Of course, one would prefer to solve the problem with no approximations, if that were possible.

Some of the discusser's doubts about the agreement between our solution and our experimental data relate to the rheological equation (5) and his estimation of $\zeta^2 b$ in that equation. Using $\zeta \sim 6$ psi and $b \sim 0.2$ (psi)$^{-2}$ as typical values, we estimate $\zeta^2 b$ as being on the order of 7. Using formulas for a and b in Appendix C, he estimates $\zeta^2 b$ (misprinted as $\xi^2 b$) as being of order 0.57. It is true that if $\zeta^2 b$ were of order 0.57, then either the plastic was nearly Newtonian, or the rheology curve fit was very poor. In fact, however, $\zeta^2 b$ was of order 7, the polystyrene plastic was highly non-Newtonian, and the curve fit was quite good. Unfortunately, the preprint of the paper contained a publication error in the formula for coefficient a, which probably caused the discusser's erroneous estimation of $\zeta^2 b$, and for this we apologize.

The rheology data were obtained from a capillary rheometer with a backpressure chamber for independent hydrostatic pressure control. The necessary calculations were made to obtain true capillary wall shear stresses and shear rates (not apparent stresses and shear rates). For the ranges of melt temperatures and shear rates occurring in the performance predictions of experiments 1 through 20 (Table 2), the rheological correlation was quite appropriate and accurate. The curve fit on shear stress was within 5 per cent from the data points, which is equivalent to about a 5 deg Fahrenheit deviation.

The authors appreciate Professor Rotem's comments on leakage flow, channel curvature corrections, and wall effects (aspect ratio). These corrections should always be made with theories or data that have proven usefulness. They can be important in pressure gradient prediction, especially when the difference between extruder output and "drag flow" is very small in comparison with the output. The state of the art in predicting these effects or corrections is certainly not complete. Some good analyses are presented in reference [14] on wall effects, references [15] and [41] for curvature effects, and reference [44] for leakage flow.

The trivial boundary value problem solution found in Appendix D is simply used as a vehicle for clarifying the assumptions and calculations made by the authors in connection with their experimental determination of average plastic temperature.

With reference to the possible errors in measured temperature rises of 10 deg F to 15 deg F, the discusser assumes that the temperature dependence of coefficients a and b is strong enough to cause gross errors in prediction due to a 15 deg error in measured temperature rise. In fact, one can easily estimate that this 10 to 15 deg F error in temperature rise results in about a 10 percent error in melt viscosity (using the given formula for a and b and typical conditions from Table 2). This error in viscosity would cause an error in total pressure rise of about 10 percent. The indirect effect of temperature change on coefficient b was included in the above estimation. A 10 percent error in total pressure rise does not appear great enough to cause any "fortuitous" agreements between the theoretical predictions and the test results as given in Figs. 3 and 4 and Table 6.

As stated in the text of the paper, the present solution should be advantageous only when the axial temperature change of the melt is not small, or the cross-channel shearing effects are significant, or both. The most constructive analysis of the authors' solution for the extruder pump problem would include numerical comparisons of this solution with other non-Newtonian solutions and with extruder test data. Nevertheless, the authors sincerely appreciate Professor Rotem's interest, his comments which they have indicated to be pertinent to the solution, and the additional references which he has supplied which makes the bibliography more complete.

The authors would like to thank Mr. J. L. Throne for his discussion. The following remarks should hopefully answer most of his questions. The polystyrene plastic was fed to the extruder as pellets, which were then melted in the screw. Complete melting was insured, at the point where the plastic entered the test section, by prior experimentation. Thermocouple probes protruding from the screw surface are the best indicators of the condition of the plastic. Both the absolute values and the variations in these temperature traces indicate whether the probes are sensing solids, fluid, or both. Also, the degree of roughness and "spikes" in the trace of each pressure transducer is a supplementary indication of melt condition. Concerning the entrance conditions of the melt for the authors' solution, it is not necessary to specify the melt velocity profile or the melt temperature profile as it enters the test section. The only velocities needed are those of the boundary screw and barrel surfaces. As initial conditions, only the average melt pressure and temperature, p_a and T_a, are needed, and these were measured as outlined in the paper.

