The Radiative Energy Loss from the Shock Front

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The radiation loss from shock waves which are generated during the collapse of protostars has a great effect on the structure of protostars. In this paper, the effects of the continuum and line radiations on the shock structure are investigated and the radiation loss rates are calculated. Plane steady shocks are assumed and the ambient conditions are taken as \( v = 60 \sim 20 \) km/sec and \( p = 10^{-8} \sim 10^{-13} \) g/cm\(^3\). The atmospheres are assumed to consist of pure hydrogen atoms which have only three levels, i.e., the ground, excited and ionized levels.

The computed results show that about 90 percent of the shock's initial kinetic energy escapes ahead of the shock front as the Balmer continuum radiation. However, in the case of low shock velocity (\( v \leq 30 \) km/sec) or high ambient density (\( p \geq 10^{-8} \) g/cm\(^3\)), the fraction of the lost energy decreases below about 80 percent.

§ 1. Introduction

At the time of the formation of protostars by rapid contraction of interstellar clouds, shock waves generated at the central core propagate toward the surface, dissociating and ionizing gas clouds.\(^1\) A part of the released gravitational energy by contraction is changed into the thermal or ionization energy of the gas, and the rest is lost in the form of radiation out of protostars. If the radiation loss is small, a protostar which has attained a quasi-hydrostatic equilibrium state has a very large radius and a high luminosity,\(^2\) and it contracts through the Hayashi phase. On the other hand, if the radiation loss rate is very large, the star comes to existence with a small radius and a low luminosity.\(^3\)

The radiation loss rate depends upon how much the radiation passes through the shock front into the preshock region and how much the collapsing gas and dust absorb the radiation in the preshock region. In many previous studies on the effects of radiation upon the shock waves in stellar atmospheres, the local thermodynamic equilibrium has been assumed between gas and radiation, and the Planck radiation has been used as the source function. Many authors have also used the diffusion approximation to the equation of radiative transfer. However, the shock front is optically thin and the local thermodynamic equilibrium does not hold. Therefore, the approximations of the Planck radiation and the diffusion cannot be used.

Whitney and Skalaifu\(\text{r}^4\) investigated the effects of radiation taking off these assumptions and found that the effect of the Lyman continuum radiation passing into the preshock gas through the shock front is large. They also investigated qualitatively the radiation loss from the atmosphere as the Balmer or higher level
continua (see Fig. 1). However, since they assumed for simplicity that the postshock region is entirely transparent to all continuum radiation and neglected the absorption, their results are far apart from the truth.

Yakubov⁶ pointed out that the postshock region cannot be transparent to the Lyman continuum and that, taking account of this fact, the ionization degree in the preshock region becomes rather low. Skalafuris⁶ investigated the radiative transfer of the Lyman continuum using a two-level model of hydrogen without assuming that the postshock region is transparent to this radiation. He has, however, assumed that this region is transparent to the Balmer and higher level continuua. His results show that 20 to 30 percent of the shock's initial kinetic energy escapes in front (see Fig. 1). If he had continued the numerical calculations beyond the region where the postshock region becomes opaque to the Lyman continuum, the radiation loss of the Balmer continuum would have increased by about 50 percent. Besides, if the assumption that the region is transparent to the Balmer continuum is removed, the radiation loss rate may become larger.

In order to investigate the radiative transfer of the Balmer continuum, the three-level approximation of a hydrogen atom is needed. Gorbatskii⁷ showed that the Lyman-α quanta play an important role in ionizing the gas behind the shock front where the radiative flow was neglected. Therefore, in this paper we solve simultaneously the equations of radiative transfer and the equations of gas flow using a model of a three-level hydrogen atom without any assumption about the optical thickness of the postshock region to continuum radiations, and investigate the shock structure, the radiation loss from the shock front and the effects of the Lyman and Balmer continuua and the Lyman-α line radiation. However, we make the simplifying assumptions that the preshock region is opaque to the Lyman continuum and transparent to the Balmer continuum.
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With this treatment the net flux vanishes in the region where the postshock gas becomes opaque to the Balmer continuum, and then the radiation will escape only forward from the shock region as the Balmer continuum (see Fig. 1). It is to be noted that most of shock's energy will not necessarily be lost since the ionized hydrogens do not completely recombine in the opaque postshock region.

In § 2, a rough estimate of the radiation loss is shown with simplified assumptions. In § 3, the basic equations, assumptions and approximations used in detailed calculations are shown. Numerical results and some discussion are presented in § 4.

§ 2. A rough estimate of the radiation loss

We shall use the following approximations which will also be used in the following sections. The shock waves are assumed to be one-dimensional plane waves since their thickness is very thin compared to the stellar radius. Since the time-scale of the change of ambient conditions are expected to be long compared with the time needed for the gas to pass through the shock, the gas flow is assumed to be steady in a co-ordinate frame moving with the shock. The gas is composed of pure hydrogen atoms, and they are assumed to have only three levels which are the ground level, (first) excited level and ionized level. The number density of the hydrogen in each level is designated by \( n_1 \), \( n_2 \) and \( n_3 \), respectively. With the ionization degree \( \alpha \) and the excitation degree \( \beta \) we have

\[
n_1 : n_2 : n_3 = 1 - \alpha - \beta : \beta : \alpha.
\]

Before entering into detailed computations, we shall estimate in this section the radiation loss with simplifying approximations. The conservation equation of the energy in front of and in the rear of a plane steady shock may be written as

\[
\frac{1}{2} v^4 + \frac{5}{2} \frac{P}{\rho} + I - \frac{F}{\rho v} = \frac{1}{2} v_0^4 + \frac{5}{2} \frac{P_0}{\rho_0} + I_0 - \frac{F_0}{\rho_0 v_0},
\]

where the quantities subscripted with 0 indicate values at great distances ahead of the shock front and the unsubscripted quantities indicates values at great distances behind the shock. In this equation \( v \) is the velocity of the gas with respect to the shock front, and \( P, \rho \) and \( I \) indicate the pressure, density and ionization plus excitation energy per unit mass, respectively, and \( F \) is the radiative flux whose sign is positive in the upstream direction.

