Under the assumption (34) for the time factor, equation (32) yields a relation of the form

$$(C_1 + C_2)e^{2\lambda t} + D_1 e^{\lambda t} = (E_1 + E_2)e^{2\lambda t} + \text{const}, \quad (39)$$

where D_1 arises from

$$p\delta W_n + W_n \delta p,$$

where δW_n, δp are due to small displacements from the equilibrium values. Such an equation can hold if $D_1 = 0$ and $E_1 + E_2 = C_1 + C_2$.

Nevertheless, equation (19) may prove very useful in saving some of the extensive integrations described in the introduction, since it gives a universal relation between F_r, F_t, V_r, V_t, and

$$Q = \int\int ph\, dx\, dy/RT.$$

If the terms involving $\dot{p}$ are not neglected, then equation (31) is replaced from equation (29) by

$$\frac{d}{dt}(K + P - 2RTQ) = -2\int\int \dot{p}h\, dS, \quad (40)$$

from which it is even more difficult to draw simple conclusions regarding rotor stability.

References

1 B. Sternlicht, H. Poritsky, and E. Arwas, "Dynamic Stability Aspects of Cylindrical Journal Bearings Using Compressible and Incompressible Fluids," First International Symposium on Gas-Lubricated Bearings, October 26–28, 1959, ACR-49, Office of Naval Research, Department of the Navy, Washington, D. C. (see Equation (A), p. 159).

2 C. H. T. Pan, "Some Basic Aspects in the Theory of Hydrodynamic Gas Journal Bearings," ONR Report, September 14, 1960.

DISCUSSION

C. H. T. Pan[2]

The author has derived a remarkable relation valid for all gas lubricated self-acting journal bearings with a continuous fluid film in the absence of journal misalignment. He needed only three assumptions:

1 The inertia of the fluid film is negligible relative to the viscous forces.

2 The gas obeys the ideal equation of state.

3 The temperature of the fluid film is uniform and constant.

The author is very kind in giving this discusser the credit of the first derivation of the one-dimensional version of this relation. Actually, Elrod and Burgdorfer (Ref. [3]), independently and at an earlier date, discussed a very similar relation with reference to the mass content of the fluid film in a journal bearing at steady-state operation. Subsequently, various investigators have extended this idea to consider the dynamical problem of an "infinitely long" (one-dimensional) journal bearing. This has been known as the Elrod-Burgdorfer constant mass condition. Equation (29) in this paper remains the most general relation available in published literature.

Further generalization to include journal misalignment and flexibility, as well as effects of external pressurization and specialty geometries such as grooves is rather straightforward. Taking these into account, one only needs supplement eq. (29) in the paper with the following equations:

$$W = \int_{-L/2}^{L/2} (v_r f_r + v_t f_t) dy; \quad (41)$$

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$$Q = \dot{Q}_e + Q_c + Q_g; \quad (42)$$

where

$$f_r = \int p \cos\theta dx; \quad (43)$$

$$f_t = \int p \sin\theta\, dx. \quad (44)$$

$\dot{Q}_e$ represents mass flow into the film through the ends as was considered in the paper. Q_c and $\dot{Q}_g$ represent mass flow into the film, respectively, from the compensator and specialty geometrical appendages such as grooves. v_r and v_t are the radial and tangential velocity components of the geometrical center of the cross section of the journal and are dependent of the axial co-ordinate in the case of a misaligned journal.

