\( P_n \) velocity anisotropy in a continental upper mantle

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Summary. Earlier interpretations of \( P_n \) travel-times from the extensive quarry-blast observation scheme in western Germany — now supplemented by explosion data from the 1972 Rhinegraben experiment — have been checked and enhanced using the new MOZAIC time-term method. The large data set (762 travel times) continues to require a considerable anisotropy of upper-mantle \( P \) velocity. The resulting estimates of the overall velocity variation — probably 0.50–0.60 km/s about a mean value of 8.05 km/s, that is, 6 to 7 per cent anisotropy — and of the direction of the maximum velocity (close to 20° E of north) are reasonably reliable. However, the detailed form of the anisotropy is obscured by various limitations of the data.

These results allow a realistic assessment of the resolving power of refraction-based studies of velocity anisotropy in the lithosphere. It is concluded that though such studies are probably adequate if the measurement of in situ anisotropy is required within the context of a generalized discussion of lithospheric dynamics they are not appropriate if a detailed specification of the anisotropy is desired.

1 Introduction

The recognition and measurement of seismic anisotropy at depth beneath continents promises to tell us much about both the dynamics of the lithosphere–asthenosphere system and the petrology of the upper mantle. However, geodynamic problems may require only very basic information concerning the anisotropy, for example an indication of the depth ranges at which or over which the phenomena is encountered, and a reliable indication of the magnitude of the anisotropy and of the direction of maximum velocity (presumed to be parallel to the maximum stress direction). On the other hand, petrological problems may require precise definition of all the elastic constants of the material (up to 21 in the general anisotropic case) so as to permit a rigorous assessment of the anisotropic predictions of different petrological models. The information resulting from any in situ anisotropy measurement must be considered with these diverse requirements in mind.

In principle, refraction-based measurements of seismic velocity as a function of direction are promisingly simple though limited from the outset to the horizontal plane and rather
Figure 1. (a) Schematic distribution of profiles and fans contributing $P_n$ data. In the case of profiles, the notation describing them is shotpoint number – approximate direction – reversing shot (if any); for fans, the notation is shotpoint – approximate distance – F. The letter V indicates observations on an array rather than a complete profile. Symbols: * shot-point (quarry blast), $\bullet$ shot-point (1972 Rhinegraben experiment) (coded by letters and numbers); –– profile or fan; $\Box$ major city (HH Hamburg, H Hannover, B Berlin, K Kologne, F Frankfurt, N Nuremberg, M Munich, S Stuttgart, St Strasbourg, Ba Basel, I Innsbruck). (b) Distribution of $P_n$ observation points. Symbols: * observation point (quarry blast), $\downarrow$ observation point (1972 Rhinegraben experiment); additional symbols as in (a). (c) Geological and geographic summary of study area.

restricted depth ranges. In practice, however, the correct separation of the effects of velocity anisotropy from those of lateral variations in both velocity and structure necessitates the use of complex patterns of shots and receivers. It is relatively easy to lay out such optimum anisotropy-measuring networks in the oceans but in continental areas the greater distances
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involved (continental $P_n$ observed, say, from 130–230 km distance compared with 20–70 km in oceans) and the difficulties of using appropriate shot-points and recording points suggest that such networks would be difficult to organize and that the resulting experiment would be both large and expensive. It is essential, therefore to make the very best use of the data that is already available from the extensive observation schemes that have been built up over a number of years in various continental areas, for example in western Europe, western USA and the USSR. These composite networks, whilst not having the advantage of being specifically designed for the purpose, should permit studies of continental velocity anisotropy and allow an assessment of the resolving power of such velocity measurements.

In a pair of previous papers (Bamford 1973, 1976a; hereafter papers I and II respectively), I demonstrated how the $P_n$ travel-time data obtained from the quarry-blast recording programme that was undertaken in western Germany during the Upper Mantle Project (i.e. up to 1971) could be rendered suitable for time-term analysis and that the clear conclusion of such analyses is that this data requires a considerable velocity anisotropy in the upper mantle. It has since become possible to extend the original $P_n$ data set by including data that was obtained during the 1972 seismic refraction experiment in the Rhinegraben (Edel et al. 1975), thus offering an opportunity to update, check and enhance the interpretations presented in papers I and II.