Here, we shall make the following approximations for simplicity: the hydrogen in the preshock gas is in the ground state because of the low temperature and most of the gas far behind the shock is also in the ground state by the radiative cooling. Therefore, the terms \( I_0 \) and \( I \) in Eq. (1) are neglected since \( \alpha_0, \beta_0, \alpha \) and \( \beta \) are much smaller than unity. The term \( 1/2 \cdot v^4 \) is also neglected because it is a few orders of magnitude lower than the term \( 1/2 \cdot v_0^4 \) as a result of cooling of the gas. Furthermore, since the gas far behind the shock would
become opaque to radiations and attains a thermodynamic equilibrium state, the radiative flux $F$ would tend to vanish.

Finally Eq. (1) is modified to

$$f = \frac{F_0}{1/2 \cdot \rho_0 v_0^2} = 1 - \frac{k}{m_H v_0^2} (T - T_0),$$

where $T$ is the temperature of the gas, and $m_H$ and $k$ are the mass of a hydrogen atom and the Boltzmann constant, respectively. The quantity $f$ is the ratio of the radiative energy flux passing forward from the shock to the kinetic energy flux of the preshock gas. As the Lyman continuum radiation would be absorbed in the preshock gas, $F_0$ is the Balmer continuum flux.

Now, we shall adopt a two-stream model for the radiation field. That is, the net radiative flux of the Balmer continuum $F_B$ is the difference between the flux $F_B^+$ in the upstream direction and the flux $F_B^-$ in the downstream direction of the gas. For $F_B^\pm$ we have

$$\frac{dF_B^{\pm}}{dx} = \mp \left( j_B - \sqrt{3} \sigma_B n_H F_B^\pm \right),$$

where the signs are in the same order, the spatial derivative is taken in the direction of the gas flow, and $j_B$ and $\sigma_B$ are the emissivity from a unit volume and the absorption cross section for the Balmer continuum photons, respectively. In the region where the optical depth is more than several of unity from the shock front, $F_B^\pm$ may be represented by

$$F_B^\pm = F_B^{\text{eq}} = \frac{j_B}{2 \sqrt{3} \sigma_B n_H}.$$  \hspace{1cm} (4)

The ionization degree would be high and $j_B$ is rather large before the thermal equilibrium is attained behind the shock front, so that the second term on the right-hand side of Eq. (3) could be neglected compared to the first term. With this approximation, $dF_B^+/dx$ becomes equal to $-dF_B^-/dx$, and since $F_B^- = 0$ at the shock front and $F_B^- = F_B^+ = F_B^{\text{eq}}$ in the thermal equilibrium region, $F_B^+$ at the shock front $F_0$ becomes equal to $2F_B^{\text{eq}}$. Hence, $F_0$ can be represented as a function of the temperature at the thermodynamic equilibrium state $T$ by substituting the numerical values into $j_B$ and $\sigma_B$ (see § 3) as follows:

$$F_0 = 2F_B^{\text{eq}} = 2.05 \times 10^{-4} \left( \frac{\gamma_B}{1/4 \cdot kT_H} \right)^x \left[ \frac{n \alpha^2}{\beta T^{1/2}} \right]_{\text{eq}}$$

$$= 2.15 \times 10^9 T \exp \left( - \frac{T_H}{4T} \right),$$

where $n$ is the number density of hydrogen and is given by $\rho / m_H$ or $n_1 + n_2 + n_3$, $T_H = \chi/k = 1.578 \times 10^9 K$, $\chi$ is the ground state ionization energy of hydrogen, and $\gamma_B$ is the average photon energy of the Balmer continuum, which is taken to be $1.2 \times 1/4 \cdot kT_H$ in this section. The last representation of Eq. (5) is obtained by
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Table I. The loss rate of the shock's energy by the Balmer continuum \((T_0=10^4\text{K})\).

<table>
<thead>
<tr>
<th>(v_0) (km/sec)</th>
<th>(\rho_0) (g/cm(^3))</th>
<th>(f = F_0/\frac{2}{3}\rho_0v_0^3)</th>
</tr>
</thead>
<tbody>
<tr>
<td>60</td>
<td>(10^{-10})</td>
<td>0.94</td>
</tr>
<tr>
<td>50</td>
<td>(10^{-10})</td>
<td>0.92</td>
</tr>
<tr>
<td>40</td>
<td>(10^{-10})</td>
<td>0.88</td>
</tr>
<tr>
<td>30</td>
<td>(10^{-10})</td>
<td>0.82</td>
</tr>
<tr>
<td>20</td>
<td>(10^{-10})</td>
<td>0.65</td>
</tr>
<tr>
<td>60</td>
<td>(10^{-11})</td>
<td>0.97</td>
</tr>
<tr>
<td>60</td>
<td>(10^{-9})</td>
<td>0.91</td>
</tr>
<tr>
<td>60</td>
<td>(10^{-8})</td>
<td>0.85</td>
</tr>
<tr>
<td>40</td>
<td>(10^{-12})</td>
<td>0.93</td>
</tr>
<tr>
<td>40</td>
<td>(10^{-9})</td>
<td>0.84</td>
</tr>
<tr>
<td>40</td>
<td>(10^{-8})</td>
<td>0.75</td>
</tr>
</tbody>
</table>

using the Saha equation for \(\left[na^2/\beta T_{1/3}\right]_{\text{eq}}\).

When the ambient conditions \(v_0, \rho_0\) and \(T_0\) are given, \(T\) and then \(f\) is obtained from Eqs. (2) and (5), and the results for several cases are tabulated in Table I. These values of \(f\) agree well with the results of the detailed calculation in the following sections showing that the approximation, \(F_0 = 2F_0\) (eq), is appropriate.

