On Contraction of Stellar System

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(Received October 28, 1972)

We consider stellar systems which include the comparable amount of gas with stars. The condition for stellar system to contract due to the dissipation of its kinetic energy through the direct and gravitational interactions between stars and gases is obtained. For a stellar system to contract within a Hubble time, it must be compact initially, and accordingly, the gas density is high. The gas cannot remain interstellar for a Hubble time and the stellar system does not contract unless the system is extremely compact initially. In conclusion, it is impossible for a stellar system to contract from the stage where the dissipation of stellar kinetic energy is ineffective to the stage where it is effective.

§ 1. Introduction

As an approach to the violent events in the nuclei of galaxies, Spitzer and Saslaw discussed the contraction of a stellar system through the star evaporation process. As the contraction proceeds, the star density becomes so high that star-to-star direct collisions become important and, as a result, a large amount of energy is released. The basic problem is, however, whether a stellar system located at the central region of a galaxy can really contract, within a Hubble time, to the stage where star-to-star collisions play a dominant role.

There are other possibilities which causes a stellar system to contract. If there is a sufficient amount of gas in a stellar system, it can contract due to the dissipation of stellar kinetic energy through direct (hydrodynamical) and gravitational interactions between stars and gases. The amount of gas which has existed in those nuclei where violent events occurred is very difficult to estimate. According to Mathews and Baker, the total mass-loss-rate from red giants, planetary nebulae, binary stars, pulsating stars and supernovae in an elliptical galaxy with mass of $10^{11} \text{M}_\odot$ could easily amount to $0.1 \sim 1.0 \text{M}_\odot$/year. At this rate $10^9 \text{M}_\odot \sim 10^{10} \text{M}_\odot$ of the gas would be ejected in a Hubble time, yet there is no proof that such amounts of gas exist. It is quite probable that a comparable amount of gas with a stellar mass of nucleus falls into the central region of a galaxy.

We consider stellar systems which in gravitational equilibrium and which contain appreciable amounts of gas.

Unno investigated the properties of gas-star system under the assumption

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that the stellar system contracts together with the gas. In his case, however, it is to be examined whether the stellar system contracts together with the gas or not.

The aim of this paper is to investigate the conditions of stellar systems under which they can contract due to the dissipation of stellar kinetic energy to the stages where enormous amounts of energy is released through the star-gas interactions and through the star-to-star collisions. In § 2 the dissipation rates of stellar kinetic energy are given, and the time-scales of contraction are obtained in § 3. In § 4 the condition for stellar system to contract is obtained and the conclusion is given in § 5. For simplicity we will assume that the systems are spherical and also that all stars are of the solar type.

§ 2. The dissipation rates of stellar kinetic energy

When a star moves through neutral gas, the dissipation rate of its kinetic energy through the direct interaction is given by

\[-(\frac{d\varepsilon}{dt})_D = C \rho_v V^2 \pi r^3,\]

where \(\varepsilon = (1/2) m_0 V^2\) is the kinetic energy of a star, \(\rho_v\) is the gas density, \(V^2\) is the root-mean-square stellar velocity, \(r\) is the solar radius and \(C\) is the constant of the order of unity. We put \(C = 1\).

When gas is ionized, the dissipation takes place through the larger cross-section. The solar-type star is considered to have a surface magnetic field whose strength is of the order of 100 gauss. If a star moves through ionized gas, the stellar magnetosphere will be formed just as the Earth's magnetosphere is formed through the interaction between its magnetic field and the solar wind. The dissipation rate of stellar kinetic energy is determined by the cross-section of its magnetosphere. The cross-section which is perpendicular to the stellar velocity can be estimated by the condition that, at the boundary of the cross-section, the magnetic pressure is equal to the static gas pressure,

\[\frac{B^2}{8\pi} = \frac{\rho_v k T}{m_H},\]

where \(B = B_0 (r/r_\odot)^{-4}\) is the stellar dipole field, \(T\) is the gas temperature and \(m_H\) is the proton mass. The radius of the cross-section is then given by

\[r = \left(\frac{m_H B^2}{8\pi \rho_v k T}\right)^{1/6} r_\odot.\]

In this case, the dissipation rate is

\[-(\frac{d\varepsilon}{dt})_D = \pi \rho_v V^2 \alpha^2 r_\odot^3\]
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with \( \alpha = (m_h B_\odot^3 / 8 \pi \rho_0 k T)^{\mu} \).

When a star moves through gas, its kinetic energy can be dissipated also through gravitational interaction. In relatively compact stellar systems as in nuclei of galaxies, \( V_i \) can be far larger than the thermal velocity of the gas. In this case, the impulsive approximation applies and the dissipation rate through gravitational interaction is given by \( \psi^5,6 \)

\[ - \left( \frac{d\psi}{dt} \right) = \frac{4 \pi \rho \psi G^2 m_\odot^2}{V_i} \ln \left( \frac{S_{\text{max}}}{S_{\text{min}}} \right), \]  

where \( S_{\text{max}} \) is the dimension of the region where the gravitational force of a single star dominates that of the whole system and given approximately by \( R/[(1 + \delta)N_i]^{1/2} \), and \( S_{\text{min}} \) is either the geometrical radius of the star or the radius at which the approximation becomes invalid.

