ADVANCED MODELING OF MIXED POPULATIONS OF HETEROtROPHS AND NITRIFIERS CONSIDERING THE FORMATION AND EXCHANGE OF SOLUBLE MICROBIAL PRODUCTS

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ABSTRACT

Biological process modeling is advanced by explicitly describing heterotroph and nitrifier biomass, incorporating formation of soluble microbial products (SMP) from both bacterial groups, and allowing degradation of SMP by the heterotrophs. Biomass decay now has two parts, endogenous respiration and formation of biomass-associated products (BAP). The model is applied to investigate interactions between heterotrophs and nitrifiers. Main attention is directed to evaluating the role that SMP produced by nitrifiers plays as a supply of organic substrate to heterotrophs and to predicting the COD concentration in the effluent. The model quantitatively describes the observed accumulation of SMP in the effluent at long SRT and at high influent substrate concentration. The significance of SMP from nitrifiers to support growth of heterotrophs is clearly elucidated through the model experiments under various operational conditions. The results indicated that a high NH₄⁺-N/COD ratio in the influent would decrease original substrate COD due to increased heterotrophs whose growth is supported by SMP from nitrifiers, but total COD increases. The minimum substrate concentration, Smin, is reduced for heterotrophs by the additional growth from SMP.

KEYWORDS

Biological treatment; multispecies model; microbial products; mixed culture; nitrifiers; substrate exchange.

INTRODUCTION

Soluble microbial products (SMP) have proved to be major components of the organic matter in effluents from biological treatment processes (e.g., Grady and Williams, 1975; Chudoba, 1967). SMP accumulation is inevitable in biological processes, because its formation is associated with microbial growth and the cell maintenance, while its biodegradability is not high (Rittmann et al., 1987). Besides contributing to effluent BOD and COD in all biological processes, SMP are especially key factors for wastewater reuse and drinking-water preparation, because they serve as possible precursors to trihalomethanes and other disinfection byproducts, competitively adsorbed to activated carbon, and create biological instability in distribution systems (Rittmann and Huck, 1989).

Two important bacterial groups work in aerobic biological processes, heterotrophs and nitrifiers. The interactions between them are becoming of much greater interest, because nitrification is an increasingly important reaction in the biological treatment of wastewater and drinking water. The competitive interaction of heterotrophs and nitrifiers for dissolved oxygen is well known. However they also interact through the exchange of organic matter. Being autotrophs, nitrifiers reduce inorganic carbon to form organic carbon in cell mass and SMP. Thus the autotrophs have the potential to supply energy and carbon for heterotrophs through their formation of SMP. Indeed, heterotrophic contamination often is found in cultures of autotrophs in which no organic-carbon substrate is supplied (Kuenen and Gottschal, 1982).

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The purpose of this paper is to quantify the interactions between heterotrophs and nitrifiers by developing a model for a multisubstrate, multispecies system in which heterotrophs and nitrifiers coexist and in which SMP are formed and degraded. The SMP aspects are based upon the experimental work of Namkung and Rittmann (1986) and advancements of Rittmann et al. (1987), both of which considered only SMP from heterotrophs. A similar model also was developed by Boero et al. (1991). In this study, we focus on the role that SMP formation by nitrifiers plays in the interactions between coexisting heterotrophs and nitrifiers. Therefore, the specific objectives of this research are:

1) to advance biological-process modeling by including the effects of SMP formation by heterotrophs and nitrifiers and SMP biodegradation by heterotrophs,
2) to investigate the effects of SMP formation and degradation on the steady-state effluent quality under various operational conditions, and
3) to apply the model to a system having a low influent COD concentration in order to evaluate heterotrophic growth supported by SMP from nitrifiers.

MODEL DEVELOPMENT

SMP Formation and Degradation

The classic study of Leudeking and Piret (1959) proposed growth-associated and nongrowth-associated product formation. The two types of products appear in our model as utilization-associated products (UAP) and biomass associated products (BAP) (Namkung and Rittmann, 1986). The sum of UAP and BAP is SMP. The UAP and BAP formation kinetics used by Rittmann et al. (1987) are employed here for heterotrophs and nitrifiers. However, a key new feature is added: BAP formation leads to loss of biomass by its solubilization. The new loss term for BAP formation can be combined with decay by endogenous respiration to form an overall biomass loss rate (b').