\section{3. Basic equations and methods of computation}

In this section we shall give the basic equations for the detailed analysis of the shock structure. In the following, the effects of viscosity, thermal conduction and electric and magnetic fields in the shock region are all neglected.

\subsection{a) The preshock region}

The conditions just at the front of the shock, which are designated by the subscript \(f\), can be derived from the conservation equations of mass, momentum and energy as follows:

\[\rho_f v_f = \rho_0 v_0,\]

\[P_f + \rho_f v_f^2 = P_0 + \rho_0 v_0^2,\]

\[\frac{1}{2} v_f^2 + \frac{5}{2} \frac{P_f}{\rho_f} - F_f = \frac{1}{2} v_0^2 + \frac{5}{2} \frac{P_0}{\rho_0} + F_0 - \frac{F_0}{\rho_0 v_0},\]

where the subscript \(0\) indicates the conditions at great distances ahead of the shock. The expressions are all the same as in the preceding section. In the preshock region the flux of the Lyman continuum radiation, \(F_L\), may be entirely absorbed and never be reemitted. The preshock gas is assumed to be transparent to the Balmer continuum radiation \(F_0\) and the Lyman-\(\alpha\) line flux is ignored, though it is a problem whether these assumptions are applicable or not. Let \(F^+\) be the
flux of the forward beam moving upstream in the direction of negative $x$. As the backward beam $F'$ is negligible, the net flux is equal to the forward flux in the preshock region. Then, we have

$$F_0 - F_f = F_{H_0} - (F_{H_f} + F_{L_f}) = - F_{L_f}^+.$$  

(9)

Since the preshock gas, as it approaches the shock front, absorbs the Lyman continuum photons and is ionized (the excitation does not occur), the difference of $I_0$ and $I_f$ in Eq. (8) is given by

$$I_0 - I_f = \frac{k T_H}{m_H} (\alpha_0 - \alpha_f) = - \frac{k T_H}{\gamma_L} \frac{F_{L_f}^+}{\rho_0 v_0},$$  

(10)

where $\gamma_L$ is the energy of the Lyman continuum photon.

From Eqs. (6) to (10) we have

$$v_f = \frac{5}{8} \left( v_0 + \frac{C_0^2}{v_0} \right) + \sqrt{\left( \frac{3}{8} v_0 - \frac{5}{8} \frac{C_0^2}{v_0} \right)^2 - \frac{1}{2} \left( 1 - \frac{k T_H}{\gamma_L} \right) F_{L_f}^+},$$  

(11)

$$P_f = \rho_0 v_0 \left( v_0 + \frac{C_0^2}{v_0} - v_f \right),$$  

(12)

$$\rho_f = \frac{\rho_0 v_0}{v_f},$$  

(13)

$$\alpha_f = \alpha_0 + \frac{m_H F_{L_f}^+}{\gamma_L \rho_0 v_0},$$  

(14)

$$\beta_f = \beta_0,$$  

(15)

where $C_0$ is the sound velocity $(P_0/\rho_0)^{1/3}$. Let $T_a$ be the temperature of hydrogen atoms and ions and $T_e$ the temperature of electrons, then the pressure is given by the equation $P_f = k/m_H \cdot \rho_f (T_{af} + \alpha_f T_{ef})$, and we assume in this region, $T_e = T_a$. If the values of $F_{L_f}$ and $\gamma_L$ are found from the results of computations in the postshock region and the ambient conditions are given, the values of the quantities just at the front of the shock are obtained by Eqs. (11) to (15).

b) The shock front

Immediately behind the shock the collisional relaxation between the external degrees of freedom occurs and it raises the atom and ion temperature $T_a$. But collisional excitation or ionization does not occur, and if the Mach number of the preshock flow is below a few tens as is usually the case, the electron temperature hardly rises because the preshock flow is subsonic to the electron gas (in this respect, the calculations of Skalafuris et al. are wrong and his subsequent paper also does not take into account this fact). Further, as the mean free paths of photons are much greater than those of gases, the absorption or emission of radiation may hardly occur.

Therefore, we find the usual Rankine-Hugoniot relations as follows:
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\[ \rho_b v_b = \rho_f v_f, \]  
\[ P_b + \rho_b v_b^2 = P_f + \rho_f v_f^2, \]  
\[ \frac{1}{2} v_b^2 + \frac{5}{2} \frac{P_b}{\rho_b} = \frac{1}{2} v_f^2 + \frac{5}{2} \frac{P_f}{\rho_f}, \]

where the subscript \( b \) denotes the quantities immediately behind the shock. Thus the quantities just behind the shock are written by those at great distances ahead of the shock in the form

\[ v_b = \frac{5}{8} (v_0 + \frac{C_0}{v_0}) - \sqrt{\left( \frac{3}{8} v_0 - \frac{5}{8} C_0^2 \right)^2 - \frac{1}{2} \left( 1 - \frac{k T_H}{\gamma_L} \right) \frac{F_{L_f}^*}{\rho_b v_0}}, \]

\[ P_b = \rho_b v_0 \left( v_0 + \frac{C_0}{v_0} - v_b \right), \]

\[ \rho_b = \frac{\rho_b v_0}{v_b}. \]

The atom temperature \( T_{ab} \) is now obtained from

\[ P_b = \frac{k}{m_H} \rho_b (T_{ab} + \alpha_b T_{eb}). \]

Here, we put \( T_{eb} = T_{ef} \) and \( \alpha_b = \alpha_f \).

c) The postshock region

In the postshock region, the gas can neither be assumed to be transparent nor opaque to radiations, and the equations of radiative transfer should be solved using the values of radiative fluxes immediately behind the shock as the boundary conditions.

We shall consider the following atomic processes by means of the three-level approximation for a hydrogen atom as in Fig. 2.

