Nonleptonic Decays and Sub-Hadronic Interaction. I
—Studies of Kaon Decays—

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From a viewpoint of the composite model of hadrons, the electromagnetic and weak interactions of pseudoscalar mesons are investigated under the following assumption: A reaction takes place only when the urbaryons (of which hadrons are composed) concentrate on one point of the space-time where and when those primary interactions take place. Especially we make investigation into the kaon nonleptonic decays by employing some types of the primary weak Hamiltonian of the nonleptonic decays at the urbaryon level. As possible candidates, three cases, i.e., $\mathcal{H}^1 = \sin \theta \cos G_0 (\tau_1 - \tau_2) \tau_3 + \text{h.c.}$, $\mathcal{H}^2 = G (\tau_1 \tau_2 \tau_3 + \text{h.c.})$ and $\mathcal{H}^3 = G (\tau_1 \tau_2 \tau_3 + \text{h.c.})$ are examined. In these expressions, $\tau_i (i=1, 2, 3)$ denotes the urbaryon triplet. It is shown that (1) neither $\mathcal{H}^1$ nor $\mathcal{H}^2$ is acceptable as the "primary" Hamiltonian; instead, (2) the case $\mathcal{H}^3 = (G_0 / \sqrt{2}) (\tau_1 \tau_2 \tau_3 + \text{h.c.})$ appears as a preferable choice among the cases $\mathcal{H}^3$. Implications of our results are also discussed.

§ 1. Introduction

Recently a considerable progress in a unified understanding of hadron reactions has been made by the consideration based on the urbaryon rearrangement diagrams (URD). It should, however, be noted that in those treatments the urbaryon lines have only the $SU(3)$ indices.

In this paper, from a viewpoint of composite model, we give discussions based on the URD, but we deal with the urbaryon lines which have not only the $SU(3)$ indices but also the Lorentz indices, in order to make more certain of particle picture of those lines.

As the first stage of our work, nonleptonic kaon decays are studied for the purpose of the investigation of the sub-hadronic interaction of nonleptonic decays. The discussions of nonleptonic hyperon decays will be given in subsequent papers.

From the viewpoint of composite model, in order to get a unified understanding of various features of hadron reactions, we are confronted with the following problems: (i) What primary interactions exist at the urbaryon level? (ii)
What dynamical structures have the hadrons as the composite systems of those urbaryons?

In this paper we try to get a reasonable answer for the question (i), without going into details of the question (ii). For example, as the primary Hamiltonian of nonleptonic decays, we may assume

\[ \mathcal{H}^i = \sin \theta \cos \theta G_0 \langle \bar{t}_s t_i \rangle \langle \bar{t}_1 v^* t_s \rangle + \text{h.c.} , \]

which can be derived from the universal current-current weak interactions,\(^*\)

\[ \mathcal{H} = G_0 \langle J_\rho + l_\rho \rangle \langle J^\rho + l^\rho \rangle^* + \text{h.c.} , \]

where

\[ J_\rho = \cos \theta \langle \bar{t}_s v_\rho t_i \rangle + \sin \theta \langle \bar{t}_3 v_\rho t_i \rangle , \]

\[ l_\rho = \langle \bar{v}_\mu v_\rho \rangle + \langle \bar{t}_3 v_\rho \mu \rangle , \]

\[ v_\rho = \gamma_\rho (1 - i \gamma_5) , \]

and

\[ G_0 = 1.502 \times (1 \pm 0.003) \times 10^{-7} m_{\pi}^{-2} . \]

On the other hand, if we take the \(|\Delta I| = 1/2\) rule seriously, we may assume

\[ \mathcal{H}^\Pi = \bar{t}_s \langle g_{\rho\rho} 1 + g_{\rho\gamma} i \gamma_5 \rangle t_s + \text{h.c.} \]

or

\[ \mathcal{H}^\Pi = G \langle \bar{t}_s O t_s \rangle \langle \bar{t}_s O^\gamma t_s \rangle + \text{h.c.} , \]

where

\[ O_{\rho} = f_{\rho\gamma} \gamma_\rho + f_{\rho\gamma} \bar{\gamma}_\rho i \gamma_5 \]

and

\[ O_{\rho} = f_{\rho\gamma} \bar{\gamma}_\rho + f_{\rho\gamma} \gamma_\rho i \gamma_5 . \]

\( \mathcal{H}^\Pi \) is the so-called “bilinear interaction”, in which the \(|\Delta I| = 1/2\) rule is assumed from the beginning. By using this “bilinear interaction” without taking account of the Lorentz-indices, the nonleptonic hyperon decays seem to be systematized phenomenologically.\(^*\) As far as we focus our attention on the URD without Lorentz-indices, both \( \mathcal{H}^\Pi \) and \( \mathcal{H}^\Pi \) lead to the same results.