For SMP biodegradation by the heterotrophs, multiple-substrate degradation kinetics are incorporated, following their introduction for steady-state processes by Rittmann et al. (1987) and successful use to describe SMP concentration changes in a nonsteady-state experiment (Chang and Rittmann, 1989).

Mathematical Expressions for a Dispersed-Growth Process

The dispersed-growth model contains eleven mass balance equations: three for original substrates (organic COD, ammonium, and nitrite), dissolved oxygen, nitrate, two types for SMP (actual SMP and SMP originally formed), and four types of biomass (heterotrophs, ammonium oxidizers, nitrite oxidizers, and inert biomass). The mass balance equations are given in Table 1.

Key features and assumptions of the model are summarized as follows.

1. The multiplicative double-Monod expression is used to describe the effects of oxygen and electron-donor concentrations.
2. Heterotrophs and nitrifiers produce SMP according to rate expressions R₁ and R₂, which are taken from Rittmann et al. (1987).
3. Only the heterotrophs degrade SMP, and they do so according to the multiple-substrate expression, R₃, which was proposed by Rittmann et al. (1987). SMP degradation gives heterotrophic cell synthesis with a yield of Yp.
4. BAP, which emanates directly from cell mass, contains organic nitrogen in the same proportion as biomass, i.e., 14 mg N per 113 mg cells (CsH7O2N).
5. Heterotrophs use ammonium and the organic nitrogen in BAP as their nitrogen source for synthesis as long as these materials are available.
6. The degradation rates of UAP and BAP are proportional to the overall SMP degradation rate according to the ratios of the originally formed UAP and BAP to originally formed SMP.
7. The "decay" loss of active biomass has two parts: decay by endogenous oxidation (b) and decay to form BAP ((1/f₉) k₂).
8. Ammonium is released from the endogenous oxidation of biomass.
9. Biomass wasting can be treated separately from biomass loss in the effluent.
10. Biomass decay results in generation of inert biomass that is not metabolically active and that does not decay further. The fraction (1 - f₉) of the active biomass can become inert through decay.
TABLE 1. Mass Balance Equations

<table>
<thead>
<tr>
<th>Component</th>
<th>Equation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Biomass</td>
<td>V = Q0X0 + Q5X5 - Q6X6 + Q7X7 - YhR1XhV + YpR1XhV + b1XhV + (1/y1)R1hXhV</td>
</tr>
<tr>
<td>Substrates</td>
<td>V = Q0X0 + Q5X5 - Q6X6 + YnR1XnV - b1XnV + (1/y1)R1nXnV</td>
</tr>
</tbody>
</table>

Substrates and Products

1. Substrate COD (S)
   dS = Q0(S0 - S) - MhXhV |

2. NH4+-N (N1)
   dN1 = Q0(N10 - N1) - MnsXnsV + MnsXnsV + YnYhXhV + YnYpR1XhV + YnYpR3XhV + (1 - Ys)R1nXnV |

3. NO2--N (N2)
   dN2 = Q0(N20 - N2) - MnsXnsV + (1 - YnYns)MnsXnsV |

4. NO3--N (N3)
   dN3 = Q0(N30 - N3) + (1 - YnYns)MnsXnsV |

5. O2 (O)
   dO = Q0(O0 - O) + KLa(Oa - O) + (1 - YhYh)R1hXhV + (1 - YpYp)R1pXpV + b1XhV + b1XpV + (1 - Ys)R1nXnV |

6. Soluble microbial Products (P)
   dP = - Q0P + (R1i + R2j)XjV - R3XhV |

7. Total of originally formed SMP (P0)
   dP0 = - Q0P0 + (R1i + R2j)XjV |

Where:
- M: Specific substrate consumption rate = qm [Sub]/(Km + [Sub])/(Km + [O2]), here [Sub] = S, N1 or N2
- R1: Specific UAP formation rate = k1 M
- R2: Specific BAP formation rate = k2
- R3: Specific SMP consumption rate = k3m(P/P0)