Collisional processes

- (elastic)
  \[ H + e \rightleftharpoons H + e, \]
  \[ H^+ + e \rightleftharpoons H^+ + e, \]

- (inelastic)
  \[ H + e \rightleftharpoons H^* + e, \]
  \[ H + e \rightleftharpoons H^* + e + e, \]
  \[ H^* + e \rightleftharpoons H^* + e + e, \]

Radiative processes

- \[ H + \gamma_L \rightleftharpoons H^* + e, \]
- \[ H^* + \gamma_B \rightleftharpoons H^* + e, \]
Fig. 2. Scheme of the collisional and radiative processes of the three-level hydrogen.

\[ H + \gamma_1 \leftrightarrow H^* \]
\[ H^* + \gamma_L \rightarrow H^+ + e \]
\[ H^* + \gamma_1 \rightarrow H^+ + e \]

Here, H, H*, H^+ and e stand for hydrogens in the ground state, excited state and ionized state and electrons, respectively. Besides \( \gamma_L, \gamma_B \) and \( \gamma_1 \) stand for photons of the Lyman continuum, Balmer continuum and Lyman-\( \alpha \) line, respectively, and we shall use the same symbols as the values of the energy of photons. The ionizing and exciting processes by atom-atom collisions are neglected because they have little effects in comparison with the processes by the electron impacts for \( \alpha \geq 10^{-4} \).

First, the energy equation of electrons is written as

\[
\frac{dE_e}{dt} + P_e \frac{dV}{dt} + \frac{d}{dt} \left( E_r + I - \frac{F}{\rho v} \right) = \left( \frac{dQ}{dt} \right)_{el} + \left( \frac{dQ}{dt} \right)_{ea}, \tag{23}
\]

where the electron energy per unit mass, \( E_e \), is given by the equation \( E_e = \frac{3}{2} P_e / \rho = \frac{3}{2} \alpha (kT_e / m_H) \), \( P_e \) stands for the electron pressure, \( V \) is the specific volume which is equal to \( \rho^{-1} \), the energy of Lyman-\( \alpha \) photons per unit mass, \( E_r \), is given by \( \frac{3}{2} \gamma (kT_H / m_H) \), \( \gamma \) being the ratio of the number density of photons \( n_r \) to that of heavy particles \( n \), and \( I \) is given by \( (\alpha + \frac{3}{2} \beta) kT_H / m_H \). The terms on the right-hand side of Eq. (23) stand for the energy gain of electrons by the elastic collisions with ions and atoms, which are written as

\[
\left( \frac{dQ}{dt} \right)_{el} = \frac{3}{2} \alpha k \frac{T_e - T_e}{m_H t_{eq}}, \tag{24}
\]

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\[
\frac{dQ}{dt} = \alpha \left( n_e \right)^{1/2} \frac{T_e - T_i}{T_e} \rho_i \langle \sigma_{\text{en}} v^3 \rangle,
\]

(25)

where \( m_e \) is the mass of the electron and \( \sigma_{\text{en}} \) is the elastic scattering cross section for the electron and the atom. According to Spitzer,\(^{19}\) \( t_{\text{eq}} \), the time of equipartition of energy between electrons and ions, is given by

\[
t_{\text{eq}} = \frac{252 T_e^{3/2}}{n_e \ln \Lambda},
\]

where \( \Lambda = 9.43 + 1.15 \log \left( T_e^{3/2}/n_e \right) \), and \( n_e \) is the number density of electrons and is equal to \( n_i \). Here, we set the ion temperature equal to the atom temperature following Skalafuris\(^{19}\) who has shown that there is little difference between them in the postshock region. From the experimental results of Brackmann et al.,\(^{10}\) we have approximately

\[
\sigma_{\text{en}} = (4 + 24 e^{-0.38 \varepsilon}) \pi a_e^2,
\]

(26)

where \( \varepsilon \) is the kinetic energy of the electron in electron volt, and \( a_e \) is the Bohr radius. From the approximation (26) we get

\[
\langle \sigma_{\text{en}} v^3 \rangle = \int_0^\infty \sigma_{\text{en}} v^3 f(v) dv
\]

\[
= 4 \left( \frac{8}{\pi} \right)^{1/2} \left( \frac{k T_e}{m_e} \right)^{1/2} \left\{ 4 + \frac{24}{(1 + 2.0 \times 10^{-5} T_e)^3} \right\}.
\]

(27)

Since \( d/dx = d/\nu dt \) from the assumption of a plane steady shock, Eq. (23) is rewritten as

\[
\frac{3}{2} \alpha T_e \frac{d}{dt} \left( T_e \frac{d
u}{dx} \right) + \frac{1}{nk} \frac{dF}{dx} - \left( T_H + \frac{3}{2} T_e \right) \frac{d\alpha}{dt} \left( \frac{dQ}{dt} \right)_{\text{ei}} + \frac{1}{k} \left( \frac{dQ}{dt} \right)_{\text{en}} \right\},
\]

(28)

Second, the equations of radiative transfer are found by use of the Eddington two-stream approximation. The opacity of the postshock gas seems to be very large to the Lyman-\( \alpha \) line radiation, so that we neglect the Lyman-\( \alpha \) radiative flux relative to the gas flow. Then, the net flux \( F \) is represented by \( (F_L^+ - F_L^-) \) and each term is calculated by the following equations:

\[
\frac{dF_L^\pm}{dx} = \left\{ \frac{j_L}{2} - \sqrt{3} \left( \sigma_L n_1 + \sigma_L n_2 \right) \right\} F_L^\pm,
\]

(29)

\[
\frac{dF_B^\pm}{dx} = \left\{ \frac{j_B}{2} - \sqrt{3} \sigma_L n_2 F_B^\pm \right\},
\]