§ 3. The time-scale of contraction

The time-scale of contraction of a stellar system is determined by the total dissipation rate of kinetic energy of stars. This rate is independent of whether the gas occupies the whole stellar system uniformly or occupies, by forming gaseous clouds, a part of the volume of a stellar system, as long as the radii of clouds are far larger than the solar radius. Using the occupation factor \( f \) of gas, the mean density of gas is \( \rho = 3M_g/(4\pi R^3 f) \) where \( M_g \) is the gas mass and \( R \) is the radius of the system. If we assume uniform distribution of stars, the number of stars which exist in gas and dissipate their kinetic energy is \( N_t f \), where \( N_t \) is the total number of stars. The total dissipation rate is

\[ \frac{dE}{dt} = \frac{d\psi}{dt} N_t f, \]  

where \( E \) is the total kinetic energy of stars. Since \( (d\psi/dt) \propto \rho \propto 1/f \) as is seen from Eqs. (1), (4) and (5), the total dissipation rate is independent of the occupation factor \( f \).

From the virial theorem, the mean-square stellar velocity is

\[ V_i^2 = \frac{G (M_t + M_g)}{2R} = \frac{G (1 + \delta) M_t}{2R}, \]

where \( M_t = N_t m_\odot \) is the total mass of stars and \( \delta = M_g/M_t \). The constant \( 1/2 \) gives a reasonable close approximation for polytropes with \( n \) in the range from 0 to 4. The time-scale of contraction due to the direct interaction between stars and gas is obtained from Eqs. (1), (6) and (7) as

\[ \tau_D = \frac{E}{(dE/dt)_D} = \frac{2 \sqrt{2 R^{1/2} \psi}}{3\delta (1 + \delta)^{1/2} m_\odot^{1/2} G^{1/2} N_i^{1/2} \sec} \]

\[ = 2.5 \times 10^{\mu} \frac{R^{n/2}}{\delta (1 + \delta)^{1/2} N_i^{n/2}} \text{ year} \]
when gas is neutral, or from Eqs. (4), (6) and (7) as
\[
\tau_D = \frac{4 \sqrt{2} (kT)^{3/2} R^{5/2}}{6^{3/2} \delta^{3/2} (1 + \delta)^{1/2} r_0^{1/2} m_\odot^{1/2} G^{1/2} N_e^{1/2} m_H^{1/2} B_0^{1/2}} \sec
\]
\[
= 8.3 \times 10^{17} \frac{R^{4/3}}{\delta^{2/3} (1 + \delta)^{1/3} N_e^{1/3}} \text{ year}
\] (9)
when gas is ionized, where \(E = (1 + \delta) G m_\odot^3 N_e^4 / 4R\) and \(R'\) is in parsec. We put \(T = 10^4 \text{K}\) and \(B_0 = 100\) gauss in Eq. (9).

The time-scale of contraction due to the gravitational interaction is given from Eqs. (5), (6) and (7) as
\[
\tau_0 = \frac{E}{(dE/dt)_0} = \frac{(1 + \delta)^{3/2} R^{3/2} N_e^{1/3}}{12 \sqrt{2} G m_\odot^{1/2} \gamma \delta} \sec
\]
\[
= 4.0 \times 10^6 \frac{(1 + \delta)^{3/2} R^{3/2} N_e^{1/3}}{\gamma \delta} \text{ year},
\] (10)
where \(\gamma = \ln(S_{\text{max}} / S_{\text{min}})\) and \(R'\) is in parsec.

§ 4. The condition for stellar system to contract

In order for a stellar system to contract within a Hubble time, \(\tau\) must satisfy the condition that \(\tau \leq 10^{10}\) years. In Fig. 1 the \(N_e R\) relations which satisfy \(\tau_D = 10^{10}\) years and \(\tau_0 = 10^{10}\) years are shown in the case of \(\delta = 1\). We put \(S_{\text{max}} = R\) and \(S_{\text{min}} = r_0\) in calculating \(\tau_0\). If a stellar system is on the left side of the shaded region initially and the gas in a system remains interstellar, it can contract within a Hubble time. For a given \(N_e\), the relation \(\tau_D = 10^{10}\) years gives a larger value of \(R'\) when gas is ionized than when gas is neutral, since the cross-section of interaction between stars and gases is larger when the gas is ionized. The same relations are shown in Fig. 2 in the case of \(\delta = 10\).

\[\text{Fig. 1.}\]
\[\text{Fig. 2.}\]

Fig. 1. The \(N_e R\) relations which satisfy \(\tau_D = 10^{10}\) years and \(\tau_0 = 10^{10}\) years are shown in the case of \(\delta = 1\). We put \(S_{\text{max}} = R, S_{\text{min}} = r_0\) in calculating \(\tau_0\) and \(B_0 = 100\) gauss, \(T = 10^4 \text{K}\) in calculating \(\tau_D\) when gas is ionized.