As for \( \mathcal{H}^\Pi \), the following extreme cases are analyzed:

(a) \( (V-A) \) V.type; \( f_{\rho} = 1, f_{\rho} = 0, f_{\rho} G = G_{\rho\rho} \) and \( f_{\rho} G = G_{\rho\rho} \), \( (\cdot \cdot 11) \)

(b) \( (V-A) \) A.type; \( f_{\rho} = 0, f_{\rho} = 1, f_{\rho} G = G_{\rho\rho} \) and \( f_{\rho} G = G_{\rho\rho} \) \( (\cdot \cdot 12) \)

\(^*\) We use ornamental writing \( \mathcal{H} \) for the primary Hamiltonian at the urbaryon level and we write in print for the effective Hamiltonian at the hadron level.

\(^*\) Hereafter, we use \( G_0 = 1.50 \times 10^{-7} m_{\pi}^{-2} \).
In this paper, we restrict our discussions to the electromagnetic and weak interactions of the \( ps \)-mesons, the reasons for which are as follows: (i) Our assumptions proposed in §2 seem to be applicable in low energy phenomena of the composite hadrons which are the s-state systems of the constituents. (ii) The matrix elements of the electromagnetic and weak interactions are relatively well known in low energies. (iii) Structures of mesons are independent of models adopted, for example, the quark model, the quartet model and so on, whereas structures of baryons depend on models.

Thus, we shall deal with only the decays of \( ps \)-mesons. Since we have not necessarily the complete knowledge as for the dynamical structures of \( ps \)-mesons, we shall replace them by some dynamical assumptions. In §2, we assume that when a hadron reaction is caused, all the urbaryons which compose those hadrons must concentrate on one point of space-time. Therefore, only the values at origin in the internal wave functions of hadrons take effect in our calculations. According to our assumptions, in §2, we will study at first the electromagnetic interaction and semileptonic decays in order to find the conditions as to the parameters introduced and to show that our assumptions are considerably reliable. In §3, nonleptonic decays of \( ps \)-mesons are studied, in particular, the \( K \rightarrow 3\pi \) decays are studied in detail. In §4, on the basis of the analysis in §3, we investigate a possible form of the “primary” Hamiltonian of nonleptonic decays and we will conclude that the “primary” Hamiltonian is

\[
\mathcal{H}_{NL} = \frac{G_0}{\sqrt{2}} (i\gamma_5 (1-i\gamma_\tau) t_\tau) (i\gamma_\tau^* t_\tau) + \text{h.c.} \tag{1.14}
\]

Lastly §5 will be devoted to conclusions and remarks.

§2. Assumptions

It is well known that there exists a universality of the coupling strengths of weak interaction. That is expressed more generally by the statement that the coupling strengths in the effective Hamiltonian, which is chosen properly, take almost the same values in the units of \( \hbar = c = m_e = 1 \).

We may regard this regularity as a reflection of the some kind of law, which may exist beyond the framework of a current theory, in the sub-hadronic level. As the first step to find out such a law, we try to analyze kaon decays phenomenologically, under the following assumptions:

(1) As for the interactions, we replace the primary Hamiltonian \( \mathcal{H} \) by the modified Hamiltonian \( \mathcal{H}' \),

\[
(\text{c}) \quad (V-A)(V-A)\text{-type}; \quad f_v = f_v' = -f_A = -f_A' = 1 \quad \text{and} \quad G = G_{V-A}. \tag{1.13}
\]
Nonleptonic Decays and Sub-Hadronic Interaction. I

\( \mathcal{H}(x) \Rightarrow \mathcal{H}'(x) = \mathcal{H}(x) \times [\zeta (\bar{t}_s(x) t_t(x))]^n, \) \hfill (2.1)

where \( n \) is the number of urbaryon lines which have no concern with the primary interaction (the so-called "spectator") and the parameter \( \zeta \) is a constant with the dimension \((\text{mass})^{-4}\).