These new results form the basis of this paper and are presented with the aim of answering the following four questions:

(i) Does the requirement for $P_n$ velocity anisotropy persist when the data set is enlarged?
(ii) Is the magnitude and direction of the anisotropy consistent over the study area or can real variations be recognized?
(iii) Is it at all possible to distinguish between various candidate forms of anisotropic variation?
(iv) Are there any ‘hidden’ constraints upon the anisotropic information resulting from this study (and, by implication, from any other similar studies)?

2 The data

The available data are summarized in Fig. 1 which shows all those refraction profiles and fans which contribute $P_n$ data to the present study (a) and the resulting distribution of observation points (b). These profiles in fact represent about half of all profiles available in western Germany (and eastern France) and a complete description of them all together with record sections and interpretations may be found in Giese, Prodehl & Stein (1976).

The quality and density of the data varies from one profile to another but, because it is all available in the form of record sections with reduced travel time, the recognition and timing of the various phases, in particular $P_n$, is straightforward. Fig. 2 shows a pair of record sections which are reasonably typical of quarry-blast results (a) and 1972 Rhinegraben results (b); although the latter show a somewhat greater degree of consistency from one seismogram to another, the reliability of the resulting $P_n$ travel-times is only marginally improved — thus $P_n$ can be recognized and timed with roughly equal facility in both the sections in Fig. 2.

In general in western Germany, the observed system of travel-time branches and the amplitude distribution along these branches, especially that of the Moho reflection ($P_{M\ P}$), indicates a crust—mantle structure within which the Moho is characterized by a sharp transition from velocities of around 6.5–6.8 km/s to velocities in excess of 7.7 km/s, for example Fig. 3(a). In special areas, however, and the Rhinegraben is such an area, the picture changes; thus Edel et al. (1975) found different velocity—depth structures just outside — Fig. 3(b) —
and inside — Fig. 3(c) — the Rhinegraben. The differences between the three functions in Fig. 3 lie primarily within the lower crust; the main result, a consequence of the change in sharpness of the crust—mantle transition, will be to radically affect the $P_mP$ phase. Although the amplitudes and travel times of the $P_n$ phases do of course depend on the properties of the lower crust (and the crust—mantle transition), a $P_n$ phase — with velocity around 8 km/s — will be generated by each of the structures in Fig. 3, and it is in no way misleading to think of these as all being the same phase. Thus one may group together as $P_n$ all first-arrivals (only rarely is $P_n$ visible at shorter ranges as a second-arrival) which share the correct features, for example, appropriate apparent velocity, consistent relationship to other travel-time branches, correct distance range etc. This very careful approach to $P_n$ recognition has the special advantage of avoiding an all-too-common assumption in the literature that any first-arrival beyond about 130 km with an apparent velocity close to or greater than 8 km/s is automatically $P_n$. In common with many recent high-resolution studies (e.g. Hrn et al. 1973; Bamford et al. 1976), $P_n$ in this study is only rarely observed beyond 250/260 km
(Fig. 4(a)); energy reaching greater distances (say, beyond about 300 km) must be associated with deeper horizons than the Moho.

The combined quarry-blast and Rhinegraben data set contains 762 $P_n$ travel times, 123 of these being derived from the Rhinegraben data. There are several reversed and crossing profiles — Fig. 1(a) — and the distributions of observations by area (Fig. 1(b)), and by distance and azimuth (Fig. 4), are all fairly even and so seem to be reasonably satisfactory for this type of study.
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The apparent velocities along individual profiles vary considerably and are suggestive, but no more than that, of velocity variation with azimuth at least to the extent that possible sectors of maximum and minimum velocity can be guessed at (Fig. 5). However, although it depends on a large number of observations, Fig. 5 takes no account of other sources of variation and so does not provide reliable evidence for even the existence of anisotropy let alone providing estimates of magnitudes and directions.