Determination of Parameter Values

Model simulations require the input of parameter values. We used the kinetic and stoichiometric parameters shown in Table 2. Most of these parameters are taken directly from the literature (e.g., Lawrence and McCarty, 1970; Knowles et al., 1965; Rittmann and Snoeyink, 1984). However, few SMP parameters are available for heterotrophs (Rittmann et al., 1987; Chang and Rittmann, 1989), and no values are available for nitrifiers. Using a radiochemical technique, Namkung and Rittmann (1986) estimated UAP and BAP rate constants for a biofilm process, but their values are not directly applicable, because no SMP degradation was considered in their model.

For the heterotrophs, we selected reasonable values of the UAP formation coefficient (k1), BAP formation coefficient (k2), and SMP degradation coefficient (k3m): 0.2 mgCODp mgCODS-1 for k1 and 0.1 mgCODp mgCODcell-1 day-1 for k2 values are similar to those previously published (Rittmann et al., 1987; Chang and Rittmann, 1989), while k3m, 1.0 mgCODp mgCODcell-1 day-1, is between 5 and 10% of the maximum specific substrate consumption rate, qm.
With respect to nitrifiers, $k_2$ was set equal to $k_2$ for heterotrophs, since BAP formation is an endogenous loss of cell mass and need not depend on the electron donor that grew the cell mass. On the contrary, UAP can be formed through substrate utilization in two ways. One is the energy yielding oxidation of the organic electron donor, and the other is cell synthesis. No quantitative information has been reported to evaluate which reaction is more related to UAP formation for heterotrophs. The $k_1$ value of 0.2 mgCODp mgCODs$^{-1}$ for heterotrophs means that 80% of electron flow goes for energy generation and cell synthesis, while 20% goes to UAP formation. The ratio is then 0.25 electrons of UAP per electron to energy plus cells. For nitrifiers, UAP is formed mainly through cell synthesis, because the electron-donor oxidation involves nitrogen and not carbon.

We selected provisional $k_1$ values for nitrifiers of 0.11 and 0.03 mgCODp mgN$^{-1}$ for ammonium and nitrite oxidizers, respectively. These values are 25% of the growth yields and indicated that 25% of the electrons flowing to synthesis of cellular carbon are lost as UAP in nitrifiers.

**TABLE 2 Values of Kinetic and Stoichiometric Parameters**

<table>
<thead>
<tr>
<th></th>
<th>NH$_4$ oxidizers</th>
<th>NO$_2$ oxidizers</th>
<th>Heterotrophs</th>
</tr>
</thead>
<tbody>
<tr>
<td>$q_m$ [mg mgCOD$_{cell}^{-1}$ day$^{-1}$]</td>
<td>1.6</td>
<td>7.0</td>
<td>15.0</td>
</tr>
<tr>
<td>$K_S$ [mg/l]</td>
<td>1.0</td>
<td>1.0</td>
<td>10.0</td>
</tr>
<tr>
<td>$K_O_2$ [mgO$_2$/l]</td>
<td>0.5</td>
<td>0.5</td>
<td>0.1</td>
</tr>
<tr>
<td>$b$ [day$^{-1}$]</td>
<td>0.05</td>
<td>0.05</td>
<td>0.10</td>
</tr>
<tr>
<td>$Y$ [mgCOD$_{cell}$ mg$^{-1}$]</td>
<td>0.44</td>
<td>0.12</td>
<td>0.5</td>
</tr>
<tr>
<td>$f_d$ [-]</td>
<td>0.8</td>
<td>0.8</td>
<td>0.8</td>
</tr>
<tr>
<td>$k_1$ [mgCODp mg$^{-1}$]</td>
<td>0.11</td>
<td>0.03</td>
<td>0.2</td>
</tr>
<tr>
<td>$k_2$ [mgCODp mg$^{-1}$ day$^{-1}$]</td>
<td>0.1</td>
<td>0.1</td>
<td>0.1</td>
</tr>
<tr>
<td>$k_3$ [mgCODp mg$^{-1}$ day$^{-1}$]</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>$Y_p$ [mgCOD$_{cell}$ mgCODp$^{-1}$]</td>
<td>-</td>
<td>-</td>
<td>0.5</td>
</tr>
</tbody>
</table>

