Nonsingular Tensor Force in Pseudoscalar Meson Theory.

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Several authors have pointed out that the so-called $r^{-3}$-singularity in the non-relativistic nuclear potential derived from the pseudoscalar meson theory is caused by the inadequacy of the way to derive the formula. We have obtained a similar conclusion, but our underlying physical consideration is somewhat different from the preceding ones.

We should remind ourselves that a $1/2$-spin particle at rest performs Schroedinger's Zitterbewegung, the amplitude of trembling being a half Compton wave length. The phase factor $\exp \left( i \frac{\mathbf{K} \cdot (\mathbf{X} - \mathbf{X}')}{\hbar} \right)$ appearing in course of calculation describes the effect of the retardation of nuclear field generated by the meson with momentum $\mathbf{K}$ which is emitted by the nucleon $(2)$ at the point $\mathbf{X}$ and absorbed by $(1)$ at the point $\mathbf{X}'$, and it is this phase factor which yields the spatial dependence of the nuclear potential. Let $(1)$, $(2)$ $\mathbf{X}, \mathbf{X}'$ denote the coordinate shifts due to the Zitterbewegung of nucleon $(1)$, $(2)$ respectively. Then for smaller separation of two nucleons, we should take $\mathbf{X}'$ into account and the phase factor will be given by

$$e^{i \left( \mathbf{K} \cdot (\mathbf{X} - \mathbf{X}' + \mathbf{X}' - \mathbf{X}) \right)} = e^{i \left( \mathbf{K} \cdot \mathbf{X} - \mathbf{X}' \right)} e^{i \left( \mathbf{K} \cdot \mathbf{X}' - \mathbf{X} \right)}$$

in first approximation. By virtue of the last two factors the singularity might reduced.

We have calculated the nuclear potential term in the Hamiltonian of two nucleons plus pseudoscalar meson field along the line stated above. (We have employed the pseudoscalar coupling exclusively.) The calculation runs in an analogous way to that in the Pauli-Fierz's solution of the electromagnetic interaction. But we have used here the "physical representation" of the Dirac particles' operators; and consequently the mathematical formulation of the physical idea was much facilitated. The result for the second order of coupling constant is approximately as follows. (The term accompanied with a nucleon pair creation was discarded. $M$: nucleon mass. $\mu$: meson mass. For simplicity the neutral meson was assumed.) If $r > \frac{\hbar}{MC}$,

$$f^2 \frac{e^{-\mu r}}{r} \left( + \left( \sigma \cdot \sigma \right) \left( \frac{1}{\mu^2 r^2} + \frac{1}{\mu r} - 3 \frac{\sigma \cdot r}{r^3} \right) \right)$$

and when $r$ becomes smaller than the nucleon Compton wave length, the form of potential gets changes, and we have for very small $r$,

$$f^2 \left( + \left( \sigma \cdot \sigma \right) \frac{1}{4r} - \left( \sigma \cdot r \right) \left( \frac{1}{r^2} \right) \frac{1}{4r} \left( 1 + 2e^{-\mu r} \right) \right)$$.  

At the beginning of calculation, we have adopted two approximations, the nonrelativistic
approximation and the neglection of all recoil effects: that is, more concretely, (i) the nucleon energy operator is taken as \( (\frac{M}{2} + \frac{p^2}{2M}) \rho \); (ii) the change of kinetic energy after the virtual meson emission is rejected; (iii) the Zitterbewegung is assumed to be equal to that in case of \( p=0 \), i.e., \( \tilde{\chi} = \frac{\hbar}{2M} \rho \sigma \); (iv) the effect due to the coordinate shift (which compensates the decrement of the transversal spin by increasing the orbital angular momentum in order to conserve the total momentum) is neglected throughout.

Fuller account of these points will appear shortly.

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Fuller account of the physical representation, including the cases of spin 1 and 0 and photon, will be published shortly in this journal.