Fig. 2. The same relations as in Fig. 1 are shown in the case of \(\delta = 10\).
As is seen from Figs. 1 and 2, a stellar system must originally be relatively compact to contract in a Hubble time and the gas density is relatively high. The gas is not in an equilibrium state but in a state of contraction in the absence of heat supply. If the gas condenses into ordinary stars, the stellar system cannot contract through gas-star interactions. If the gas forms a single supermassive object (SMO), the SMO goes into free-fall at first and goes into the phase of Kelvin contraction subsequently. If any energy source other than the gravitational energy release from the contraction of the SMO is not enough to stop the contraction, the SMO becomes unstable and will go into collapse at last due to the general relativistic effect. Since the life-time of SMO in this case is much shorter than a Hubble time, the stellar system cannot contract due to the dissipation of the kinetic energy of stars which exist in the SMO.

If, however, the dissipation of stellar kinetic energy in the SMO supplies the luminosity $L$ of SMO, the stellar system contracts together with the SMO. The condition $(dE/dt)_D \geq L$ gives

$$N_c \geq 1.2 \times 10^{11} \frac{R^{19/13}}{\delta^{1/4} (1 + \delta)^{1/4}}.$$  \hspace{1cm} (11)

In Fig. 3 the line $(dE/dt)_D = L$ is shown.

There is another possibility which will cause a stellar system to contract. A stellar system can dissipate its kinetic energy due to star-to-star direct colli-
sions. According to Spitzer and Saslaw, the average dissipation rate is \( \frac{de}{dt} \) = \( V_\ast \xi m_\odot / t_c \), where \( t_c \) is the mean collision time and \( \xi \) is the average mass of gas liberated in a stellar collision in unit of \( m_\odot \). The time scale of the contraction is

\[ \tau_c = 2.0 \times 10^7 \frac{R^{n_1}}{(1 + \delta)^{n_2} N_i^{n_2}} \text{ year} \]  

and it is almost impossible to contract within a Hubble time under reasonable values of \( N_i \) and \( R' \). To make this situation clear, we show the line given by \( (de/dt)_D = (de/dt)_c \) in Fig. 3, only in the upper part of which \( (de/dt)_c \) dominates \( (de/dt)_D \). In calculating \( (de/dt)_c \) we have used the value of \( \xi \) computed with criterion 2 by Spitzer and Saslaw.

To see the dominant type of dissipation of the stellar kinetic energy, the line given by the relation \( (dE/dt)_D = (dE/dt)_o \) is shown in Fig. 3. The liberated energy by star-to-star direct collisions dominates in region I and the dissipation through the direct interaction dominates in regions II and III. In region IV the dissipation through the gravitational interaction dominates.

The time-scale of contraction due to the evaporation process is longer than \( \tau_D \) under the various values we have used.

§ 5. Conclusion

We have obtained the condition for a stellar system to contract due to the dissipation of stellar kinetic energy through the gas-star interactions. The results are shown as \( N_i - R \) relations in Fig. 3 in the case of \( \delta = 1 \). When a system is on the left side of the shaded region initially, the time-scale of contraction is shorter than a Hubble time if we assume that the gas in a stellar system remains interstellar. When a system is in region III or IV, however, the gas cannot remain interstellar for a Hubble time, but it either condenses into ordinary stars or forms an SMO. If the gas condenses into ordinary stars, the star-gas interactions disappear and the stellar system does not contract. If the gas forms an SMO, the stellar system does not contract either since the life-time of SMO is much shorter than a Hubble time. In region II the dissipation of stellar kinetic energy supplies the luminosity of SMO, \( L \), and the stellar system contracts together with the SMO. In region I the liberated energy by star-to-star direct collisions supplies \( L \) and the stellar system contract. Thus, the stellar system contract only when it is extremely compact initially so that the energy generation is violent.

In conclusion, it is impossible for a stellar system to contract from the stage where the dissipation of stellar kinetic energy through the direct or the gravitational interaction between stars and gases is ineffective to the stage where the dissipation is effective. The other value of \( \delta \) does not change the conclusion essentially if it is of the order of unity.
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Acknowledgments

The main part of the present work was completed while the author stayed at the Goddard Institute for Space Studies, New York, New York, U.S.A., as a Resident Research Associate of the National Research Council-National Academy of Sciences. He wishes to express his sincere gratitude to Dr. R. Jastrow for his hospitality during the time spent there.

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