(2) As for the wave function of the \( ps \)-meson composed of an urbaryon and an anti-urbaryon, we use

\[
\langle 0 | T(t_\alpha(x_1) \bar{t}_\beta(x_2)) | q \rangle = \theta_q(x_1, x_2) \epsilon_{\alpha\beta} \frac{1}{\sqrt{(2\pi)^2 \omega_q}} \frac{e^{-i x \cdot q}}{4} \left[ (a(x) \gamma q + b(x)) i \gamma S_{\alpha\beta} \right], \tag{2.2}\]

where

\[ X = (x_1 + x_2)/2 \quad \text{and} \quad x = x_1 - x_2. \tag{2.3} \]

(3) Reactions are caused by rearrangements of the urbaryons which are the constituents of hadrons, that is, disconnected diagrams are omitted.

It should be noticed that the assumptions can be expressed in other words; a reaction takes place only when the urbaryons composing the hadrons concentrate on one point of space-time where and when the primary interaction takes place. We expect that these assumptions are valid in low energy reactions of \( ps \)-mesons which are the s-state systems.

For instance, if we want to obtain the electromagnetic interaction of \( ps \)-mesons (Fig. 1(a)), we calculate the amplitude as shown in Fig. 1(b) under our assumptions. That is, we replace the primary Hamiltonian

\[
\mathcal{H}_{\text{em}}(x) = \sum_{i=1}^3 e_i (\bar{t}_i(x) \gamma_p t_i(x)) A^p(x) \tag{2.4}
\]

by the modified sub-hadronic Hamiltonian

\[
\mathcal{H}_{\text{em}}'(x) = \sum_{i=1}^3 \zeta e_i (\bar{t}_i(x) \gamma_p t_i(x)) (\bar{t}_f(x) t_f(x)) A^p(x), \tag{2.5}
\]

where \( e_1 = 2e/3 \) and \( e_2 = e_3 = -e/3 \) for the quark model and \( e_1 = e \) and \( e_2 = e_3 = 0 \) for the quartet model. Then we can obtain the amplitude as follows:

\[
A(P_i^f(q) + \gamma(k) \rightarrow P_i^f(p)) = \zeta e_i \left( \frac{1}{4} \left[ (a(0) \gamma q + b(0)) i \gamma S_{\alpha\beta} (\bar{t}_f(a(0) \gamma p + b(0))) \right]_{\alpha\beta} \right)_{\beta} 
\]

\[
+ \zeta e_i \left( \frac{1}{4} \left[ (a(0) \gamma q + b(0)) i \gamma S_{\alpha\beta} (\bar{t}_f(a(0) \gamma p + b(0))) \right]_{\alpha\beta} \right)_{\alpha} 
\]

\[ = \zeta a(0) b(0) (e_i - e_f) [(p + q) \cdot e]. \tag{2.6} \]

Therefore we have a condition

\[ \zeta a(0) b(0) = 4. \tag{2.7} \]

Thus, we derive the amplitudes of hadron reactions by employing the modified sub-hadronic Hamiltonian \( \mathcal{H}' \) defined in (2.1), but in this paper, we
call $\mathcal{H}$ in (2.1) the “primary” Hamiltonian.

From the analysis of semi-leptonic decays, we obtain the values

$$a(0) = 0.980 \times (1 \pm 0.001) m_\pi \approx m_\pi, \quad (2.8)$$

$$\cos \theta = 0.973 \pm 0.034 \quad (2.9)$$

and

$$\sin \theta = 0.231 \pm 0.021. \quad (2.10)$$

Here we have employed the “primary” Hamiltonian (1.2).

§ 3. Nonleptonic decays of kaons

In Tables I and II, we show the amplitudes for $K \to 2\pi$ and $K \to 3\pi$ decays, respectively. Since it is clear that $\mathcal{H}^1$ and $\mathcal{H}^2$ transform as a member of the octet, only the amplitudes for $K_S \to \pi^+\pi^-$ and $K^+ \to \pi^+\pi^+\pi^-$ decays are shown in the case of $\mathcal{H}^1$ or $\mathcal{H}^2$. In Table II, we write the $K \to 3\pi$ decay amplitudes as the following form:

$$A(K \to \pi^i\pi^j\pi^k) = c(ijk) \left[ 1 + a(ijk) \frac{3T - Q}{Q} \right], \quad (3.1)$$

which are the linear approximation of decay amplitude which appears to be in good agreement with experimental data. $i, j$ and $k$ refer to the pion isospin indices, $T$ is the odd pion kinetic energy, $Q$ is the $Q$-value of the relevant process.

