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Discussion

On the Maximum Value of the Maximum Degree of Kinematic Chains

Lung-Wen Tsai[1]. The authors are to be congratulated for the derivation of the equations for the maximum value of the maximum degree of kinematic chains.

The discusser, however, would like to point out that the same results can also be derived by applying the well-known Euler's formula which states that $L=e-v+2$ for planar graphs, where L denotes the number of faces or loops, including the peripheral face, e denotes the number of edges and v denotes the number of vertices. Since the degree of a vertex in a planar graph equals the number of loops passing through that vertex, the maximum possible degree of a vertex occurs when all the loops pass through that vertex. Hence,

$$d=e-v+2 \tag{1}$$

or, in terms of kinematic chains, we have

$$M=J-N+2 \tag{2}$$

where d denotes the maximum degree of a vertex, M the maximum number of joints on a link, J the number of simple joints, and N the number of links in a kinematic chain. This condition was pointed out earlier by Buchsbaum and Freudenstein in 1970 [1].

However, we should also keep in mind that a vertex can have at most $(v-1)$ incident edges in a graph with v vertices. For kinematic chains, this implies that the value of M in equation (2) should never exceed $N-1$.

Substituting $M=N-1$ into equation (2), we obtain $J=2N-3$. Hence, the following conditions should be imposed:

$$M=N-1, \qquad \text{for } J>2N-3 \tag{3}$$

Thus, we have arrived at the same conditions derived by Yan and Harary.

Furthermore, if we limit ourself to those joints with up to two degrees of freedom for planar mechanisms, and with up to three degrees of freedom for spatial mechanisms, then for all mechanisms with positive degree-of-freedom [2],

$$J \le 2N-3 \tag{4}$$

Hence, for practical purposes, only equation (2) is sufficient.

The following special cases are worth mentioning:

(1) For one degree-of-freedom planar linkages with binary joints, the degree-of-freedom equation reduces to:

$$J=3N/2-2 \tag{5}$$

Substituting equation (5) into (2) yields

$$M=N/2 \tag{6}$$

This is in complete agreement with the result given by Rosenauer and Willis (listed as reference 5 on the paper).

(2) For one-degree-of-freedom epicyclic-gear trains, the number of turning-pair edges equals the number of links minus one, and the number of geared edges equals the number of turning-pair edges minus one. Therefore, the degree-of-freedom equation reduces to

$$J=2N-3 \tag{7}$$

Substituting equation (7) into (2) yields

$$M=N-1 \tag{8}$$

Hence, it is always possible to construct an N-link, one degree-of-freedom epicyclic-gear train with the maximum permissible number of joints on a link.

It is a pleasure to commend the authors for their fine work.

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Authors' Closure

The authors would like to thank Professor L. W. Tsai for his discussion.

It is clear that the expression for the maximum value of the maximum degree of planar kinematic chains with revolute pairs and with positive number of degrees of freedom can be derived based on Euler's formula as pointed out by Professor Tsai. This fact was also derived early by Paul [5] and Mayourian [6] as indicated in the introduction of the paper.

In our approach, we identified that a "block" in graph theory is corresponding to a "kinematic chain" in kinematics. Here, a kinematic chain refers to a generalized kinematic chain which is connected, closed, without any cut link, and with simple generalized joints only. Based on the concept of block, we derived the generalized mathematical expressions for the maximum value of the maximum degree of generalized kinematic chains. We hope this approach can stimulate more studies on the application of the theory of blocks to kinematic number synthesis.

Discussion

An Application of Dual Graphs to the Automatic Generation of the Kinematic Structures of Mechanisms, Vol. 108, pp. 392–398 (86-DET-1) by W. J. Sohn and F. Freudenstein

Jerry T. Pugh[1]. The authors introduce the idea of using

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