(30)

where

\[
\begin{align*}
    j_L &= 4.501 \times 10^{-22} T_e^{-1/2} n_e^2, \\
    j_B &= 4.501 \times 10^{-22} T_e^{-1/2} n_e^2 / 8, \\
    \sigma_L &= 7.906 \times 10^{-19} \varphi_L (kT_H/\gamma_L)^3,
\end{align*}
\]
\[
\sigma_{LB} = 7.906 \times 10^{-14} \bar{g}_B (kT_H/\gamma_B)^3/32
\]

and

\[
\sigma_B = 7.906 \times 10^{-14} \bar{g}_B (kT_H/\gamma_B)^3/32
\]

in c.g.s. system of units. Here, \( \bar{g}_L \) and \( \bar{g}_B \) are the gaunt factors for the bound-free absorption in the ground state and the excited state, respectively, and from the tables of Karzas et al.\textsuperscript{10} we adopt \( \bar{g}_L = 0.8 \) and \( \bar{g}_B = 0.9 \) through the calculations. The photon energies of the Lyman and Balmer continua, \( \gamma_L \) and \( \gamma_B \), depend on the electron temperature of the region where the photons are emitted, and are given by

\[
\gamma_n = \frac{kT_H}{n^2} \frac{1}{\theta_n} e^{-\theta_n} \left[ -E_\ell (-\theta_n) \right], \quad (31)
\]

where \( \theta_n = T_H/n^2T_e \) and \( n \) is the principal quantum number of the level of a hydrogen atom. By use of the approximate equation for the exponential integral,\textsuperscript{12} 

\[-E_\ell (-\theta_n), \]

which is given by \( \int_{\theta_n}^\infty (1/t) e^{-t} dt \), Eq. (31) is rewritten as

\[
\gamma_n = \frac{kT_H}{n^2} \frac{\theta_n^2 + 3.331\theta_n + 1.682}{\theta_n^2 + 2.335\theta_n + 0.251}. \quad (32)
\]

In the following computations, we adopt the values of \( \gamma_n \) at the points where the net fluxes, \( F_L \) and \( F_B \), become 1/2 of their maximum values in the postshock region, respectively.

Next, we shall assume that a radiative equilibrium holds between the Lyman-\( \alpha \) line radiation and the gas, i.e.,

\[
n = \frac{A_n}{\sigma_1c} \frac{n_1}{n_1 - (g_1/g_2)n_2}, \quad (33)
\]

where \( A_n \) is the spontaneous de-excitation transition probability, \( \sigma_1 \) is the absorption cross section for the Lyman-\( \alpha \) radiation, \( c \) is the light velocity and \( g_1 \) and \( g_2 \) are the statistical weights of the ground state and the excited state, respectively. Although \( \sigma_1 \) depends on the atom temperature \( T_a \) we assume for simplicity that it takes a constant value such that

\[
\frac{A_n}{\sigma_1c} = 5 \times 10^{41} \text{ (cm}^{-1}).
\]

Differentiating Eq. (33) with time, we get

\[
\frac{1}{\gamma} \frac{d\gamma}{dt} + \frac{1}{n} \frac{dn}{dt} = \frac{1}{1 - \alpha - (5/4)\beta} \left( \frac{d\alpha}{dt} + \frac{1 - \alpha}{\beta} \frac{d\beta}{dt} \right). \quad (34)
\]

On the other hand, the equations for \( \alpha, \beta \) and \( \gamma \) have the forms

\[
\frac{d\alpha}{dt} = C_{11} + C_{21} + R_{11} + R_{21}, \quad (35)
\]
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\[ \frac{d\beta}{dt} = C_{12} - C_{22} + R_{12} - R_{22}, \]  

\[ \frac{d\eta}{dt} = -R_{12} - R_{133}, \]  

where \( R_{22} = R_{B22} + R_{L22} + R_{133} \). Each term on the right-hand sides of Eqs. (35) to (37) stands for the collisional and radiative reaction rate mentioned above in Fig. 2. The collisional ionization rate from the ground state is obtained by the experiments of Fite and Brackmann as

\[ C_{12} = 1.06\pi a_{0}^{2} \sqrt{\frac{8kT_{e}}{\pi m_{e}}} \left( 1 + \frac{2T_{e}}{T_{H}} \right) \exp\left( -\frac{T_{H}}{T_{e}} \right) \alpha \left\{ n_{1} - n_{2} \left( \frac{h^{2}}{2\pi m_{e}kT_{e}} \right)^{\frac{3}{2}} \exp\left( \frac{T_{H}}{T_{e}} \right) \right\}. \]  

The collisional excitation rates from 1S to 2P and 2S are obtained by the experiments of McGowan et al. and by Lichten and Schultz, respectively, and their results can be approximated as

\[ C_{13} = 5.7\pi a_{0}^{2} \sqrt{\frac{8kT_{e}}{\pi m_{e}}} \left( 1 + \frac{8T_{e}}{3T_{H}} \right) \exp\left( -\frac{3T_{H}}{4T_{e}} \right) \alpha \left\{ n_{1} - n_{2} \left( \frac{h^{2}}{2\pi m_{e}kT_{e}} \right)^{\frac{3}{2}} \exp\left( \frac{3T_{H}}{4T_{e}} \right) \right\}. \]  

As for the collisional ionization rate from the excited state we adopt the approximate formula obtained theoretically by Mihalas as

\[ C_{23} = (19.987 - 0.589 \times 10^{-4}T_{e} - 2.819 \times 10^{-4}T_{e}^{-1} + 0.544 \times 10^{8}T_{e}^{-2}) \times \pi a_{0}^{2} \sqrt{\frac{8kT_{e}}{\pi m_{e}}} \exp\left( -\frac{T_{H}}{4T_{e}} \right) \alpha \left\{ n_{3} - n_{4} \left( \frac{h^{2}}{2\pi m_{e}kT_{e}} \right)^{\frac{3}{2}} \exp\left( \frac{T_{H}}{4T_{e}} \right) \right\}. \]  