Table I. $K \to 2\pi$ decay amplitudes. $M$ is kaon mass and $m$ is pion mass. $x = b(0)/a(0)$ and $G_{pv} = G_{pv}/4$ are defined.

<table>
<thead>
<tr>
<th>Primary Hamiltonian</th>
<th>Process</th>
<th>Amplitude</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\mathcal{H}^1$</td>
<td>$K_S \to \pi^+\pi^-$</td>
<td>$\sqrt{2} \sin \theta \cos \theta G_{\pi\pi}(M^2 - m^2)$</td>
</tr>
<tr>
<td>$\mathcal{H}^2$</td>
<td>$K_S \to \pi^0\pi^0$</td>
<td>$-\sin \theta \cos \theta G_{\pi\pi}(M^2 - m^2)$</td>
</tr>
<tr>
<td>$\mathcal{H}^2$</td>
<td>$K^+ \to \pi^+\pi^+\pi^-$</td>
<td>$\sqrt{2} \sin \theta \cos \theta G_{\pi\pi}(M^2 - m^2)$</td>
</tr>
</tbody>
</table>

$\mathcal{H}^2 = G_{\pi\pi}(I_\pi I_\pi I_\pi) + h.c.$

$\mathcal{H}^2 = 2G_{pv}a(2x^2 - M^2 + 2m^2)$

$\mathcal{H}^2 = G_{\pi\pi}(I_\pi I_\pi I_\pi) + h.c.$

$\mathcal{H}^2 = (3/\sqrt{2}) G_{\pi\pi\pi}(M^2 - m^2)$

$\mathcal{H}^2 = (1/\sqrt{2}) G_{\pi\pi\pi}(M^2 - m^2)$

$\mathcal{H}^2 = G_{\pi\pi\pi}(M^2 - m^2)$

$\mathcal{H}^2 = (3/\sqrt{2}) G_{\pi\pi\pi}(M^2 - m^2)$

Obtained amplitudes for $K \to 3\pi$ decays are quartic functions of meson four momenta. In Table II, we have used the approximation $E \approx (Q + 4m)E/2 - (Q + 2m)m/2$ where $E$ is the odd pion energy, $M$ the kaon mass, $m$ is the pion mass and $Q = M - 3m$. 

Fig. 1. The electromagnetic interaction of meson caused by the primary Hamiltonian (a) $\mathcal{H}^1 = \sum_{i=1}^{4} \epsilon_{\alpha}(I_{\pi}(x)\tau_{\pi}(x)) A^\alpha(x)$

or (b) $\mathcal{H}^2 = \sum_{i=1}^{4} \epsilon_{\alpha}(I_{\pi}(x)\tau_{\pi}(x)) \times (I_{\pi}(x)\tau_{\pi}(x)) A^\alpha(x)$. 

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Table II. $K \to \pi\pi$ decay amplitudes. $x=b(0)/a(0)$ and $G_{pe} = \zeta g_{pe}/4$ are defined.

