Acausality and Renormalizability

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A few years ago, Velo and Zwanziger\cite{1} made an interesting observation: The covariant wave equation of an interacting higher-spin particle generally violates the Einstein causality. The covariant wave equation of a free higher-spin particle consists of equations of motion and constraints. When interaction is switched on, the constraints also are modified nontrivially. After eliminating them, therefore, the propagation character of the wave equation, which is determined by its highest-derivative terms, will be different from that in the free case. Then the action can generally propagate faster than light velocity, that is, we encounter acausality. No such phenomenon takes place for the spin-0 and spin-1/2 cases. The spin-1 case is marginal; the acausality depends on the form of interaction. The Proca equation for a vector field $u^\mu$ having a self-interaction $\mathcal{L}_{\text{int}}=g(U^\mu U_\mu)^2$ was shown to be acausal for $g<0$.

If the interaction of the higher-spin particle is due to an external field, the above acausality is not very troublesome because the existence of the external field already violates the Lorentz invariance of the theory. If, however, we consider a manifestly covariant quantum field theory, though the Velo-Zwanziger argument does not rigorously apply to it, the acausality trouble is very serious. Because of Lorentz invariance, super-light-velocity propagation implies propagation to the past in some other Lorentz frames.

If quantum field theory is acausal, Heisenberg operators will not commute (or anticommute) at spacelike separations, so that $T$-products cannot be defined covariantly. On the other hand, we know that perturbation theory satisfies local commutativity in every order. The above two aspects are compatible only if the order of the singularity on the light cone increases indefinitely for higher-order perturbation terms. Since the order of the light-cone singularity is bounded in a renormalizable theory, it is natural to conjecture that a renormalizable field theory cannot be acausal in the sense of Velo and Zwanziger. [Of course, the converse is not true because the $\phi^4$ theory is causal but unrenormalizable.] In the following, we present an instructive example to support the above conjecture.

Recently the present author\cite{2} proposed an indefinite-metric quantum theory of a massive vector field, which is a straightforward extension of the Landau-gauge quantum electrodynamics.\cite{3} In contrast with the Proca formalism, this theory is manifestly renormalizable if the coupling constant is dimensionless. The field equations are

\begin{align}
\partial^\mu U_\mu &= 0, \\
(\Box + m^2)U_\mu - \partial_\mu B &= j_\mu, \\
\Box B &= -\partial^\mu j_\mu,
\end{align}

where $U_\mu$ denotes the vector field having a bare mass $m$, $B$ being an auxiliary scalar field of negative norm and $j_\mu$ is the current. As seen from (1), if $\partial^\mu j_\mu$ involves no second (or higher) derivatives, then the highest derivatives are the d'Alembertians alone; hence our field equation is causal. The above condition is satisfied if $j_\mu$ contains no derivatives, for example, if $\mathcal{L}_{\text{int}}=g(U^\mu U_\mu)^2$. Hence this renormalizable theory is causal in conformity with the above conjecture. This result should be compared with the fact that the corresponding Proca theory is unrenormalizable and acausal as mentioned at the beginning.
The above consideration also suggests that the acausality trouble may be avoided by introducing a formalism containing auxiliary fields even for higher-spin cases.

2) N. Nakanishi, Phys. Rev. D5 (1972), 1324;
   See also P. Ghose and A. Das, Nucl. Phys. B41 (1972), 299.