Further, the radiative reaction rates are given by

\[ R_{11} = -\frac{1}{n_{1}^{L}} \{ j_{L} - \sqrt{3}\sigma_{L} n_{1}(F_{L}^{+} + F_{L}^{-}) \}, \]  

\[ R_{B22} = -\frac{1}{n_{B}^{L}} \{ j_{B} - \sqrt{3}\sigma_{B} n_{2}(F_{B}^{+} + F_{B}^{-}) \}, \]  

\[ R_{133} = \frac{1}{n_{1}^{L}} \sqrt{3}\sigma_{L} n_{1}(F_{L}^{+} + F_{L}^{-}) \]  

and

\[ R_{133} = 7.906 \times 10^{-3} \beta c n_{1} \]  

and \( R_{11} \) is obtained from Eqs. (34), (36) and (37).

With the conservation laws the quantities in the postshock region (denoted with no subscript) are related to the quantities just behind the shock as

\[ \rho v = \rho_{s} v_{b}, \]  

\[ P + P_{r} + \rho v^{2} = P_{s} + \rho_{s} v_{b}^{2}, \]  

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\[
\frac{1}{2} v^2 + \frac{5}{2} \frac{P}{\rho} + 4 \frac{P_v}{\rho} + I - \frac{F}{\rho v} = \frac{1}{2} v_b^2 + \frac{5}{2} \frac{P_b}{\rho_b} + I_b - \frac{F_b}{\rho_b v_b},
\]

where \( P_r \), the pressure of the Lyman-\( \alpha \) radiation, is given by \( (1/4) n_e kT_H \), which has been ignored in front of the shock. The values of all quantities in the post-shock region may be obtained by solving Eqs. (23) to (47), simultaneously.

\section*{Methods of computation}

In the procedure of computations, we first give the values of ambient conditions, i.e., \( \rho_0, v_0, P_0 \) (or \( T_0 \)), \( \alpha_0 \), and \( \beta_0 \), in front of the shock. Next, by taking the trial values of the fluxes at the shock front, \( F_{L_f}^+ \) and \( F_{B_f}^+ \), and assuming the values of \( \gamma_L \) and \( \gamma_B \), the conditions just behind the shock are all fixed. Then, from this point we can calculate downwards with the equations derived above.

If the value of \( F_{L_f}^+ \) or \( F_{B_f}^+ \) is unfit, the values of the flux may become unstable around the region where the gas becomes opaque to the Lyman or Balmer continuum radiation. In this case, we correct the value of \( F_{L_f}^+ \) or \( F_{B_f}^+ \), and the values of \( \gamma_L \) and \( \gamma_B \), and calculate again from the beginning. Iterating this procedure, we may approach the correct solution. The time step, which is equivalent to the space step in the steady shock, in integrating the difference equations has been taken so that the variation rates of the physical quantities \( T_a, T_\alpha, \alpha, \beta, 1-\alpha - \beta, \gamma, F_L \) and \( F_B \), are all less than 1 percent.

\section*{Numerical results and discussion}

\subsection*{Ambient conditions}

In calculations we have used the following values as the conditions at great distances ahead of the shock. The velocity of the gas flow, \( v_0 \), may be nearly equal to the free-fall velocity at the surface of the central core of protostars, which is of the order of \( (GM/R)^{1/2} \), where \( M \) and \( R \) are the mass and radius of the core of protostars, respectively. Provided that \( M \) is about \( 1 M_\odot \) and \( R \) is about \( 100 R_\odot \), \( v_0 \) becomes about 40km/sec. Therefore, we have investigated the cases of 60 to 20km/sec.

As the ambient density \( \rho_0 \) may be equal to or somewhat lower than the atmospheric densities of red giants, we have used \( 10^{-8} \) to \( 10^{-19} \)g/cm\(^3\) as \( \rho_0 \). The ambient temperature \( T_0 \) has little effect on the shock structure unless it is above several thousand degrees. In this work we have adopted 10K. The excitation degree \( \beta_0 \) and the ionization degree \( \alpha_0 \) are determined by the values of \( \rho_0 \) and \( T_0 \), but they may also be affected by, for example, cosmic rays. When \( \alpha_0 \) and \( \beta_0 \) are not so high, they have little effect on the shock. Therefore, we can take \( \alpha_0 = 10^{-4} \) and \( \beta_0 = 10^{-8} \sim 10^{-9} \) without any influence on the results.

\subsection*{Numerical results}

Some values of the computational results are tabulated in Table II. The shock structures are shown in Figs. 3, 5 and 7 for the cases \( \rho_0 = 10^{-10} \)g/cm\(^3\)
Table II. The physical quantities of the shock waves for various ambient conditions on $v_0$ and $\rho_0$ with $\alpha_0 = 10^{-4}$ and $T_0 = 10^4$K. The flux $F_{\text{H}}$ is the minimum energy flux needed to ionize the gas completely and is equal to $kT_0\rho_0v_0/m$. The lengths $l_\alpha$ and $l_B$ are the distances from the shock front to the points where $F_{\text{H}}$ becomes nearly equal to the local equilibrium flux, i.e., $F_{\text{H}} = j_\alpha/4\sqrt{3} (\sigma_{e\text{e}}n_1 + \sigma_{i\text{e}}n_2)$, and $F_{\text{B}}$ becomes $j_B/4\sqrt{3} \sigma_B n_2$, respectively, and $\rho_0$ is the density at the point where $(F_{\text{B}}^*-F_{\text{B}})/(F_{\text{B}}^*+F_{\text{B}}^*)$ decreases to 1/100. The maximum value of, for instance, the electron temperature in the postshock region is denoted by $(T_e)^{\text{max}}$.