<table>
<thead>
<tr>
<th>primary Hamiltonian</th>
<th>process</th>
<th>$c(ijk)$</th>
<th>$a(ijk)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\mathcal{H}^1 = \sin \theta \cos \theta G_0 (f_3 v, f_3) (f_1 v^* p t_2)$</td>
<td>$+++$</td>
<td>$\sqrt{2} \sin \theta \cos \theta G_0 \frac{4M^2}{3} \left{ \frac{1}{x^3} - \frac{m(M-2m)}{x^3} \right}$</td>
<td>$Q \frac{-7x^3 - 2m(M+m)}{2M} x^3 - m(M-2m)$</td>
</tr>
<tr>
<td>+ h.c.</td>
<td>$+00$</td>
<td>$\sqrt{2} \sin \theta \cos \theta G_0 \frac{4M^2}{3} \left{ \frac{1}{x^3} - \frac{m(M-2m)}{x^3} \right}$</td>
<td>$Q \frac{-4x^3 + m(M+m)}{2M} x^3 - m(M-2m)$</td>
</tr>
<tr>
<td></td>
<td>$+0$</td>
<td>$\sin \theta \cos \theta G_0 \frac{4M^2}{3} \left{ \frac{1}{x^3} - \frac{m(M-2m)}{x^3} \right}$</td>
<td>$Q \frac{8x^3 + m(M+m)}{2M} x^3 - m(M-2m)$</td>
</tr>
<tr>
<td></td>
<td>$000$</td>
<td>$\sqrt{2} \sin \theta \cos \theta G_0 \frac{4M^2}{3} \left{ \frac{1}{x^3} - \frac{m(M-2m)}{x^3} \right}$</td>
<td>0</td>
</tr>
<tr>
<td>$\mathcal{H}^2 = g_{pe} (f_3 v p t_2) + h.c.$</td>
<td>$+++$</td>
<td>$\sqrt{2} G_{pe} \left{ 6x^2 + (M^2 + 3m^2) - \frac{2M^2 m(M-2m)}{x^3} \right}$</td>
<td>$Q \frac{-4M^2 {4x^2 + m(M+m)}}{2M} 6x^4 + (M^2 + 3m^2) x^2 - 2M^2 m(M-2m)$</td>
</tr>
<tr>
<td>$\mathcal{H}^3 = G_{YY} (f_{3v} f_{3}^* p t_2) + h.c.$</td>
<td>$++-$</td>
<td>$\sqrt{2} G_{YY} \left{ \frac{2x^2 - (5m^2 - 3m^2)}{3} + \frac{4M^2 m(M-2m)}{3x^3} \right}$</td>
<td>$Q \frac{-5M^2 {2x^2 - m(M+m)}}{2M} 6x^4 - (5m^2 - 3m^2) x^2 + 4M^2 m(M-2m)$</td>
</tr>
<tr>
<td>$G_{AA} (f_{3v} f_{3}^* p t_2) + h.c.$</td>
<td>$+++$</td>
<td>$\sqrt{2} G_{AA} \left{ \frac{2x^2 - (M^2 - 3m^2)}{3} \right}$</td>
<td>$Q \frac{-M^2 {14x^2 + 3m(M+m)}}{2M} 6x^4 - (M^2 - 3m^2) x^2$</td>
</tr>
<tr>
<td>$G_{PV} (f_{3v} p t_2) + h.c.$</td>
<td>$++-$</td>
<td>$\sqrt{2} G_{PV} \left{ \frac{4M^2}{3} \left{ \frac{1}{x^3} - \frac{m(M-2m)}{x^3} \right} \right}$</td>
<td>$Q \frac{-x^2 + 2m(M+m)}{2M} x^2 - m(M-2m)$</td>
</tr>
</tbody>
</table>
and $a(ijk)$ is the slope parameter.

Here, we review in brief the isospin kinematics$^{10}$ of $K \rightarrow 3\pi$. If the $3\pi$ final state in $K$-decays is pure $I=1$, it leads to the following predictions for $c(ijk)$ and $a(ijk)$:

$$
\frac{c(+-)}{c(00)} = 2, \quad \frac{c(+0)}{c(00)} = \frac{\sqrt{2}}{3}, \quad \text{and} \quad \frac{a(+-)}{a(00)} = -\frac{1}{2}.
$$

(3.2)

The predictions of the $|\Delta I|=1/2$ rule are

$$
\frac{c(+-)}{\sqrt{2}c(+0)} = \frac{\sqrt{2}c(+0)}{c(-0)} = \frac{\sqrt{2}}{3}, \quad \frac{c(00)}{c(+0)} = 1
$$

(3.3)

and

$$
\frac{-2a(+-)}{a(+0)} = \frac{a(+0)}{-2a(+0)} = 1.
$$

(3.4)

Finally, we will concentrate our attentions on the present experiments,$^{11}$ which are shown in Table III.

Table III. Experiments of decay parameters for $K \rightarrow 2\pi$ and $3\pi$ decays.

<table>
<thead>
<tr>
<th>Experiments$^{11}$</th>
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<tbody>
<tr>
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<tr>
<td>$</td>
</tr>
<tr>
<td>$\frac{1}{\sqrt{2}}</td>
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<tr>
<td>$\sqrt{2}</td>
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<tr>
<td>$</td>
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<tr>
<td>$\frac{\sqrt{2}}{3}</td>
</tr>
<tr>
<td>$a(+-)$</td>
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<tr>
<td>$a(+0)$</td>
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<tr>
<td>$a(+0)$</td>
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<tr>
<td>$a(+-)$</td>
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<tr>
<td>$a(+0)$</td>
</tr>
</tbody>
</table>

§ 4. Sub-hadronic interaction of nonleptonic decays

By comparing our results in Tables I and II with experiments in Table III, we try to determine the "primary" Hamiltonian at the baryon level from which the nonleptonic decays are caused at the hadron level.

