Inequalities in Neutral K-Meson Decay

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In this Letter, we present restrictions on \((\text{Im} A)^2\) and CP-violating parameter \(u\) in the general frame of Wu and Yang's phenomenological analysis.

In order to derive these restrictions, we make use of Schwarz's inequality which is of the form

\[
\left[ \sum_n (\text{Re} a_n \text{Im} a_n)^2 \right] \leq \left[ \sum_n (\text{Re} a_n)^2 \right] \times \left[ \sum_n (\text{Im} a_n)^2 \right].
\]  

(1)

In (1) and the following relations, we use the notations \(a_n\) as amplitude for \(K \to n\) and \(M_n = m_n - (i/2) \gamma n\) for \(J = S\) or \(L\). \(\text{Re} a_n^*\) and \(\text{Im} a_n^*\) in (1) are calculated from the unitarity relation

\[
\sum_n a_n^* a_n = -iu(M_L^* - M_S).
\]  

(2)

and relations

\[
\sum_n (a_n^* a_n)^2 = \frac{u^2}{1-u^2} |M_S - M_L|,
\]

\[
\sum (a_n^* a_n)^2 = \frac{1}{1-u^2} \frac{|M_S - M_L|}{|M_S - M_L|} \times \{(M_S^* - M_S) + u^2(M_S^* - M_L^*)\},
\]

\[
\sum (a_n^* a_n)^2 = \frac{1}{1-u^2} \frac{|M_S - M_L|}{|M_S - M_L|} \times \{(M_L^* - M_L) + u^2(M_S^* - M_L)\}.
\]  

(3)

Relations in (3) are derived by the same procedures as in Ref. 1. If we want to get the relations in superweak theory, we must add the condition \(t/su = 2(\Delta m/\Delta \gamma)\) to (2) and (3), where \(\Delta m = m_S - m_L\) and \(\Delta \gamma = \gamma_S - \gamma_L\). \(u\) in (2) and (3) must be taken to be 0 in the case of CP-invariance. But in the following discussions, we are concerned only with the case of CPT-invariance.
Case I: If we apply (1) to the case in which the summations are taken over all possible channels, we get a restriction on $u^2$ for both $J = S$ and $L$,

$$
\frac{4\gamma L_s \gamma L_s}{(\gamma S + \gamma L_s)^2 + 4(m_S - m_L)^2} \geq u^2.
$$

(4)

Case II: If we apply (1) to the case in which the summations are taken over only 2π channel, we have inequalities

$$
A + \frac{|t|\sqrt{B}}{4(1-u^2)} \geq (\text{Im} A_2) \geq \frac{A - |t|\sqrt{B}}{4s^2}.
$$

(5)

for $J = S$ and

$$
A + \frac{|t|\sqrt{B}}{4s^2} \geq (\text{Im} A_2) \geq \frac{A - |t|\sqrt{B}}{4s^2}.
$$

(6)

for $J = L$, where

$$
A = (1-u^2)(\gamma S + \gamma L)^2 - s(\gamma S - \gamma L)^2,
$$

$$
B = 4\gamma S \gamma L - u^2(\gamma S + \gamma L)^2.
$$

(7)

Further we have a restriction on $u^2$ as follows for both $J = S$ and $L$,

$$
\frac{4\gamma S \gamma L}{(\gamma S + \gamma L)^2 + 4(m_S - m_L)^2} \geq u^2.
$$

(8)

Case III: If we apply (1) to the case in which the summations are taken over all possible channels but for 2π channel, we have inequalities for (Im $A_2$) in the same forms as in (5) and (6) for $J = S$ and $L$, respectively, except for the substitutions of

$$
A = -2ut(Am) + (1-u^2)
$$

$$
\times (\gamma S^2 + \gamma L^2) - s(\gamma S^2 - \gamma L^2),
$$

$$
B = 4\gamma S \gamma L - u^2(\gamma S^2 + \gamma L^2),
$$

$$
-\gamma S \gamma L - (\gamma S - \gamma L)^2. \text{(9)}
$$

Also the inequality (8) in case II should be replaced by

$$
\frac{4\gamma S \gamma L}{(\gamma S - \gamma S^2) + (\gamma L - \gamma L^2)^2} \geq u^2.
$$

(10)

In deriving inequalities (4)−(10), we have used the relations

$$
\gamma S = [A_5 + (\text{Re} A_5)^2 + (\text{Im} A_5)^2] + [A_5 + (\text{Re} A_5)^2 - (\text{Im} A_5)^2]s + 2t[\text{Re} A_5 \cdot \text{Im} A_5],
$$

$$
\gamma L = [A_5 + (\text{Re} A_5)^2 + (\text{Im} A_5)^2] - [A_5 + (\text{Re} A_5)^2 - (\text{Im} A_5)^2]s - 2t[\text{Re} A_5 \cdot \text{Im} A_5].
$$

(11)

If we substitute the experimental data into (4) and (8), we have $5.97 \times 10^{-4} \geq |u|$ and $4.08 \times 10^{-3} \geq |u|$, respectively. This is consistent with $u_{\text{exp}} \approx 2 \cdot \text{Re} \epsilon_{\text{exp}} = + (3.20 \pm 0.34) \times 10^{-3}$ which comes from the charge asymmetry, $\delta_L$, in semileptonic decay of $K_L$ meson. On (Im $A_2$), the inequality (5) is approximately equivalent to (6) if we assume that both $|u|$ and $|t|$ are of orders $10^{-3}$, respectively. Also we have the following restrictions on (Im $A_2$) from (5) and (6) by assuming $u = 3.20 \times 10^{-3}$; $1.35 \geq |\text{Im} A_2| \times 10^{-2} \geq 0.58, 2.11 \geq |\text{Im} A_2| \times 10^{-2} \geq 0.17, 2.87 \geq |\text{Im} A_2| \times 10^{-2} \geq 0.93$ for $|t| \times 10^3 = 1, 3, 5$, respectively. It is sure that the numerical results here contain an error by virtue of the uncertainty of experimental data. In the above discussions, we do not deal with case III, because we do not know the value of $\gamma S - \gamma L_{S^2}$ at the present stage of experiment.