3 Methods of analysis

3.1 MOZAIC TIME-TERM ANALYSIS

The time-term approach is known to be an appropriate method for the interpretation of travel-time data in terms of realistically complex structures but, in its basic form (Willmore & Bancroft 1960; Bamford 1971), the method demands that shot-points and receivers be organized into strongly interleaved homogeneous patterns which do not exist in the composite observation scheme of the essentially heterogeneous type available in western Germany. However in paper II I introduced the concept of MOZAIC time-term analysis which is especially designed to deal with data from such heterogeneous networks. The MOZAIC method is
based on the premise that within the region of study there exist small areas over which the delay time to a particular refraactor is constant and that the distribution of such areas may be intelligently guessed at using ancillary information such as gravity, or geological, maps. It is then possible to construct from a heterogeneous observation scheme a MOZAIC network to which more or less conventional time-term analysis can be applied; for a shot-point lying in an area \( i \) and a receiver in area \( j \), the theoretical refraction travel-time \( t_{ij} \) between these points may be then written as

\[
t_{ij} = a_i + a_j + \frac{D_{ij}}{V_p}
\]  

where \( a_i \) and \( a_j \) are the delay times of the \( i \)th and \( j \)th areas respectively, \( D_{ij} \) is the distance between shot-point and receiver, and \( V_p \) is the refraactor velocity. We are not concerned here with the mechanics of time-term solutions — these are adequately discussed elsewhere (papers I and II) and should be relatively familiar.

The refraactor velocity \( V_p \) can take one of several forms; it may be assumed to be constant, or to vary laterally over the survey region, it may increase as rays penetrate deeper into the refraactor or it may vary with direction. The representation of velocity anisotropy in (1) is central to the discussion in this paper.

### 3.2 REPRESENTATION OF VELOCITY ANISOTROPY

The theoretical treatment of body-wave propagation in anisotropic elastic media is complex (Crampin 1977); for example, in the most general anisotropic case, 21 independent elastic constants are required for complete specification. However, functions which adequately describe the variation of refraactor velocity with direction are relatively simple.

Backus (1965) showed that a weak anisotropy of \( P \) refraactor velocity may be represented by

\[
V_p^2 = c_p^2 + A + C \cos 2\phi + D \sin 2\phi + E \cos 4\phi + F \sin 4\phi
\]  

where, in \( this \) paper’s convention, \( \phi \) is the azimuth measured clockwise from north. The coefficients \( A, C, D, E \) and \( F \) are derived from just seven of the elastic constants (Backus 1965, equation (22)): \( c_p \) is the assumed isotropic velocity. ‘Weak anisotropy’ is normally understood to imply a total velocity variation of less than approximately 10 per cent; the amounts encountered in upper mantle studies fall well within this range.

Crampin (1977, equations (12)–(17)) shows that, if angles can be measured from a direction of sagittal symmetry, then (2) reduces to

\[
V_p^2 = c_p^2 + A' + B' \cos \psi + C' \sin 2\psi
\]  

where \( \psi \) is measured from the direction of sagittal symmetry. However, in a real situation, the direction of symmetry will not be known in advance and hence equation (2) is the correct form at least in the early stages of interpretation. Once a symmetry direction has been identified, even if only approximately, then it may be both appropriate and desirable to use equation (3).

Given that equation (2) will correctly represent any weak anisotropy, the theoretical travel-time equation (1) must be rewritten to take account of this anisotropy in a form suitable for least-squares analysis. This is simple to do (e.g. Raitt et al. 1969) and need not be reproduced fully here; however, various consequences of this rewriting become significant when considering the limitations of the results and should be identified.
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Equation (2) may be rewritten as a perturbation $d(V_p^2)$ about a mean $V_0^2$ thus

$$V_p^2 = V_0^2 + d(V_p^2)$$

(4)

where

$$V_0^2 = c_p^2 + A$$

and

$$d(V_p^2) = C \cos 2\phi + D \sin 2\phi + E \cos 4\phi + F \sin 4\phi$$

This perturbation can then be introduced into the theoretical travel-time equation by a simple first-order series expansion of the delay times and $1/V_p$ in equation (1). After additional manipulation and rearrangement, and introduction of the parameter 'offset distance' (the horizontal distance between a point on the surface and the point on the refractor at which the correspondence ray undergoes critical refraction), we ultimately obtain

$$t_{ij} = a_i + a_j + \frac{D_{ij}}{V_0} + \frac{1}{2V_0^3} (R_i + R_j - D_{ij}) (C \cos 2\phi + D \sin 2\phi + E \cos 4\phi + F \sin 4\phi)$$