Since the parity violating and conserving parts of the "primary" Hamiltonian contribute to $K \rightarrow 2\pi$ and $K \rightarrow 3\pi$ decays, respectively, the coupling strengths of
the parity violating and conserving parts of the "primary" Hamiltonian are determined from the different experiments. Here, let us assume that the coupling strength of the parity violating part of the "primary" Hamiltonian is nearly equal to that of the parity conserving part. If we take the standpoint of the composite model of urbaryons where the urbaryons are made of leptons and the primary Hamiltonian at the urbaryon level originates in the pure leptonic weak interaction $H_w = G_{
u l} l^* l$, this assumption may be reasonable. (We discuss in Appendix a case of the Hiroshima model\(^{14}\) as an example.)

First, if the "primary" Hamiltonian of nonleptonic decays is given by $\mathcal{H}^I$ which is expected from the semi-leptonic decays, we find that (i) for $K \rightarrow 2\pi$ the pure $|\Delta I| = 3/2$ process (i.e., $K^+ \rightarrow \pi^+ \pi^0$) can be caused with strength about 20 times greater than the observed value, and the $|\Delta I| = 1/2$ processes are, on the contrary, caused only partially about $1/20$\(^{14}\) (ii) for $K \rightarrow 3\pi$ the slope parameter $a(++)$ cannot be made consistent with the experimental value, irrespectively of the parameter $x^2 = (b(0)/a(0))^2$, as shown in Fig. 2(a) and (iii) although $\mathcal{H}^I$ leads to relations (3.3) for $c(ijk)$, it leads to a sum rule

$$4a(++) + 3a(00) + 5a(-0) = 0 \quad (4.1)$$

for the slope parameters, which is quite inconsistent with (3.4).

Secondly, if the "primary" Hamiltonian of nonleptonic decays is given by $\mathcal{H}^I$, we cannot find $x^2$ which guarantees the condition $g_{\nu l} = g_{\nu e}$ and gives the experimental value of $a(++)$. (See Fig. 2(b).)

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**Fig. 2.** (Figure captions are printed on the next page below.)
In the case of $\mathcal{A}^\Pi$, we assume $f_A/f_V = -1$. (See Appendix.) Under this condition, we analyze the cases a) $(V-A)V$, b) $(V-A)A$ and c) $(V-A)(V-A)$.

a) Case of $(V-A)V$
As shown in Fig. 2(c), we can find $x^2=1.00$ so as to get
\[ G_{\tau\tau} = G_{\alpha\tau} = 0.74G_0 \times (1 \pm 0.01) \]  
(4.2)
and
\[ a(++) = 0.48. \]  
(4.3)

b) Case of $(V-A)A$
As shown in Fig. 2(d), we have always $a(++) < 0$, irrespectively of $x^2$.

c) Case of $(V-A)(V-A)$
As shown in Fig. 2(e), we can find $x^2$, at which the decay widths of $K\to 2\pi$ and $K\to 3\pi$ can be reproduced by a common value of $G_{r-A}$. Unfortunately, this value of $x^2$ leads to $a(++)>1$ and $a(+-)<0$.

Considering the present experimental situation, we should not be too strict for our results to agree well with the slope parameter $a(++)$, although the sign and order of $a(++)$ might be expected to be explained. Thus we shall put a criterion as $0<a(++)<1$. From the discussions stated above, we find that only the case of $(V-A)V$ can get over this criterion.