The author has challenged the validity of stability studies based on the infinitely long bearing theorem. This discusser, as a matter of personal taste, shares the author's misgiving about the infinitely long bearing theorem, but cannot agree with the argument used by him. His case was built upon a very dubious argument that $\partial p/\partial t$ can be disregarded. It is very easy to disprove this point by a specific example. Consider the case that the journal center is undergoing a steady translatory whirl motion about the center of a plain cylindrical bearing with the angular speed $\dot{\alpha}$. After complete decay of any transient condition, the "steady-state" film pressure is stationary with respect to co-ordinates with their origin at the journal center and rotating with $\dot{\alpha}$, and $\dot{Q}$ vanishes regardless of the length of the bearing. Fig. 1 of the paper actually suggests adoption of the rotating co-ordinates. In principle, this pressure can always be expressed as a Fourier series of the angular variable θ of the rotating co-ordinate system. That is

$$p = \text{Re}\left[\sum_{n=0} P_n(y)e^{in\theta}\right] \quad (45)$$

Since

$$\frac{d\theta}{dt} = -\dot{\alpha}, \quad \text{therefore}$$

$$\frac{\partial p}{\partial t} = -\dot{\alpha}\,\text{Re}\left[\sum_{n=1} inP_n(y)e^{in\theta}\right] \quad (46)$$

Consequently, the time dependent term in eq. (29), instead of vanishing, should be

$$\int\int h\dot{p}dS$$

$$= \int\int -\dot{\alpha}C(1 + \epsilon\cos\theta)\,\text{Re}\left[\sum_{n=1} inP_n(y)e^{in\theta}\right] dS$$

$$= -C\epsilon\dot{\alpha}\int\int \cos\theta\,\text{Re}\,[iP_1(y)e^{i\theta}]dS$$

The last integral can be readily shown to be the same as

$$\int\int \sin\theta p dS = F_t$$

and since $V_t = C\epsilon\dot{\alpha}$, one obtains

$$\int\int h\dot{p}dS = -F_t V_t \quad (47)$$

Substituting the last result into eq. (29) one actually obtains the uninteresting, nevertheless correct relation

$$W = F_t V_t \quad (48)$$

In fact, according to detail analysis (Ref. [4]), it can be shown that for the steady whirl problem, $W = F_t V_t > 0$ only when $0 < \dot{\alpha}/\omega < 0.5$ for bearings of any length. The last conclusion has been amply borne out by experimental evidence (Ref. [5]). This example clearly disputes the author's assumption of negligible $\partial p/\partial t$.

Additional References

3 H. G. Elrod, Jr., and A. Burgdorfer, "Refinements of the Theory of the Infinitely Long, Self-Acting, Gas-Lubricated Journal Bearing," Office of Naval Research Contract Nonr-2342(00), Task NR 097-343, January, 1960, Franklin Institute Interim Report No. I-A2049-10.

4 C. H. T. Pan and B. Sternlicht, "On the Translatory Whirl Motion of a Vertical Rotor in Plain Cylindrical Gas-Dynamic Journal Bearings," Journal of Basic Engineering, Trans. ASME, Series D, vol. 84, 1962, pp. 152–158.

5 R. C. Elwell, R. J. Hooker, and B. Sternlicht, "Gas Bearing Stability Study—Vertical Rotor Investigation," General Electric T.I.S. Report No. 60GL88, May 20, 1960.

Author's Closure

I am indebted to Dr. C. H. T. Pan for calling my attention to Elrod and Burgdorfer's earlier utilization of the "mass content" Q of infinitely long bearings.

The proof, derived from infinitely long bearing theory, of small displacement stability near the position of journal center corresponding to any applied load, is disconcerting. Dr. Pan ascribes the difficulty to neglect of the term $\partial p/\partial t$. As evidence, he presents the case of a steady, circular whirl of a rotor In this case (unless I am mistaken) he suggests adopting axes with center at the bearing center, and rotating with the same angular velocity as $\dot{\alpha}$ at the journal. Relative to these axes the rotor center is stationary, the velocity W_n vanishes, and after the transient has died down, $\partial p/\partial t$ vanishes; however, equation (1) no longer applies, due to the velocity $-a\dot{\alpha}$ of the bearing surface. In deriving equations (46)–(48), Dr. Pan must have had stationary axes in mind.