Viscous dissipation in the vicinity of and due to the melt thermocouple probes was considered by the authors. For the extruder tests described in Table 2, the local velocities and shear rates created a relatively small error in the measurement of T_a. If, however, the velocities or viscosities had been an order of magnitude higher than in these tests, the viscous dissipation at the thermocouple probes would have caused significant errors in measuring T_a (see, for example, Van Leewan, reference [25]). Therefore, in our extruder tests, the measurement of T_a should not have had any significant effect upon pressure prediction or performance. Polystyrene, by the way, is rather viscous; i.e., its viscosity is of the same order of magnitude as other thermoplastics, under processing conditions.

The thermoplastics of greatest consequence in extrusion today, namely PVC, polyethylene, and polystyrene, are basically similar in melt elastic response. None of these three plastics would be considered to be highly elastic. If the material being pumped in the extruder is very much more elastic than polystyrene, then the authors extruder solution could not be considered to be adequate. A comprehensive extruder theory for the pumping of highly elastic fluids is not presently available.

The discussion by Professor J. G. Williams and Professor R. T. Fenner is appreciated by the authors. The problem solved in the authors' paper consisted of solving for both axial and radial temperature gradients, for a complex geometry, including cross flow effects. To do this, viscosity was evaluated at the mean radial temperature at each cross-section. However, prior non-Newtonian, non-isothermal solutions contained more approximations. References [3], [4], and [34], for example, each neglect axial temperature gradients completely. Evaluating the rheology coefficient, a, at a mean radial temperature may not be perfectly realistic, but it seems preferable to completely neglecting axial heat convection for general extruder pump prediction. In some cases, the mean temperature method will be rather unrealistic. However, our experience has been that most extruder problems apparently suffer much less from this approximation than they do from the absence of axial convection in the energy balance. A more general solution, when it appears, will certainly be appreciated.

The Rabinowitsch "law" nearly always fits thermoplastic melt stress-strain rate data better than a simple one-term "power law" representation. Equation (5) contains both linear and cubic terms in shear stress. When $b = 0$, the description is linear; i.e., the slope is 1 on a plot of log ζ versus log ϵ. But by increasing b, the slope approaches $^1/_3$ on a plot of log ζ versus log ϵ. The advantage of equation (5) over a power law description is that, on a log ζ versus log ϵ plot, equation (5) can be nearly straight or very curved, but the power law plot must be a straight line. The disadvantage of equation (5) is that it is limited to a slope of $^1/_3$. Fortunately, many thermoplastics have slopes between 1 and $^1/_3$ on a log ζ versus log ϵ plot. However, in a case where slopes up to $^1/_5$ are required to fit the rheology data, equation (5) can easily be modified by adding one term as follows:

$$\epsilon = \frac{\alpha}{2} (1 + b\zeta^2 + c\zeta^4)\zeta \tag{28}$$

Equations (6) and (9) are then replaced by the following equations:

$$\epsilon_{ij} = \frac{\alpha}{2} (1 + \frac{b}{2} \zeta_{rs}\zeta_{rs} + \frac{c}{4} [\zeta_{rs}\zeta_{rs}]^2)\zeta_{ij} \tag{29}$$

$$\frac{\partial q_z}{\partial y} = \alpha\zeta_{yz}[1 + b(\zeta_{yz}^2 + \zeta_{xy}^2) + c(\zeta_{yz}^2 + \zeta_{xy}^2)^2]$$
$$\frac{\partial q_x}{\partial y} = \alpha\zeta_{xy}[1 + b(\zeta_{yz}^2 + \zeta_{xy}^2) + c(\zeta_{yz}^2 + \zeta_{xy}^2)^2] \tag{30}$$

The solution using equations (30) follows that presented in the paper, but the algebra is more lengthy.

In all nonisothermal extruder solutions, the knowledge of the screw temperature profile is tacitly assumed. In actual extruder screw design, one cannot normally specify the screw temperature profile, unless both screw temperature control and measurement are available. To the best of our knowledge, no theoretical solution for computing screw temperature has been published. In theory, T_S can be computed by additional heat transfer equations, but in practice T_S is usually empirically related to T_a, and they are solved for simultaneously at each cross section.

In the actual design of plasticating extruders, the melt pressure (and temperature) at the beginning of the pumping zone is required. This information is found by writing and solving the equations describing the melting zone of the extruder. A melting zone analysis was written by one of the authors (H. K.) in 1967 and has been used with fairly good success on polystyrene materials. Of course, the melting zone analysis requires more approximations and empirical data correlation than the pumping zone analysis. The melting zone analysis is inappropriate in cases of severely intermittent solids flow and melting, or in case of partial screw channel blockage by the solids.