(5)

where $R_i$ and $R_j$ are the offset distances at $i$ and $j$ respectively, an equation suitable for least-squares analysis. To achieve this two sacrifices have been made. Firstly, only the first two terms in the series expansions have been included — the resulting approximation will not result in serious error provided the anisotropy remains small (say less than 10 per cent total velocity variation). Secondly, and definitely more seriously, the offset distances $R_i$ and $R_j$ have been introduced into (5) as observed quantities; however they depend on the structure above the refractor — in the case of $P_n$, on the crustal velocity-depth function and the depth to Moho (itself possibly calculated from the delay times). Thus the resolution of refractor velocity anisotropy depends on the information that is available about the overlying structure; although the exact significance of this dependence will not become clear until specific examples are considered, one could predict that it would represent a serious weakness as the offset distances enter (5) in company with the observation distance $D_{ij}$ and errors in the latter are usually very damaging.

4 MOZAIC solutions

Many different MOZAIC solutions of the available $P_n$ data have been carried out so as to test as severely as possible the hypothesis that velocity anisotropy is required by the data, to test the stability of the resulting anisotropy functions both in terms of intrinsic error and regional variation and to assess the influence of factors such as offset distance.

At first sight it might seem that there existed a very large number of different viable combinations of the observations summarized in Fig. 1. However, a few trial solutions indicate that the number of different combinations is in fact very severely restricted by the requirement (for a successful solution) that the included observations should have a relatively even distribution by area and, especially, by azimuth: a glance at Fig. 1(b) shows that these conditions will be satisfied by only a few sub-networks. Nevertheless, in addition to a straightforward comparison of the original quarry-blast data set with the new augmented set, the distribution of observations with respect to the principal geological features — Fig. 1(c) — does suggest certain viable data divisions of tectonic interest. For example, one could try to concentrate on the relatively uncomplicated area known as the South German Triangle (bounded by the Alps, the Rhinegraben and the Bohemian Massif) by excluding observations made north of the Vogelsberg, or include data from the area of the Rhinegraben only (or
exclude this data completely). At this point another fundamental problem of continental $P_n$ anisotropy studies is encountered; typical $P_n$ observation distances range from less than 120 km to more than 250 km (Fig. 4). Thus typical ‘anisotropy networks’ will tend to cover an area of at least these dimensions and involve an assumption, in a MOZAIC solution, of consistent velocity anisotropy over this area; the scale of tectonic variations may however be less than this. Hence, although the $P_n$ observations have been combined in various different ways that concentrate upon different areas (Fig. 6), these areas correlate only roughly with possibly different geological provinces and so the resulting study of regional variations in anisotropy is likely to be fairly crude.
Sub-division of the observations by area allows an assessment of the stability (or regional variability) of MOZAIC solutions when different data sets are analysed. It is equally important to examine the stability of solutions within sub-divisions when the way in which the observations are linked together (basically, the definition of areas of equal delay time) is altered. Thus, for each data sub-division, two different MOZAICs were designed, the first based on geology (Geologisches Karte der Bundesrepublik Deutschland 1:1000000 1973) and the second on Bouguer gravity (Gerke 1957); both were influenced by reviews of the dominant trends of tectonic features of the area (e.g. Edel 1975). As far as possible, a conscious attempt was made to avoid correlation between the patterns of the different MOZAICs.

The results of solutions of ten different MOZAICs will be described here: these were made up as follows:

Two MOZAICs including only quarry-blast data; A1 (geology-based) and A2 (gravity-based).
Two MOZAICs including all available data (Fig. 6); B (geology) and C (gravity).
Two MOZAICs excluding all observations north of the Vogelsberg (Fig. 6); D (geology) and E (gravity).
Two MOZAICs including only data ‘in the area of the Rhinegraben’ (Fig. 6); F (geology) and G (gravity).
Two MOZAICs excluding data ‘in the area of the Rhinegraben’ (Fig. 6); H (geology) and I (gravity).

Although the crustal structure in the area is perhaps better known than anywhere else in the world (see Giese et al. 1976) the crustal velocity—depth functions are not sufficiently well known at each observation point to permit a rigorously correct assessment of offset distance. However, typical offset distances, based on velocity—depth functions like those in Fig. 3, are between 30 and 40 km; hence, a preliminary assessment of the effect of offset distance was made by solving each of MOZAICs B–I three times for velocity anisotropy, using uniform offset distances of 30, 35 and 40 km in turn.