In order to investigate $(V-A)V$ interaction in more detail, we study the following "primary" Hamiltonian:
\[ \mathcal{H}_{NL} = G(t^\ast \tau_3(1-i\tau_3)t_3) \]
\[ (\bar{t}^\ast(1+\lambda i\tau_3)t) + \text{h.c.}, \]  
(4.4)
whose results are shown in Fig. 3 as functions of $\lambda$ in cases of $x^2=1.00$ and 1.06. From Fig. 3, we find that if $x^2=1.00$, we obtain
\[ G = \frac{1}{1.5}G_0 \times (1 \pm 0.01) \]  
(4.5)
and
\[ a(++) = 0.16 \]  
(4.6)
at $\lambda=0.34$. This result is sensitive to $x^2$. For example, if $x^2=1.06$, we obtain
\[ G = \frac{1}{1.35}G_0 \times (1 \pm 0.001) \]  
(4.7)
and
\[ a(++) = 0.51 \]  
(4.8)
Thus it is probable that we can obtain desirable solution at least $x^2 = 1$. This condition implies $a(0) = b(0) = 1$ in unit of pion mass. This result may have connection with the so-called universality of the weak interactions in pion mass unit pointed out in Ref. 6). From the above discussions, we may conclude that the "primary" Hamiltonian of nonleptonic decays is $(V - A)V$ type.

§ 5. Conclusions and remarks

In summary, we comment on our results. (i) We have proposed the calculation method which enables us uniquely to obtain all amplitudes which can be caused from the "primary" Hamiltonian at the urbbaryon level. The "universality of the weak interactions" is understood in our theory by the fact, $a(0) = b(0) = 1$ in unit of pion mass. It is to be noticed that the universality of weak interactions has been discussed by setting up the effective Hamiltonian except for leptonic decays. (ii) By considering Figs. 2 and 3, we find that the expected "primary" Hamiltonian is $(V - A)V$ type with $G = G_0 / \sqrt{2}$. As for this interaction, a possible speculation is given in Appendix.

Next we discuss some problems which have appeared when we obtain the results stated above. (a) Our calculation method is based on an assumption such as an interaction concentrated upon one point. Although this method is conventionally introduced in order to simplify the calculations of composite system, the fact that we get $a(0) = b(0) = 1$ in pion mass unit and that $a(0)$ and $b(0)$ may be considered as constants throughout all the reactions implies that our method is not conventional but may have connection with something of sub-hadronic matter. As for this point, it will become clear if we apply our method to the higher spin mesons. (b) Instead of our spectator $[\zeta (\vec{t})]$, Ishida had introduced $\exp\{\zeta (\vec{t})\}$ (which he called "connector") from a viewpoint different from ours. If we adopt his "connector" instead of our spectator, we have amplitudes for $K^\pm$ and $K \to 3\pi$ (except for $K^\mu$) whose magnitude is equal to one half of those of the amplitudes we have got hitherto. Therefore, $G/G_0$ in Fig. 3 must be two times. Then we can find by considering the universality of weak interactions, that the expected "primary" Hamiltonian of nonleptonic decays is

$$\mathcal{H}_{NL} = G (\bar{t} i \gamma_\mu (1 - i \gamma_5) t_\nu) (\bar{t} i \gamma_\nu (1 - i \gamma_5) t_\mu) + \text{h.c.},$$

where

$$G \approx G_0.$$

Even then, it should be noticed that there exists a solution only at $x^2 = 1.00$. At $x^2 = 1.00$ and $\lambda = -1$, we get from Fig. 3

$$a(+ + -) = 1.26,$$

which is about ten times greater than experiments.
We can see that the values of the internal wave functions at the origin, \(a(0)\) and \(b(0)\), correspond to parameters \(a\) and \(b\), respectively, which have been introduced by the present authors,\(^{17}\) who have proposed the following "translation rules": 
\[
(\tilde{t}\tilde{t}\tilde{t}) \Rightarrow a\partial_s P^i_j, \quad (\tilde{t}\tilde{t}\tilde{t}\tilde{t}) \Rightarrow ibP^j_i \quad \text{and} \quad (\tilde{t}\tilde{t}\tilde{t}) \Rightarrow 0 \quad \text{for} \quad \Gamma \neq \gamma_\rho_i\gamma_\sigma_j \text{and} \gamma_\sigma_j.
\]

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Appendix

Let us discuss roughly that if urbaryons are composite particles which contain leptons, then what sub-hadronic interactions result from the primary interactions at the lepton level. Here, we give discussions on the basis of the Hiroshima model\(^{16}\) and deal with weak interactions.