Unit of substrate: mgCOD for heterotrophs, mgN for nitrifiers

**RESULTS AND DISCUSSIONS**

**Effluent Quality Changes with SRT**

Steady-state analyses were carried out to evaluate the validity of model with respect to conventional parameters (eg., effluent substrate and mixed liquor biomass) and to investigate the effects of solids retention time (SRT) on species interactions for a typical wastewater-treatment application. In all cases reported here, the dissolved oxygen concentration was high enough to preclude any oxygen limitation. In addition, the influent contained organic substrate (COD$_S$) and ammonium nitrogen (NH$_4^+$-N) as electron donors but no biomass or SMP.

Figure 1 shows the predicted changes of COD$_S$ and total COD in the effluent and mixed liquor heterotrophs ($X_h$) concentration when SRT is increased. Total COD (indicated as COD) means the sum of COD$_S$ and the COD from SMP. Whereas COD$_S$ continues to decline for increasing SRT, due to increased $X_h$, the total soluble COD in the effluent remains relatively constant between 15 and 20 mg/l for SRT >1 day, because SMP dominates the effluent soluble COD. The prediction describes well the generally observed tendency to accumulate COD that has low biodegradability with increasing SRT in effluent from biological wastewater treatment.

Figure 2, which presents the corresponding predictions for nitrification, shows two drops of NH$_4^+$-N concentration and a peak of NO$_2^-$-N. The first drop corresponds to NH$_4^+$-N consumption for synthesis of heterotrophs at low SRT. The second drop in NH$_4^+$-N occurs when nitrification starts (around 2 days), which also coincides with nitrite build-up. Once nitrifiers are well maintained in the process, NH$_4^+$-N and NO$_2^-$-N concentrations are governed by nitrification, which is nearly complete for SRT>4 days.

Figures 3 presents the steady-state COD mass balance. Once heterotrophic activity is fully established (SRT>0.3 days), most of the entering COD$_S$ leaves as biomass or is consumed by electron transfer to oxygen (O$_2$), producing CO$_2$ and H$_2$O. The transfer to O$_2$ increases with increasing SRT, due to increasing endogenous respiration. For very large SRT's, effluent SMP becomes almost as important as wasted biomass as a COD sink. The total outflow of COD is greater than the influent COD$_S$ (=200mg/l) for SRT>2 days, where nitrification begins, because some fraction of electrons in NH$_4^+$-N and NO$_2^-$-N are converted to organic cell materials and SMP. This increase in the COD flow illustrates one aspect of the interaction between nitrifiers and heterotrophs: the increase to the COD flow in carbon from CO$_2$ reduction by nitrification.
Figure 4, the nitrogen balance, illustrates that, for SRTs too low to allow nitrification, the nitrogen exists in the system exclusively as NH₄⁺-N and biomass nitrogen. Once nitrification is established, NO₃⁻-N is the major sink for nitrogen. The soluble organic nitrogen (SON) in the effluent increased with increasing SRT, due to more formation of BAP. Parkin and McCarty (1981) experimentally measured SON (0.26 to 0.63 mg/l) in the effluent for a SRT of 6 days and a volumetric load of 1.0 gCOD l⁻¹ day⁻¹ for an activated sludge process. The predicted SON of 0.6 mg/l at the same SRT was in the reported range. This agreement indicates that the given k₂ value and the assumption for the nitrogen content of BAP were reasonable.