<table>
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<tr>
<th>case</th>
<th>$v_0$ (km/sec)</th>
<th>$\rho_0$ (g/cm$^3$)</th>
<th>$F_{\text{H}}^*/F_{\text{H}}$</th>
<th>$\sigma_f$ $F_{\text{L}}^*/F_{\text{H}}$</th>
<th>$T_e$ (10$^4$K)</th>
<th>$T_B$ (10$^4$K)</th>
<th>$v_B$ (km/sec)</th>
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<td>1</td>
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<tr>
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<th>$(\beta)_{\text{max}}$</th>
<th>$(\gamma)_{\text{max}}$</th>
<th>$(\alpha)_{\text{max}}$</th>
<th>$l_\alpha$(m)</th>
<th>$l_B$(km)</th>
<th>$\rho_\alpha/\rho_0$</th>
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<td>1.4·10$^3$</td>
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Table III. The ionization degrees at the shock front ($\rho_0 = 2.4 \times 10^{-9} \text{g/cm}^3$).

<table>
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<th>$v_0$ (km/sec)</th>
<th>60</th>
<th>40</th>
</tr>
</thead>
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<td>our model</td>
<td>0.19</td>
<td>0.0019</td>
</tr>
<tr>
<td>Yakubov</td>
<td>0.27</td>
<td>0.0096</td>
</tr>
<tr>
<td>Skalafuris</td>
<td>0.066</td>
<td>0.0096</td>
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</table>

with $v_0 = 60, 40$ and $25\text{km/sec}$, respectively, and the rates of the atomic reactions in the shock waves for each case are shown in Figs. 4, 6 and 8. Space profiles of the fractional populations in various states are shown in Fig. 9 for the case $v_0 = 60\text{km/sec}$ and $\rho_0 = 10^{-10} \text{g/cm}^3$. The values of $f=F_{\text{B}}^+/2\rho_0 v_0^3$ have shown good agreement with the estimates in § 2. As is seen from Table II most of the shock’s kinetic energy is lost as the Balmer continuum, but when $v_0 \lesssim 25\text{km/sec}$, the value of $f$ decreases below 70 percent. The thickness of the shock waves varies nearly proportional to the inverse of $\rho_0$ but the other properties of the shock waves are not much affected by $\rho_0$. For $\rho_0 \gtrsim 10^{-7} \text{g/cm}^3$, however, the radiation loss decreases considerably.

The physical processes characterizing each region of the shock waves can be summarized as follows.

The Lyman continuum radiation passing through the shock front ionizes the preshock gas up to a few tens percent in the case $v_0 = 60\text{km/sec}$, and a few percent for $50\text{km/sec}$. For $v_0 \lesssim 30\text{km/sec}$, however, it has little effect on the shock structure. The amounts of this effect are given in Table II in the sixth column headed “$\alpha_f$”. When $v_0 \gtrsim 70\text{km/sec}$, the ionization degree of precursor approaches unity and then the radiative recombinations and the collisional processes in this region, which have been ignored in this work, would have considerable effect on the shock structure. Table III shows the ionization degrees at the shock front for the case $\rho_0 = 2.4 \times 10^{-9} \text{g/cm}^3$ (which is obtained by interpolation between the cases $\rho_0 = 10^{-8} \text{g/cm}^3$ and $10^{-9} \text{g/cm}^3$) together with the estimate of Yakubov

Considering that the approximations are quite different from each other, it may be said that these values agree rather well with each other.

When the preshock gas is highly ionized as in the case $v_0 = 60\text{km/sec}$, the electron temperature at the front, $T_\text{ef}$, rises to some extent from the ambient temperature $T_\circ$. Immediately behind the shock front, the velocity of the gas flow decreases to about one-fourth of the preshock velocity. Owing to the dissipation of the shock’s kinetic energy, the temperature of atoms and ions, $T_\alpha$, rises rapidly, but the electron temperature $T_e$ does not. Thereafter, $T_e$ begins to rise through the elastic atom-electron and ion-electron collisions, but the drop of $T_\alpha$ is not remarkable as far as the ionization degree is low. When $T_e$ is low, the major atomic process is the radiative ionization by the Lyman continuum in the case $v_0 \gtrsim 30\text{km/sec}$. As $T_e$ increases, the collisional excitation gradually becomes ac-
The Radiative Energy Loss from the Shock Front

Fig. 3. Profile of the shock structure for the case \(v_0=60\text{km/sec}\) and \(\rho_0=10^{-10}\text{g/cm}^3\) (case 1 in Table II). Fluxes are plotted in the unit of \(F_H\), which is defined as \(kT_H\rho_0v_0/m_H\).

Fig. 4. Plots of various reaction rates for the case \(v_0=60\text{km/sec}\) and \(\rho_0=10^{-10}\text{g/cm}^3\) (case 1).
Fig. 5. Same as Fig. 3, but $v_0 = 40$ km/sec, $\rho_0 = 10^{-10}$ g/cm$^3$ (case 5 in Table II).

Fig. 6. Same as Fig. 4, but $v_0 = 40$ km/sec, $\rho_0 = 10^{-10}$ g/cm$^3$ (case 5).
The Radiative Energy Loss from the Shock Front

Fig. 7. Same as Fig. 3, but \( v_0 = 25 \text{km/sec}, \rho_0 = 10^{-10} \text{g/cm}^3 \) (case 8 in Table II).

Fig. 8. Same as Fig. 4, but \( v_0 = 25 \text{km/sec}, \rho_0 = 10^{-10} \text{g/cm}^3 \) (case 8).
Fig. 9. Space profiles of the fractional populations in various states for the case $v_0=60\text{km/sec}$ and $\rho_0=10^{-10}\text{g/cm}^3$ (case 1). The asterisk denotes the values in thermodynamic equilibrium at the local conditions.

tive, and as a number of hydrogen atoms is populated in the excited state, the collisional ionization from this state rapidly becomes active, coupled with the temperature increase. The collisional ionization from the ground state and the radiative ionization from the excited state by the Balmer continuum are not effective.