In the Hiroshima model, urbaryons are considered as the three-body composite system composed of lepton, antilepton and \(B\) particle. We assume, for convenience, that the antilepton and the \(B\) particle construct \(J^P=0^+\) state, which is denoted by \(\Phi(\tilde{B})\). If there is a lepton interaction given by a matrix element \((\tilde{I}'\tilde{I})\), then the sub-hadronic interaction as illustrated in Fig. A1 will be as follows:

\[
(\tilde{I}'\tilde{I}) \Rightarrow (\tilde{I}'\tilde{I}) (\bar{\psi}\nu) (\tilde{B}B) = (\tilde{I}'\tilde{I}) (\nu\tilde{B})_{\alpha\beta} (B\bar{\nu})_{\beta\alpha} \\
\Rightarrow \tilde{I}'\tilde{I} \text{Tr}(\tilde{\Phi}(\nu\tilde{B})\Phi(\nu\tilde{B})) t. \quad (A\cdot1)
\]

In order to give electromagnetic and semileptonic weak interactions suitably, we put

\[
\text{Tr}(\tilde{\Phi}(\nu\tilde{B})\Phi(\nu\tilde{B})) = 1. \quad (A\cdot2)
\]

Now, let us deal with the interaction as illustrated in Fig. A2 which causes nonleptonic decays. On the basis of our assumption, the primary interaction of nonleptonic decays is expected to be given by

\[
(\bar{e}\nu_\mu) (\bar{\nu}_\mu\nu_\mu) \Rightarrow (\bar{e}\nu_\mu) (\tilde{B}B) (\bar{\nu}_\mu\nu_\mu) \\
\Rightarrow \tilde{t}\nu_\mu\Phi(\nu\tilde{B}) \Phi(\nu\tilde{B}) \nu_\mu t_{\mu}. \quad (A\cdot3)
\]

Putting

\[
\tilde{\Phi}(\nu\tilde{B}) \Phi(\nu\tilde{B}) = A + B, \quad (A\cdot4)
\]
where the matrices $A$ and $B$ consist of only the terms of odd and even numbers of $\gamma$-matrices, respectively, we have

$$v_\rho \Theta (\bar{v}_e B) (\bar{v}_\mu B) v^\rho = v_\rho A v^\rho = A' (1 - i \gamma_5), \quad (A-5)$$

where the matrix $A'$ consists of only the terms of odd number of $\gamma$-matrices. Therefore, it is not expected that such a two-body transition has the interaction form as

$$\tilde{t}_1 (g_{\rho e} 1 + g_{\rho \mu} \gamma_5 t_5) + h.c. \quad (A-6)$$

Next, let us consider that the sub-hadronic interaction of nonleptonic decays is given by Fig. A3, where we assume vector and/or axial-vector bosons which interacts only with $B$ particle. Then, the sub-hadronic interaction will be given by

$$(\bar{v}_e \nu_e) (\bar{v}_\mu \nu_\mu) (\bar{B} O_e B) (\bar{B} O_\mu B) = (\bar{v}_e \nu_e) (\bar{B} O_e B) (\bar{v}_\mu \nu_\mu) (\bar{B} O_\mu B)$$

$$\Rightarrow (i \bar{v}_e \nu_e) (\bar{B} O_e B) (\bar{B} O_\mu B) (i \bar{t}_1 O^* t_5) \sim (i \bar{t}_1 O^* (1 - i \gamma_5) t_5) (i \bar{t}_1 O^* t_5). \quad (A-7)$$

Conversely, if the sub-hadronic interaction of nonleptonic decays has the form (A-7), it seems that this interaction suggests the existence of $SU(3)$ singlet-vector boson and/or axial-vector boson which possibly interacts as constructive force between urbaryons. In fact, we have found that the expected primary Hamiltonian is $(V - A)V$ type. Therefore, it is reasonable that there is not any particle to be composed of $t_1$ and $t_5$ or $\bar{t}_1$ and $\bar{t}_5$ whereas there exist mesons to be composed of $\bar{t}_1$ and $t_5$.

Although our discussions are based on the particular dynamical model, it may be expected that our conclusions are general ones.

**References**

3) For example, A. Murayama, Prog. Theor. Phys. **39** (1968), 1546.
G. Zweig, CERN preprint.
Nonleptonic Decays and Sub-Hadronic Interaction. I

   Peking Group, preprint (The 1966 Summer Physics Colloquium of the Peking Symposium).
8) Particle Data Group, Rev. Mod. Phys. 43 (1971), Part II.
10) For example, R. H. Dalitz, Rev. Mod. Phys. 31 (1959), 823.
    T. Hayashi and M. Nakagawa, Prog. Theor. Phys. 35 (1965), 515.