A Note on the C-Meson Hypothesis

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Several years ago, Sakata and Pais proposed independently a device which is effective in making finite the second order self-energy of an electron calculated on the basis of the positron theory. They assume that electrons are necessarily surrounded by a neutral scalar meson field—the C-meson field—besides the ordinary electromagnetic field. It is found that when the coupling constant \( f \) between C-meson field and the electron field satisfies the condition

\[
      f^2 = 2e^3
\]  

(1)

the second order self-energy of an electron becomes finite, and thus the scalar field has some feature of cohesive force acting on the charge of the electron.

In spite of the difficulties brought about by the introduction of a new particle (the C-meson) associated with this field, various attempts have since made to justify this hypothesis both theoretically and experimentally. This method of mixing fields has been extended to analyze the difficulties of the present quantum theory of fields, especially the problem of vacuum polarization. On the other hand, it has also been remarked frequently that the C-meson itself can hardly be observed in the present experiments if it exists at all. Therefore it may not be useless to ask once again whether or not the C-meson hypothesis is a satisfactory one from the theoretical viewpoint. The fourth order self-energy of an electron, for example, may thus give a crucial test for the validity of this hypothesis with which we shall be concerned in this note.

Diagrams corresponding to the fourth order self-energy of an electron due to the interaction with the surrounding electromagnetic field and the C-meson field are drawn immediately as follows:

These are second order corrections to the graph of the second order self-energy. Here the dotted line represents either a virtual photon or a virtual C-meson and we must of course take account of all possible diagrams of orders \( e^4, e^2f^2 \), and \( f^4 \). Matrix elements associated with them are evaluated by the usual method described by Feynman and Dyson. (It is convenient to make use of the relativistic cut-off of Feynman in actual computations.)

Direct calculation of the matrix representing the graph \( a \) gives rise to the contribution \( \delta x_{4\alpha} \) to the fourth order self-energy of an electron;

\[
   \delta x_{4\alpha} = -\frac{1}{2} \frac{1}{2x} (\partial x_{\nu})^2 + B \delta x_{\alpha}
\]  

(2)

where the first term is linearly divergent for free electrons and does disappear if the self-mass is amalgamated with the mechanical mass. \( B \) is a logarithmically divergent term of the charge renormalization type. \( \delta x_{\nu} = \delta x_{\nu\nu} + \delta x_{\nu\nu'} \) is the self-mass of an electron in the second order which is known to be finite on employing the condition (1). \( \delta x_{\nu\nu} \) and
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\[ \delta x_s = A \delta x_i + B \delta x_i + C \delta x_i \]

where \( A \) diverges logarithmically and is removed by the mass renormalization. \( C \) is finite. There arises thus no difficulty in diagrams \( a \) and \( b \) if mass and charge are renormalized to the second order. The necessity of mass renormalization in the case of the \( C \)-meson theory was once stressed by the present author\(^1\).

The third graph \( c \) shows rather different features from the above ones. It has the form

\[ \delta x_s = B' \delta x_i + B'' \delta x_i + D \delta x_i \]

where \( B', B'' \), and \( D \) are logarithmically divergent. The charge-renormalization factors in (4) does not cancel those of (2) and (3) contrary to the case where the electromagnetic field only is taken into account. This is related to the circumstance that in our case the meaning of the word “correction to a vertex” is not unique at all as in the usual electrodynamics.

After the leading terms are removed by the mass renormalization (of the \( C \)-meson field alone) and the charge renormalizations (of both kinds) the modified \( D_p \)-function gives rise to a logarithmically divergent term belonging to the graph \( d \). This term arises due to the polarization of the electronic vacuum and therefore it may not be cunous even if this is divergent in the \( C \)-meson theory.

Collecting the above results, we find that the condition (1) can not insure the convergence of the fourth order self-energy and thus success of the \( C \)-meson hypothesis in the second order approximation is no more than an accident. The reason for the existence of the \( C \)-meson is thus very much weakened, though one can not exclude the possibility that it still describes a sort of particle which really exists in nature. It must however highly be appreciated that the \( C \)-meson theory has been instrumental in analyzing the present complicated features of the theories of elementary particles.

This consideration strongly suggests that various theories of mixing fields, especially those which have been developed relating to the divergences of vacuum polarization, are to be scrutinized by similar methods as sketched here. The regularization procedure of Pauli and Villars may not be rejected\(^6\), since it concerns only with a single type of fields contrary to the “realistic” theories in which mixtures of various types of fields are considered. It is desirable to make these circumstances clearer from the general point of view.

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F. J. Dyson, Phys. Rev. 75 (1940), 486, 1736.
6) W. Pauli and F. Villars, Rev. Mod. Phys. 21 (1949), 484.