Figure 5 shows that, for this typical wastewater example, heterotrophs generate much more SMP than nitrifiers and that BAP became more important for longer SRTs. SMP formation from nitrification depends on the available amounts of NH₄⁺-N and NO₂⁻-N. If influent NH₄⁺-N were higher, larger amounts of SMP would be formed through nitrification.
Effects of Influent NH\textsubscript{4}\textsuperscript{+}-N/COD\textsubscript{S} on Effluent Quality

Figure 6 shows that SMP from nitrification becomes more important when the ratio of influent NH\textsubscript{4}\textsuperscript{+}-N to COD\textsubscript{S} is increased by decreasing the COD\textsubscript{S}. When influent COD\textsubscript{S} is reduced from 200 to 20 mgCOD/l, originally formed SMP by nitrification is almost equivalent to half of influent COD\textsubscript{S}. Thus, SMP formation through nitrification has a more important role in carbon or carbonaceous COD flow at higher ratios of NH\textsubscript{4}\textsuperscript{+}-N/COD\textsubscript{S} in the influent.

To more thoroughly investigate the interactions when the NH\textsubscript{4}\textsuperscript{+}-N/COD\textsubscript{S} ratio is relatively high, we use an application having a low influent COD\textsubscript{S} concentration, such as drinking-water treatment or wastewater reuse. The parameter values in Table 2 are again used, but the influent concentration of COD\textsubscript{S} and NH\textsubscript{4}\textsuperscript{+}-N are set to much lower values. The concentrations range from 0.01 to 10 mg/l. For the predictions shown here, SRT is set to 100 days, a long value representative of biofilm processes used in these low-concentration applications. The $S_{\text{min}}$ value ($=K_s b'/(Yqm-b')$) of nitrifiers is 0.27 mg/l; hence, no nitrifiers exist for an influent NH\textsubscript{4}\textsuperscript{+}-N of 0.1 mg/l.

![Graph showing ratio of originally formed SMP from nitrifiers to influent COD\textsubscript{S}](image)

Fig. 6. Ratio of originally formed SMP from nitrifiers to influent COD\textsubscript{S}

- UAP
- BAP

Figure 7 shows how changes in influent COD\textsubscript{S} affect the effluent COD\textsubscript{S} under various conditions of influent NH\textsubscript{4}\textsuperscript{+}-N concentration. The first trend is that the effluent COD\textsubscript{S} increases continuously and approaches a fixed value of 0.20 mgCOD/l as influent COD\textsubscript{S} increases, as long as nitrification occurs. This value, 0.20 mgCOD/l, corresponds to the effluent COD concentration for SRT=100 days when heterotrophic biomass is grown only through utilization of COD\textsubscript{S} and its own SMP. A second trend is that effluent COD\textsubscript{S} decreases as the influent NH\textsubscript{4}\textsuperscript{+}-N increases. The trends are rooted in the same cause: an increase in the influent ratio of NH\textsubscript{4}\textsuperscript{+}-N to COD\textsubscript{S} means heterotrophic biomass is being more and more supported by degradation of SMP from nitrifiers. Figure 8 proves this cause: heterotrophic biomass increases with increasing influent COD and NH\textsubscript{4}\textsuperscript{+}-N. More input of NH\textsubscript{4}\textsuperscript{+}-N brings about more nitrification, which forms more SMP available for growth of heterotrophs. When nitrifiers are absent, COD\textsubscript{S} increases to the fixed value and remains there until all the nitrogen for synthesis is consumed, at around COD\textsubscript{S}^0=7 mg/l; then, heterotrophs are nitrogen-limited, and effluent COD\textsubscript{S} rises.
The top lines in Fig. 9 show the changes in total effluent COD, while the solid areas represent COD$_S$. They demonstrate that the total effluent COD increases when influent COD$_S$ or NH$_4^+$-N increases and that the major fraction of COD is SMP at high influent NH$_4^+$-N. The effect is created by SMP formation, which increases when either substrate has an increased concentration.

Fig. 7. Changes of effluent COD$_S$ with increasing influent COD$_S$ concentration (SRT=100 days, HRT=0.25 days)

Fig. 8. Changes of biomass concentration with increasing influent COD concentration (SRT=100 days, HRT=0.25 days)
Fig. 9. Changes of COD and COD$_S$ in the effluent with increasing influent COD$_S$ concentration (SRT=100 days, HRT=0.25 days)

The effects of influent NH$_4^+$-N/COD$_S$ ratio also are investigated by continuously changing influent NH$_4^+$-N at a constant influent COD$_S$ of 2 mgCOD/l. Figure 10 shows the changes in COD$_S$ and COD in the effluent and the heterotrophs concentrations. The growth of heterotrophs is limited by the availability of a nitrogen source at very low influent NH$_4^+$-N. In this region, increasing influent NH$_4$-N gives more heterotrophic growth and results in lower COD$_S$ in the effluent. Then, the COD$_S$ and COD in the effluent remain constant until nitrifiers exist in the process (at about 0.3 mgN/l). COD$_S$ decreases, but COD increases when nitrifiers are active in the process.