When the cooling of the electron gas by the collisional excitation and ionization begins to work, the increase of $T_e$ weakens and finally a quasi-equilibrium state is attained. At the same time the collisional de-excitation occurs actively. Therefore, the populations of atoms in the excited and ground states are determined by the collisional equilibrium. Let $n_1^*, n_2^*$ and $n_3^*$ be the number densities of hydrogens in the corresponding states when the gas is in the thermal equilibrium with the local density and the temperature equal to the local electron temperature, then we have

$$\frac{n_2}{n_1} = \frac{n_2^*}{n_1^*} - \Delta \quad \text{and} \quad \frac{n_3}{n_1} \ll \frac{n_3^*}{n_1^*}. $$

The deviation from the thermal equilibrium, $\Delta$, is small. When the ionization degree approaches unity as is the case for $v_0=60\text{km/sec}$, the excitation degree begins to decrease according to the decrease of $n_1$, as is seen from the above
The Radiative Energy Loss from the Shock Front

On the other hand, the increase of the ionization degree, i.e., the increase of the impact electrons makes the reactions more active, but at the same time it decreases the gas thermal energy. Therefore, the temperature of heavy-particles, $T_a$, drops to $T_e$ and decreases further together with $T_e$. Then, since the ratio of $n_2^*$ to $n_1^*$ decreases, the excitation degree decreases further. The collisional reaction rates begin to decrease due to the drop of $T_e$. As for the radiative reactions, emission becomes active in place of absorption owing to the increase of $\alpha$.

Afterwards a quasi-equilibrium state is attained among the collisional excitation, the collisional ionization from the excited state and the radiative recombination to the ground and excited states. When $v_0$ is high, the peak value of the ionization degree approaches unity and the opacity of the Lyman continuum radiation becomes very small. Therefore, the emission region of the Lyman continuum is thick and the flux becomes large. On the contrary the flux of the Lyman continuum radiation is negligibly small when $v_0$ is small.

When the optical depth of the Lyman continuum becomes several of unity from the shock front, the fluxes $F_{1+}$ and $F_{1-}$ become equal to the intensity in the radiative equilibrium $F_L(eq)$. If the ionization degree takes a maximum value in the region between the front and this equilibrium region as is the case for $v_0 > 50 \text{km/sec}$, the flux at the front, $F_{1+}$, is larger than $F_L(eq)$. On the other hand, when $v_0 \leq 50 \text{km/sec}$, the value of $\alpha$ is still growing near the radiative equilibrium region, and then $F_{1+}$ is smaller than $F_L(eq)$. However, the region opaque to the Balmer continuum lies far behind the peak region of $\alpha$ in all cases for $v_0$. Therefore, the forward flux, $F_B^+$ is not much absorbed on the way. This is the reason why the approximation in §2 was appropriate. In the equilibrium region for the Lyman continuum $F_L(eq)$ decreases with the distance from the shock front due to the decrease of $\alpha$. The radiative recombination to the ground level is nearly completely compensated by the ionization by the Lyman continuum radiation. Therefore, the main process to change the physical situation of the gas is the radiative recombination to the excited state which is partially compensated by the collisional ionization. The collisional transitions between the excited and ground states are very active, but the net collisional de-excitation rate $-C_{1s}$ is small and nearly equal to the difference between the radiative recombination rate to the excited state, $-R_{1s}$, and the collisional ionization rate $C_{2s}$. Thus the gases gradually recombine and are de-excited with the decrease of the temperature. This situation lasts until the Balmer continuum radiation attains a radiative equilibrium.

In the final thermodynamic equilibrium state, the shock's initial kinetic energy has mostly been changed into the Balmer continuum radiation, and the gas kinetic temperature and the ionization degree are brought back nearly to the initial values and the velocity of mass motion drops to a few hundredths of the initial velocity.
c) Concluding remarks and discussion

The rate of the collisional ionization from the ground state, $C_{1s}$, is so small as to be a few hundredths of $C_{2s}$, and this process does not play an important role from the beginning to the end. This result disagrees with the model of Skalafuris.\textsuperscript{9}

The Lyman-$\alpha$ photons also have little effect on the shock waves for all cases we have investigated. The effect of the Lyman-$\alpha$ photons increases as $\rho_0$ decreases. However, even in the case $\rho_0 = 10^{-19}$g/cm$^3$ and $v_0 = 60$km/sec, the photo-ionization rate from the excited state by the Lyman-$\alpha$ photons is less than about $1/30$ of the collisional ionization rate $C_{2s}$. Therefore, even if Gorbatskii's value\textsuperscript{7} is used for $A_{2s}/\sigma C$ which is $20/3$ times greater than ours and is rather too large, the Lyman-$\alpha$ photons would not play a major role except in the case that the gas density is as low as that of circumstellar clouds.

The main purpose of this work was to investigate the energy loss rate from the shock front. In this respect, we have found that most of the shock's initial kinetic energy is lost by recombination radiations to higher levels. However, in the case $\rho_0 \geq 10^{-7}$g/cm$^3$ or $v_0 \leq 25$km/sec, the loss rate has been found much smaller.

This cannot, however, be regarded as a final conclusion because the loss rate for these cases may increase if the higher levels of hydrogens, negative hydrogens or heavy elements are considered. Besides, it is important to investigate whether or not the Balmer continuum radiation is absorbed much by the dust grains in the collapsing gas clouds when protostars are formed. It will be the next question to investigate a non-steady shock including the initial and boundary conditions.

Further, the numerical results indicate that the postshock region is not very thick to the Lyman-$\alpha$ line radiation when the shock velocity is as high as 60km/sec. Therefore, it is necessary to consider the transfer of this radiation in the case of a high shock velocity, though it may have little effect on the shock structure as the energy of the Lyman-$\alpha$ radiation is small.

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