Case II

$$A_r = \frac{1}{16} \qquad p_0 = 430.6 \text{ psf}$$

$$L = 0.375 \text{ in.} \qquad C_D = 0.63$$

$$\frac{L}{a} = 6 \qquad f = 0.050$$

$$V_0 = 9.0 \text{ fps} \qquad q'(0) = 0.128$$

Case III

$$A_r = \frac{1}{4} \qquad p_0 = 265.7 \text{ psf}$$

$$L = 0.049 \qquad C_D = 0.617$$

$$\frac{L}{a} = 0.39 \qquad f = 0.050$$

$$V_0 = 12.0 \qquad q'(0) = 0.140$$

The zero points for pressure and time are noted in Fig. 9. It should be pointed out that the zero point could not be preset since the oscilloscope was set to trigger the sweep internally on a certain vertical deflection of the beam. As less total deflection occurred it took a greater time to reach the minimum input to trigger the scope. However, the traces could be extrapolated back until they intersected the zero pressure line and this is how the zero point was determined. In each case the zero point was about 1.5 ms from the peak pressure which occurs at the instant the valve is completely closed. This agrees with the known 1.5 millisec valve closing time.

Comparison of Experimental and Analytical Results

The three cases corresponding to experimental conditions were solved using the closed form solution of equation 16 for instantaneous valve closure. The results for the pressure response are shown in Fig. 10. There is a considerable discrepancy between these and the experimental data shown in Fig. 9. However, since the decay times are on the order of the closing time, such a discrepancy is to be expected. To more closely approach experimental conditions, it was necessary to use the modified plane wave analysis which considers the valve closing over a specified time interval. This analysis assumes that the flow through the valve was linearly decreased from its initial value to zero in 1.5 millisec. The analytical results for the three cases are shown in Fig. 11. The experimental data presented in Fig. 9 are also shown on these curves.

It can be seen that the analytical and the experimental results are in good agreement. The maximum pressures are compared in Table 3.

The primary difference between the predicted and observed results is that the calculated pressure peak is very sharp while that observed is quite rounded. There are several reasons for this. One is that the valve closure function is not completely known and would actually be somewhat different than the linear one assumed. Some of the discrepancy could be partly due to nonlinear compressibility of the liquid. Also some errors may be introduced because f and C_D were assumed constant for varying flow conditions. In general, it is felt that the comparison is satisfactory and that equation (12) is a reasonably accurate description of the dynamics of short lines and orifices.

Table 3 Maximum predicted and observed pressure surges

Case	Observed surge (psi)	Predicted surge (psi)
I	320	328
II	140	146
III	45	47

Conclusions

A time dependent description of the dynamic response of orifices and short lines is formulated. This expression gives results which compare well with experimental data. The good agreement between the predicted and observed results indicates that the various assumptions made in the analysis are acceptable. The study shows that significant transient effects can occur due primarily to the inertia of the fluid in the vicinity of, and within the orifice. However, these effects are short term and decay rapidly. As the axial dimension of the orifice decreases the decay time becomes very small. It is, therefore, likely that in many practical engineering situations the use of steady-state characteristics to describe the transient behavior of the orifice is acceptable. However, in systems with rapidly occurring transients or those requiring a rapid response, it may be necessary to consider the dynamic behavior of orifices or other hydraulic components in the system.

Acknowledgment

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DISCUSSION

C. F. Holt[3] and J. M. Robertson[4]

As the authors note, in certain cases the transient response of an orifice can differ greatly from the generally assumed instantaneous steady state. We have been analyzing the transient compressible flow occurrences in positive displacement pumps of fluid power systems and find that the assumptions made for such a transient response can markedly affect the results. The authors' paper presents a welcome discussion of this phenomena. Since our transient analysis differs somewhat from the authors' approach, these comments are based on our familiarity with the flow problem. Specifically we note some contradiction in the use of a discharge coefficient for an outflow assumed to be a source. A discharge coefficient of the order of the usual value (about 0.6) implies a separated jet flow downstream which is not permitted in the model chosen. An alternative approach permits one to separate the two concepts.

In considering the transient response of an orifice one may apply the unsteady Bernoulli equation from a point for upstream

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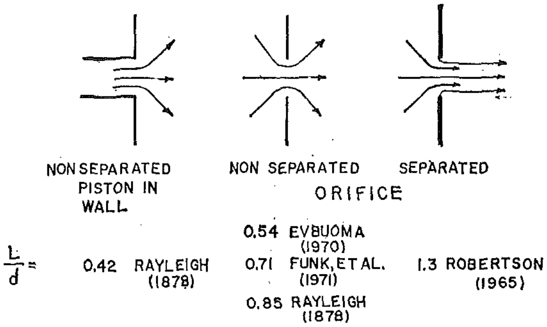

Fig. 12 Values of the inertial length coefficient for various flow conditions

to the region of minimum flow area at or just down stream of the orifice, as indicated by Robertson [10].[5]

