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Transition to Meandering Rivulet Flow in Vertical Parallel-Plate Channels

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While there have been many studies of gravity-driven rivulet flow over an external surface, treatment of internal rivulets has been sparse. A notable exception is the analytical and experimental study by Anand and Bejan [1]; they derived an approximate solution for a laminar rivulet developing between two parallel vertical planes and employed it to examine the transition to a meandering flow.

For rivulets flowing between closely spaced plates, the velocity becomes ''slow'' (i.e., ''low'' Reynolds number) and an approximately fully-developed flow at its terminal velocity can be reached in a short distance. Conceptually, this value can be derived from the analysis of Anand and Bejan. However, for cases where the inlet volumetric flow rate Q is sufficiently high that the rivulet becomes wide relative to the plate spacing, a simple treatment can describe the resulting flow.

For this treatment, we orient the coordinates as $x=$vertical, $y=$spanwise and $z=$normal to the vertical plates and assume:

- an incompressible fluid with constant properties
- steady, laminar flow (except at the leading edge outside the region of interest)
- fluid density much greater than density of its surroundings (so $p_s\{x\}\approx$constant)
- negligible interface shear with surroundings at edges ($\rightarrow \partial u/\partial y\approx 0$)
- fully-developed flow away from the inlet
- rivulet width in spanwise direction (W) considerably larger than edge region formed by curvature of free surfaces at the edges

Under these assumptions and approximations, the one-dimensional governing equations reduce to:

$$\text{continuity:}\quad \dot{m}=\rho Q\approx\rho V_b(Ws)=\text{constant}$$

$$x\text{-momentum:}\quad \rho g s=2g_c\tau_W$$

when applied to a horizontal element. If one starts with a wide rivulet at a velocity lower than terminal as Anand and Bejan did, the frictional force is less than the body force so the flow accelerates under gravity, making the cross sectional area less until the balance of these two terms suppresses the advective term (not shown). For steady laminar flow between parallel plates [2], the development length is of the order of $(x_L/s)\approx 0.02\,\text{Re}$ so the lower the Reynolds number Re is, the shorter the distance required for the following treatment to become reasonable.

For a fully developed laminar flow between parallel plates, the wall shear stress may be non-dimensionalized as $f=24/\text{Re}$ with the Reynolds number based on the hydraulic diameter of twice the plate spacing (e.g., as by Schade and McEligot [2], Kays and Crawford [3] and many others). Substitution and rearrangements lead to:

$$\text{bulk velocity,}\quad V_b\approx gs^2/(12\nu)$$

and

$$\text{Reynolds number,}\quad \text{Re}=(\rho V_b D_h/\mu)\approx gs^3/(6\nu^2)$$

depending on the plate spacing and fluid properties and independent of the total flow rate. The rivulet width then is determined by the volumetric flow rate provided at the inlet as

$$W\approx 12Q\nu/(gs^3)\quad\text{or}\quad (Wgs^3/(Q\nu))\approx 12$$

Provided that the assumptions and approximations above are approached so that this treatment is valid, the results offer the possibilities of employing this configuration as a viscometer or flowmeter by measuring the appropriate quantities. For such applications the sensitivity to plate spacing makes its accurate measurement important. Alternatively, the measurement of the rivulet width, flow rate and viscosity can be used to estimate the plate spacing and, as a first approximation, its variation or the degree to which the plates are parallel.

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Nomenclature

D_h = hydraulic diameter, $2s$
f = Fanning friction factor, $2g_c\tau_W/(\rho V_b^2)$
g = acceleration of gravity
g_c = units conversion factor, e.g., 1 kg m/(N s^2)
$\dot{m}$ = mass flow rate
p_s = static pressure of surroundings (typically air)
Q = volumetric flow rate
Re = Reynolds number, $\rho V_b D_h/\mu$
s = spacing between plates
u = vertical velocity component
V_b = bulk or mixed mean vertical velocity
W = rivulet width
x = vertical coordinate
y = spanwise coordinate
z = normal coordinate
μ = absolute viscosity
ν = kinematic viscosity, μ/ρ
ρ = fluid density
τ_W = wall shear stress

References

[1] Anand, A., and Bejan, A., 1986, ‘‘Transition to Meandering Rivulet Flow in Vertical Parallel-Plate Channels,’’ ASME J. Fluids Eng., **108**, pp. 269–272.

[2] Schade, K. W., and McEligot, D. M., 1971, ‘‘Cartesian Graetz Problems with Air Property Variation,’’ Int. J. Heat Mass Transfer, **14**, pp. 653–666.

[3] Kays, W. M., and Crawford, M. E., 1980, *Convective heat and mass transfer, 2nd edition*, McGraw-Hill, New York.