The results of all these anisotropy solutions are summarized in Table 1—the variance quoted is of course indicative of the quality of fit of the solution. Note that, at any statistically significant level, the variance obtained is independent of the offset distance used. This variance may be compared with the equivalent value for a uniform velocity solution.

The main general feature of these solutions is the dramatic improvement in the fit quality of solutions when velocity anisotropy is allowed, no matter which data or combination of data is used. Solutions which allow other possible variations in refractor velocity, for example vertical velocity gradient or lateral velocity variation, do not give any significant improvement in quality of fit over the uniform velocity solutions; the variances quoted for the uniform velocity solutions therefore represent all solutions that do not allow velocity anisotropy. This confirms the result established in papers I and II, namely that the data requires velocity anisotropy to be present.

The most important point about the anisotropy coefficients themselves is that the computed coefficients vary significantly as the data set is altered and that, in comparison, the influence of differing combinations of the same data or of choice of offset distance are of secondary importance; compare, for example, results for B/C with those for F/G in Table 1. A minor point is the apparently low significance of the sin 4θ terms.

The delay times computed in these solutions are themselves of little significance to the present discussion and are not detailed here. However the values obtained are relatively consistent from one solution to another and do give a reasonable picture of the Moho in the area of study (see Bamford 1976b); these two facts will be utilized in the next Section in which
Table 1. MOZAIC solutions: summary of principal results.

<table>
<thead>
<tr>
<th>MOZAIC</th>
<th>Uniform velocity solution variance (s^2)</th>
<th>General anisotropy results: coefficients of anisotropy*</th>
</tr>
</thead>
<tbody>
<tr>
<td>(based on)</td>
<td></td>
<td>C/2V_o^2</td>
</tr>
<tr>
<td>A1 (geology)</td>
<td>0.089</td>
<td>0.031</td>
</tr>
<tr>
<td>A2 (gravity)</td>
<td>0.090</td>
<td>0.032</td>
</tr>
<tr>
<td>B (geology)</td>
<td>0.080</td>
<td>0.030</td>
</tr>
<tr>
<td>C (gravity)</td>
<td>0.069</td>
<td>0.026</td>
</tr>
<tr>
<td>D (geology)</td>
<td>0.074</td>
<td>0.027</td>
</tr>
<tr>
<td>E (gravity)</td>
<td>0.087</td>
<td>0.029</td>
</tr>
<tr>
<td>F (geology)</td>
<td>0.061</td>
<td>0.021</td>
</tr>
<tr>
<td>G (gravity)</td>
<td>0.067</td>
<td>0.023</td>
</tr>
<tr>
<td>H (geology)</td>
<td>0.089</td>
<td>0.025</td>
</tr>
<tr>
<td>I (gravity)</td>
<td>0.106</td>
<td>0.025</td>
</tr>
</tbody>
</table>

* Coefficients as in equation (5); C/2V_o^2 is coefficient for \cos 2\phi, D/2V_o^2 for \sin 2\phi, E/2V_o^2 for \cos 4\phi, F/2V_o^2 for \sin 4\phi.
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we consider the constraints placed by these MOZAIC solutions upon our knowledge of upper-mantle velocity anisotropy in this area.

5 Velocity anisotropy — magnitude, direction and form

Important though it undoubtedly is, the simple statistical demonstration that velocity anisotropy is required by the data tells us nothing that is really useful to studies of either the dynamics or the petrology of the lithosphere: the essential information is the magnitude, direction and form of this anisotropy. This information is best displayed by graphing the velocity—azimuth curves which may be computed from the coefficients in Table 1. Fig. 7 is a montage of all the anisotropy curves resulting from MOZAIC solutions in which a uniform 30-km offset distance was assumed. There are ten such curves and we immediately see that whereas the majority (solid lines) are reasonably consistent one with another as regards magnitude and direction (though not in details of form), two curves (broken lines) are inconsistent with the rest, most noticeably regarding the direction of maximum velocity but also in magnitude. These two curves are in fact the pair from MOZAICs F and G, that is the two MOZAICs for which data selection was biased toward the area of the Rhinegraben (Fig. 6) and the question arises as to whether or not this inconsistency reflects a real, and therefore highly significant, difference in anisotropy local to the Rhinegraben. However, consider Fig. 8 in which the respective anisotropy curves and the azimuthal distribution of observations for MOZAICs E and G are compared. Now the fitting of any non-linear function, even