If we make the assumption that the uniform one dimensional velocity at each station x can be written as

$$u(x, t) = u(x_2, t)\,\frac{u(x, \infty)}{u(x_2, \infty)}$$

then the unsteady Bernoulli equation written from x_1 to x_2 appears as

$$L\,\frac{du(x_2, t)}{dt} + \frac{u^2(x_2, t) - u^2(x_1, t)}{2} + \frac{P_2 - P_1}{\rho} = 0 \qquad (27)$$

with

$$L \equiv \int_{x_1}^{x_2} \frac{u(x, \infty)}{u(x_2, \infty)}\,dx$$

The integral L, is termed the inertial length and represents the characteristic length of a column of fluid being accelerated through the orifice. It is noted that L may be based on steady inviscid flow. The integration of equation (27) is then straightforward with the only difficulties arising in the determination of L. Robertson [10] has evaluated the inertial length coefficient for a few cases. Thus for a rounded pipe entrance a half diameter long, the inertial length (the lengths to be added to that of the tube) is about one diameter. Similar calculations of L for a circular orifice in the side of a large reservoir indicates a value (for separated flow) of about one and one third diameters. Recently one of our co-workers (S. N. Evbuoma) has attempted solution of the axisymmetric nonseparated orifice flow and found L for this case of about $0.54d$. The inertial or effective length concept was introduced by Rayleigh [11] in his explanation of the difference between predicted and measured impedance of a Helmholtz resonator. In order to obtain agreement with experimental observations it was found necessary to assume an apparent increase in the length of the entrance tube of the resonator. This length was found to be related to the diameter of the opening, d, as

$$L = l_t + 0.85d$$

Here l_t represents the thickness of the orifice or tube and $0.85d$ represents a correction of $0.85d/2$ added to each end. In the case of an orifice with zero thickness $0.85d$ represents the total effective length. This result assumes that the opening is surrounded by an infinite baffle at both ends. Many investigators [12, 13, 14] have confirmed the Rayleigh end correction as being accurate within limited flow regimes. Thurston, et al. [12, 13], have shown that this end correction is valid for flow through an orifice and a short tube within the linear region where resistance is independent of volume flow rate. They did not evaluate the inertial correction for the nonlinear region. Bolt, et al. [14] have shown that within the linear region, for an orifice placed in a long tube of radius R the effect of the tube walls is to alter the Rayleigh end correction from the maximum given as $0.85d$ with $d/R < 0.2$ to zero for $d/R = 1$.

For the present paper an inertial length can be calculated from the equations presented by the authors. Thus equation (27) is of the same form as the authors' equation (10) which can be rearranged to give:

$$\frac{P_1 - P_2}{\rho} = \frac{C_D A_0}{\sqrt{\dfrac{C_D A_0 \pi}{2}}}\,\frac{du}{dt} + \frac{u^2}{2}, \qquad \text{where } u = \frac{q}{C_D A_0}$$

The coefficient of the unsteady term can be identified as the inertial length which may be written as:

$$L = \left(\frac{C_D}{2}\right)^{1/2} d \qquad \text{where } A_0 = \frac{\pi d^2}{4}$$

For $C_D = 0.625$ we have $L = 0.556\,d$. This result applies to an orifice of assumed zero thickness. The inertial length depends on the discharge coefficient and implies that L for separated flow is less than L for nonseparated flow. With $C_D = 1$, one gets $L = 0.707\,d$ which is close to the Rayleigh value of $0.85\,d$. Our approach indicates that flow details quite near the finite sized orifice have a significant effect on L and yet this seems to have been ignored in the authors' approach. Fig. 12 compares calculated inertial lengths and their associated flow fields. For the orifice in a large wall without flow separation, the authors' value is seen to lie between the other two values. Certainly it is much more easily found than by our method. The exact value is probably not significant.

Flow separation is difficult to incorporate into the analysis, particularly when flow reversal is occurring. It seems to have a definite effect on the inertial length, but depending on the transient conditions, may not always be present. Flow starting from rest begins as an ideal fluid without separation then viscous effects must enter, leading to separation and jet formation. As the flow further develops in time, turbulence appears downstream, although the jet and upstream flow may be laminar. Currently we are attempting to assess the rapidity of appearance of separation and downstream jet flow as affected by viscosity. It is difficult to say how the inertial length varies with high rates of flow. The one case calculated shows that L can be above the Rayleigh value, ($L = 1.3d$) when the flow separates. J. W. Daily, et al., in a study of the resistance of orifices and tubes in unsteady flow concluded that "with acceleration the resistance is appreciably less than for the equivalent steady state, with deceleration the resistance is appreciably more than for the equivalent steady state and for intense jet action as obtained with small orifice to tube diameter ratios, it appears that unsteadiness produces an internal flow structure that is no longer comparable to any steady state condition."

At this stage one must conclude that the transient response of an orifice or short tube can be difficult to predict under certain conditions where separated jets might occur. The inertial length coefficient is a function of the flow field as well as the geometry and as such depends on the strength of the transient. The good agreement obtained by the authors for their verification experiments seems to indicate that for the conditions tested, the inertial correction for the orifice is slightly less than the Rayleigh value. At higher flow rates or transients which cause flow reversals and nonlinearities, complications may arise.

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Authors' Closure

We would like to thank Professor Robertson and Mr. Holt for their discussion of our paper. Their points are well taken and the additional references are appreciated.

The use of the discharge coefficient was motivated by our desire to obtain a relatively simple dynamic description which would also satisfy the well-known steady state orifice flow-pressure drop relationship.

To calculate the inertial length we would prefer to write equation (10) as

$$\frac{p_1 - p_2}{\rho} = \frac{A_0}{\sqrt{\frac{C_D A_0 \pi}{2}}} \frac{du}{dt} + \frac{u^2}{2C^2{}_D}, \quad \text{where} \quad u = \frac{q}{A_0},$$

which does satisfy the steady state relationship. The inertial length is then

$$L = \left[\frac{1}{2C_D}\right]^{1/2} d$$

which, for $C_D = 0.625$, yields

$$L = l_t + 0.89d$$

The difference between this result and that presented by the discussers is a consequence of using different reference velocities. We agree with the discussors that the exact value is probably not significant.

Rather than ignoring the flow details near the finite sized orifice, which may have a significant effect on the inertial length, we have, in order to achieve the desired simplicity, made an a priori assumption concerning the flow field. We agree that under other flow conditions, and particularly for the case of flow reversal, our approach would have to be modified.