The trends of decreasing COD$_S$ and increasing COD with increasing influent NH$_4^+$-N are caused by enhanced heterotrophic growth supported by SMP from nitrifiers. Thus, nitrification significantly affects the effluent quality in terms of COD, as well as nitrogen, when the influent COD$_S$ is low.

Fig. 10. Changes of COD, COD$_S$, and $X_h$ with increasing influent NH$_4^+$-N concentration (COD$_S^0$=2mgCOD/l, SRT=100 days, HRT=0.25 days)
Role of SMP from Nitrifiers in Lowering $S_{\text{min}}$ of Heterotrophs

The concept of $S_{\text{min}}$, the minimum substrate concentration to support a steady-state biomass (Rittmann, 1987) is useful for describing the loss of heterotrophs at low influent COD$_S$ concentration. Traditionally, $S_{\text{min}}$ is defined as

$$S_{\text{min}} = K_s \frac{b}{Y q m - b}. \quad (1)$$

Since BAP formation is an additional biomass loss in the model here, $b$ should be replaced by $b' = b + (1/Y_s)k_2$ to estimate $S_{\text{min}}$. The model developed here also includes, for heterotrophs, an additional growth term $[Y_p k_3 m(P/P^0)X_h]$ through SMP utilizations. In that case, $b$ becomes $b + (1/Y_s)k_2 - Y_p k_3 m(P/P^0)$. Combining biomass loss from BAP production and biomass growth from SMP utilization gives this new equation for $S_{\text{min}},$

$$S_{\text{min}} = K_s \left[ b + \left( \frac{1}{Y_s} \right) k_2 - Y_p k_3 m(P/P^0) \right] Y q m - b - \left( \frac{1}{Y_s} \right) k_2 + Y_p k_3 m(P/P^0) \right]$$

When nitrifiers produce SMP that can be utilized by heterotrophs, $S_{\text{min}}$ decreases, although the amount of the decrease depends on $(P/P^0)$. In the limit, $(P/P^0)$ goes to unity, and $S_{\text{min}}$ is expressed by the following equations:

If $b + (1/Y_s)k_2 > Y_p k_3 m$, $S_{\text{min}} = K_s \left[ b + \left( \frac{1}{Y_s} \right) k_2 - Y_p k_3 m \right] Y q m + Y_p k_3 m - b - (1/Y_s)k_2$$ \quad (3a)$

If $b + (1/Y_s)k_2 \leq Y_p k_3 m$, $S_{\text{min}} = 0 \quad (3b)$

When $Y_p k_3 m$ is greater than $b + (1/Y_s)k_2$, heterotrophs are sustained only by SMP formed by nitrifiers. This phenomenon was demonstrated already for COD$_S^0 = 0$ mg/l in Fig. 8. When SMP from nitrifiers is not available (i.e. NH$_4^+$/N$^0$ = 0.1 mg/l), heterotrophs do not grow at all. However, at NH$_4^+$/N$^0$ of 2 and 5 mg/l, heterotrophs exist even when COD$_S^0$ = 0 mg/l, because they grow by utilizing SMP from nitrifiers. Thus, SMP formation by nitrifiers reduces $S_{\text{min}}$ for heterotrophs, and heterotrophic activity can exist with concentrations lower than regular $S_{\text{min}}$ value. In the limit, heterotrophic activity exists with no input COD$_S$.