![Figure 7. Anisotropy functions for all MOZAICs, 30-km offset solutions.](https://academic.oup.com/gji/article-abstract/49/1/29/563055)
a very simple one, is likely to be most reliable when there is a relatively even distribution of observations, otherwise the function may become distorted in regions where data is scarce. We note that for both E and G, the high velocities occur within the sector $340-070$ degrees; however, whereas the azimuthal distribution for E is relatively uniform over this sector, that for G has a clear scarcity of data in the sector $000-020$ degrees. Thus, one might suspect that in the case of E, the even data distribution would provide an anisotropy function whose maximum was close to the real maximum whereas in G the position of the function maximum might be controlled much more by the presence of large numbers of observations adjacent to the maximum; if, as appears from the solid lines in Fig. 7, the real maximum actually lies in the sector $000-020$ degrees, the result for G could totally be unpredictable and incorrect. Note also that the position of the minimum of the functions does

Figure 8. Comparison of azimuth distribution (above) and anisotropy curves (30-km offset solutions) for MOZAICs E and G.
not alter much between E and G; in the appropriate sector (100–140 degrees), the number of observations is reduced uniformly between E and G, that is, no new gaps appear in the distribution.

Thus the broad consistency of the other solutions together with the above, admittedly qualitative, argument does tend to suggest that the F and G curves are systematically incorrect and do not reflect real variations in the anisotropy. This said, however, one must recognize that any uneveness in the azimuthal distribution of observations might introduce small systematic errors into computed anisotropy curves; for example, the clear minimum in the distribution in Fig. 4(b) over 080–100 degrees may be important. Therefore, whilst the solid lines in Fig. 7 (and the results in Table 1) allow the following summary to be made with reasonable confidence

(i) total anisotropy 0.50–0.55 km/s if 30 km offset distance assumed; varies in proportion to offset distance

and

(ii) maximum velocity direction approximately 10°–20° E of north,

the results obtained so far have two serious weaknesses. Firstly, the anisotropy functions may be skewed by fluctuations in the azimuthal distribution of observations, and secondly the effect of offset (and this really means the effect of crustal structure) has not been correctly allowed for; these are assumed to be the root causes of the small differences in the relatively consistent anisotropy functions in Fig. 7 (the solid lines therein) which lead to uncertainty in details of the anisotropy.

Raitt et al. (1969) described an approach which, with some modifications by this author, can be used to partially overcome these problems. If the anisotropic travel-time equation (5) is rewritten so that the anisotropic effect is described simply in terms of the perturbation \(d(V^2_p)\) about the mean \(V^2_p\) as in equation (4), then

\[
 t_{ij} = a_i + a_j + \frac{D_{ij}}{V_0} + \frac{1}{2V_0^2} (R_i + R_j - D_{ij}) \cdot d(V^2_p) \tag{6}
\]

and a velocity perturbation can be calculated directly (and independently) for each observation in the following manner:

(i) \(t_{ij}\) and \(D_{ij}\) are the observed travel-time and distance respectively,

(ii) the delay times \(a_i\) and \(a_j\) may be calculated from the several independent estimates of the delay time at every shot-point and observation point contained in the solutions of MOZAICs A–I. The median of these estimates can be regarded as the best estimate of the delay time at each point,

(iii) the offset distances \(R_i\) and \(R_j\) depend on the depth to Moho and on the crustal velocity–depth function; they may be calculated from the delay times (following (ii) above) if the crustal velocity is known, and

(iv) the mean velocity \(V_0\) can be calculated from the \(1/V_0\) values computed for all the MOZAIC solutions (Table 1).