It is possible, under the assumptions of infinitely long bearing theory, to compute the effect of $(\partial p/\partial t)$-term. To this end we replace the term $2\rho W_n$ in the equation (1) by

$$2\rho \frac{\partial h}{\partial t}, \tag{49}$$

(in which form it is applicable even when the bearing surface is moving normally). Then, equation (1), upon integration over the bearing surface, leads to

$$0 = \int \left(\rho \frac{\partial h}{\partial t} + h \frac{\partial \rho}{\partial t}\right) d\theta = \frac{d}{dt} \int (\rho h) d\theta, \tag{50}$$

provided, again, that the terms involving $\partial/\partial y$ and representing the flow across the journal ends are neglected. Multiplication by La converts the last integral into Q, and equation (50) into constant mass equation

$$0 = \frac{dQ}{dt} \tag{51}$$

As pointed out by Elrod and Burgdorfer, equation (51) holds even for nonisothermal gas behavior and applies even to a liquid lubricant (provided that it does not cavitate).

From (50) it follows that the term $h\dfrac{\partial \rho}{\partial t}$, when integrated, yields a term which is equal and opposite to that of the integral resulting from the term $\int \rho W_n d\theta$. In place of (10), one now arrives at the "corrected" relation, which also follows from equation (29):

$$V_r F_r + V_t F_t = -\int \frac{\partial p}{\partial t}\, h dS. \tag{52}$$

This verifies Dr. Pan's equation (47) for the circular whirl.

The difficulty in applying the above lies, of course, in the fact that $\partial p/\partial t$ is generally unknown. With the $\partial/\partial t$ term retained, equations (1), (3) become nonlinear partial differential equations involving t as well, and the difficulties of integrating them mount, even if one assumes that past history for $t < 0$ can be forgotten, through the attainment of some steady state at $t = 0$.

For the above or other reasons, a great deal of bearing theory *is* based upon the assumption that $\partial p/\partial t$ in (1) can be neglected. For instance, this assumption is usually made in attempts to integrate (1) or (3) for p, in order to obtain F_r, F_t as functions of V_r, V_t and of the other pertinent parameters. If equation (3), representing the infinitely long bearing, is used, the value of F_r, F_t so computed should satisfy (10), and to them the small displacement stability proof contained in equations (30)–(38) should apply.

Continuing with small displacements near a steady deflection position, a possible procedure for including the effect of the $\partial p/\partial t$-term is as follows. For small displacements about the steady position corresponding to the given external load, the coordinates and velocities of the journal center vary with time as $e^{\lambda t}$, where λ is a constant. Hence, it might be proper to assume that the pressure increment Δp likewise varies as $e^{\lambda t}$, thus putting

$$\frac{\partial p}{\partial t} = \frac{\partial p_0}{\partial t} + \frac{\partial(\Delta p)}{\partial t} = \lambda \Delta p, \tag{53}$$

where $p_0(\theta)$ is the steady-state pressure, determined by solving (3) for $\epsilon = \epsilon_0$, $V_r = V_t = 0$, and Δp the added pressure due to the (small) motion of the journal center. This Δp is solved from the linearized equation (1), putting

$$\left.\begin{aligned} \epsilon &= \epsilon_0 + \epsilon_1 e^{\lambda t}, \\ \alpha &= \alpha_0 + \alpha_1 e^{\lambda t}, \\ V_r &= C\epsilon_1 \lambda e^{\lambda t}, \\ V_t &= C\epsilon_0(\alpha_1 \lambda) e^{\lambda t}, \\ p &= p_0(\theta) + \Delta p e^{\lambda t}, \end{aligned}\right\} \tag{54}$$

with the $\partial/\partial y$ terms neglected, for any assumed ϵ_1, α_1, and λ. Next ΔF_r, ΔF_t are computed. The dynamical rotor equations are then set up, $e^{\lambda t}$ is canceled, and ϵ_1, α_1 are eliminated. This leads to an equation for the determination of λ. The stability is then determined by examining the real part of λ.

It seems unlikely that similar use of the equation which is now replacing (52) could determine stability conditions without actual integration of the Reynolds equations.