CONCLUSIONS

Activated sludge modeling is advanced by including heterotrophic biomass, nitrifying biomass, SMP production by both types of biomass, and SMP degradation by heterotrophs. The model is very useful to evaluate the interactions between heterotrophs and nitrifiers from the aspect of SMP from nitrifiers. The model is able to predict effluent qualities properly and evaluate effects of SMP formation on them. At high SRT, effluent COD remains almost constant, because the major fraction is SMP. The effects of SMP from nitrifiers depends on the influent NH$_4^+$/N$^0$/COD$_S$ ratio, and they became more significant in controlling the carbonaceous COD flow at higher ratios.

SMP formed by nitrifiers promotes heterotrophic growth, which then utilize more original organic substrates and produces more of its own SMP. In addition, supply of the external SMP reduces the minimum substrate concentration ($S_{\text{min}}$) for heterotrophs. Heterotrophs can grow with very low influent organic substrate concentration, if the SMP is available from nitrifiers. However, SMP formation causes higher COD in effluent for higher concentration of influent COD$_S$ and NH$_4^+$/N$^0$.

ACKNOWLEDGEMENT

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NOTATIONS

- $b$ : Decay rate constant [day$^{-1}$]
- $b'$ : Overall decay rate constant = $b + (1/Y_s)k_2$ [day$^{-1}$]
- COD : COD concentration = COD$_S$ + COD$_P$ [mgCOD/l]
- COD$_P$ : COD concentration of SMP [mgCOD/l]
- COD$_S$ : COD concentration of organic substrate [mgCOD/l]
- $f_d$ : Biodegradable fraction of active biomass [-]
\[ k_1 : \ \text{UAP formation rate constant [mgCOD_p mgCOD_s^{-1}]} \]
\[ k_2 : \ \text{BAP formation rate constant [mgCOD_p mgCOD_{cell}^{-1}\text{ day}^{-1}]} \]
\[ k_{3m} : \ \text{Multiple substrate degradation rate constant [mgCOD_p mgCOD_{cell}^{-1}\text{ day}^{-1}]} \]
\[ K_s : \ \text{Saturation constant for substrate (electron donor) [mg/l]} \]
\[ K_o : \ \text{Saturation constant for oxygen (electron acceptor) [mgO}_2/l]} \]
\[ K_{L,a} : \ \text{Overall oxygen transfer coefficient [day}^{-1}]} \]
\[ O : \ \text{Dissolved oxygen [mgO}_2/l]} \]
\[ O_s : \ \text{Saturation concentration of oxygen [mgO}_2/l]} \]
\[ P : \ \text{SMP concentration [mgCOD_p/l]} \]
\[ P_0 : \ \text{Originally formed SMP concentration [mgCOD_p/l]} \]
\[ q_m : \ \text{Maximum specific substrate consumption rate [mg mgCOD_{cell}^{-1}\text{ day}^{-1}]} \]
\[ Q : \ \text{Flow rate [l day}^{-1}]} \]
\[ S : \ \text{Substrate concentration [mg/l]} \]
\[ SON : \ \text{Soluble organic nitrogen [mgN/l]} \]
\[ V : \ \text{Reactor volume [l]} \]
\[ X : \ \text{Biomass concentration [mgCOD/l]} \]
\[ Y : \ \text{Growth yield [mgCOD_{cell} mg^{-1}]} \]
\[ Y_p : \ \text{Growth yield associated with SMP degradation [mgCOD_{cell} mgCOD_p^{-1}]} \]
\[ \beta : \ \text{Oxygen demand for substrate consumption [mgCOD mg}^{-1}]} \]
\[ \gamma_s : \ \text{Conversion coefficient of biomass concentrations to COD [mgCOD_{cell} mgCOD^{-1}]} \]
\[ \gamma_n : \ \text{Nitrogen content in biomass [mgN mgCOD_{cell}^{-1}]} \]

Subscript:
- \( h \): Heterotrophs or heterotrophic oxidation
- \( n \): Nitrifiers
- \( ns \): NH₄ oxidizers or NH₄ oxidation
- \( nb \): NO₂ oxidizers or NO₂ oxidation
- \( e \): Steady-state condition

Superscript:
- \( 0 \): Influent flow, except for \( P \)
- \( e \): Effluent flow
- \( w \): Wasted flow

REFERENCES