For the present purpose, the delay times and \(V_0\) (8.06 km/s) were calculated from all the MOZAIC solutions wherein a uniform 30-km offset distance had been assumed, and then individual offset distances were calculated from the delay times by assuming a uniform mean crustal velocity of 6 km/s, a reasonable approximation (Fig. 3). A \(d(V^2_p)\) perturbation was then computed for each observation and the resultant total velocity plotted as a function of azimuth in the ‘velocity scattergram’ shown in Fig. 9(a).
Figure 9. (a) Velocity scattergram. (b) Best-fitting curves for the velocity—azimuth data in (a) for both general and reduced (22.5°) forms of anisotropy.

The information contained in this velocity scattergram has the considerable advantage of being one stage removed from any premature assumptions regarding the form of the anisotropy, and does allow rough estimates of magnitude and direction. However, fitted curves are more informative and so several have been fitted to the data in Fig. 9(a), one in which all angles are measured from zero, that is, the general anisotropy equation (2) is fitted once, and a series in which the reduced equation (3) is fitted with various assumed directions of sagittal symmetry close to the apparent maximum velocity direction. The results obtained are summarized in Table 2: note that velocity squared has been fitted and so the quoted standard deviations are in these terms (thus a 'standard deviation' of 1.600 corresponds to
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**Figure 9 (b)**

![Graph showing anisotropy curves](image)

**Table 2. Anisotropy curves* fitted to velocity data.**

<table>
<thead>
<tr>
<th>Curve</th>
<th>Standard deviation (km/s)$^2$</th>
<th>$V_0^2$</th>
<th>$\cos 2\phi$</th>
<th>$\sin 2\phi$</th>
<th>$\cos 4\phi$</th>
<th>$\sin 4\phi$</th>
</tr>
</thead>
<tbody>
<tr>
<td>General case*</td>
<td>1.600</td>
<td>64.778</td>
<td>3.177</td>
<td>3.246</td>
<td>0.647</td>
<td>-0.172</td>
</tr>
<tr>
<td>Reduced case,* 10°</td>
<td>2.151</td>
<td>64.621</td>
<td>4.452</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Reduced case,* 12°</td>
<td>2.014</td>
<td>64.592</td>
<td>4.575</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Reduced case,* 14°</td>
<td>1.894</td>
<td>64.578</td>
<td>4.659</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Reduced case,* 16°</td>
<td>1.794</td>
<td>64.578</td>
<td>4.707</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Reduced case,* 18°</td>
<td>1.718</td>
<td>64.591</td>
<td>4.722</td>
<td></td>
<td></td>
<td>-0.065</td>
</tr>
<tr>
<td>Reduced case,* 20°</td>
<td>1.670</td>
<td>64.616</td>
<td>4.709</td>
<td></td>
<td></td>
<td>-0.140</td>
</tr>
<tr>
<td>Reduced case,* 21°</td>
<td>1.656</td>
<td>64.632</td>
<td>4.693</td>
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<td>-0.176</td>
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<tr>
<td>Reduced case,* 22°</td>
<td>1.650</td>
<td>64.651</td>
<td>4.671</td>
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<td>-0.524</td>
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* General case, $V_p^2 = V_0^2 + C \cos 2\phi + D \sin 2\phi + E \cos 4\phi + F \sin 4\phi$ (equation (2)) where $\phi$ is the azimuth east of north.

* Reduced case, $V_p^2 = V_0^2 + B' \cos 2\psi + C' \sin 4\psi$ (equation (3)) where $\psi$ is the angle measured from the direction of symmetry (given in degrees).
a velocity uncertainty of roughly 0.1 km/s). The standard deviations are suitable guides to
the fit quality of the various curves. The main features of these curves are as follows:

Statistically the very best fit is obtained by using the general anisotropy equation (2); the
resultant curve — Fig. 9(b) — indicates a mean velocity of 8.05 km/s and an overall
velocity variation of 0.58 km/s — the direction of maximum velocity is 16° E of north. On
the other hand, the quality of fit obtained using the reduced form with an assumed sym-
metry direction 22.5° E of north is only slightly inferior and gives insignificantly different
results (mean velocity 8.04 km/s, overall variation 0.58 km/s): the corresponding curve is
also shown in Fig. 9(b). It could be argued therefore that the solutions do not provide com-
pelling evidence that other than the reduced equation (3) is required, and elsewhere
(Crampin & Bamford 1977) we argue that it is difficult to think of any likely geophysical
process that will not produce an anisotropy with at least one vertical symmetry plane.
However, whatever form the anisotropy takes, the overall magnitude (0.58 km/s) and mean
velocity (8.05 km/s), and the direction of maximum velocity (close to 20° E of north) are
reasonably well determined. Further details of the anisotropy are obscured by the approxi-
mately ±0.1 km/s uncertainty in the fitted velocity at any one azimuth.

6 Discussion
It is interesting to consider to what extent the results obtained above might be improved
upon. Let us begin by considering the scatter in Fig. 9(a): it is considerable and is
undoubtedly derived from two sources.

First, travel-time measurements errors, random fluctuations in structures, errors in delay
times (and hence in offset distances) and so forth are all absorbed, through equation (6),
into the calculation of the velocity value for each observation and so introduce considerable
scatter. An interesting point about the measurement errors is they depend upon the strength
of the refracted signal which in turn depends, amongst other things, upon the velocity con-
trast at the Moho; thus in the presence of velocity anisotropy at the Moho the measurement
errors will themselves be directionally dependent. A very qualitative study of the record
sections indicates that the weakest signal directions may correspond roughly to the mini-
mum velocity direction of Fig. 9(a); it is possible therefore that some of the very low
velocity values on the scattergram, which could correspond to systematically overestimated
travel-times, may have been predictable.

The second source of error is due to continuing deficiencies in taking account of offset
distances. Although their calculation from the delay times represents a considerable improve-
ment over an assumption of a uniform value, the further assumption that the same mean
crustal velocity, in this case 6 km/s, applies everywhere is undoubtedly not true.

As far as the present data set is concerned, uncertainties due to travel-time measurement
errors, and also to the unevenness of the azimuthal distribution of observations, are more or
less built in; for example, for many profiles the record sections used in this study represent
the only readily available version of the data and so no dramatic improvement in the reliability
of the travel-times can be expected. In contrast, it should be possible to effect an
improvement in the quality of the crustal information by a careful re-assessment of existing
data and results. It would then be worthwhile to carry out a much larger number of
MOZAIC solutions to produce much more reliable estimates of the delay times. A more
coherent scattergram might then reveal genuine regional variations in anisotropy, for
example in \( V_0 \), in addition to allowing greater resolution of magnitude, direction and form.

Conclusions
The results presented here clearly demonstrate that \( P_n \) velocity anisotropy is required by
the data, that the overall velocity variation is probably 0.50—0.60 km/s (i.e. 6—7 per cent)
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about a mean value of 8.05 km/s and that the maximum velocity direction is close to 20° E of north: these might be termed first-order results. Second-order results, which might include a clearer definition of the anisotropy curve, and hence greater resolution of magnitude and direction, might become available if the scatter in Fig. 9 can be reduced. If not, then one would be forced to accept only the first-order results which, whilst being quite adequate for a discussion of the implication of in situ anisotropy for the dynamics of the lithosphere (e.g. Fuchs 1977), do not place very severe constraints on the petrological models that are plausible candidates for an anisotropic upper mantle (Crampin & Bamford 1977).

These conclusions have important implications for future studies of velocity anisotropy beneath the continents. Although the west German network was not designed specially for anisotropy studies, few of the deficiencies in the results are a direct consequence of this. In a special experiment it might be possible to achieve an even azimuth distribution of observations by designing an optimum network but travel-time measurements are unlikely to be dramatically improved simply because special explosions were used (in any case some of the data used here was from special explosions) although it might be possible to boost weak signals if the weak signal direction was known beforehand. On the other hand, it is highly unlikely that any realistic (in terms of size and cost) experiment would generate the amount of data available in the west German network which took several years to build up. Crustal information is vital whether or not the anisotropy network is specially designed.

It is realistic to conclude therefore that the quality of information in Fig. 9 is fairly typical of any purely refraction-based study of velocity anisotropy and certainly the oceanic results of Raitt et al. (1969) and Morris, Raitt & Shor (1969), obtained in specially designed experiments and with the effects of variations in offset distance probably of reduced significance in an oceanic situation, are not of markedly superior quality (see also Crampin & Bamford 1977). Thus, if other than first-order information on in situ anisotropy is required, it is possible that an alternative measurement method must be sought.

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