Since long ago, it has been obscure whether the second maximum of Rossi curve really exists or not. If the maximum exists, it is very important to explain what kind of showers gives rise to such a maximum. Bothe and others ascertained this phenomenon by their experiment and attempted to explain it as due to hard or knock-on showers. However, Janossy maintained it from his experiment as a spurious effect, while, on the other hand, there exists some experiments which contradict with his result. Although the experiments and the interpretation of the above authors are partly convincing, they seem to be not free from such ambiguity and inconsistency that we are forced to take up this problem on the ground of the later development of cosmic ray physics.

Recent experiment carried out by Kameda and Miura seems to establish the evidence of the existance of the second maximum, and they inferred that this maximum is caused by the nucleonic component on the absorption law of agent rays and of the initial increase of the shower frequencies in lead and paraffin. The similar result was also obtained by Clay, but he attributed it to knock-on showers. His interpretation may, however, not be accepted because of the following reasons:

1) Primary rays show the characteristic feature of nucleons as already seen.
2) According to the theoretical and experimental reasons, the saturation of knock-on showers should take place in the much smaller thickness of the absorber of the second maximum.
3) The frequency of the second maximum can not be explained consistently by knock-on hypothesis for both narrow and wide zenith angle of the agent rays, as discussed in later section (§ 4).
4) The absorption coefficient of meson is so small that the knock-on shower can not produce such a sharp maximum.

In this paper, first we describe briefly the experiment of Kameda and Miura (§ 2), and present the further argument for their nucleonic hypothesis (§ 3). Then we show that this assumption gives right order of the frequency of the shower and inquire some conditions to give rise to the appreciable maximum (§ 4). We also discuss some feature of the secondary rays referring to the shape of
the transition curve and study whether or not this feature favours other experimental results (§5). But our interpretation contain some ambiguity and may not be conclusive (§6).

§2. Summary of experimental results

In this section, we give an outline of experimental results obtained by Kameda and Miura. The apparatus is shown in Fig. 1, where 1, 2, 3, 4, 5, 7, 8, are used for coincidences and 6 for anti-coincidence.

The role of the trays 1, 2 and 6 makes essentially their experiment prefer to others. The anti-coincidence (12–6) selects only vertical primary rays and can avoid some ambiguity caused by local and air showers.

We denote the frequency of the various types of showers by the following abbreviation.

NS, (1234—6) : Narrow angle showers produced in \( \Sigma_1 \) without \( \Sigma_0 \).

NPS, (12347—6) : NS accompanied by the discharge of at least two counters in tray 7. This was measured for \( \Sigma_2 = 10 \text{cm Pb} \).

A, (1234—6) : absorption curve of shower producing rays, measured for \( \Sigma_1 = 10 \text{cm Pb} \) and 17 cm Pb. Hereafter, we denote them by \( A_0 \) and \( A_\Pi \).

The experimental results of NS and NPS are shown in Fig. 2, where the contribution of knock-on showers is estimated by Kameda and Miura making use of the data of Brown et al.

§3. Feature of primary rays

Kameda and Miura have already pointed out that the second maximum is caused by nucleonic component on the following two bases.

(1) Subtracting the contribution of knock-on showers, the shape of curve A in Figure 2 can be represented by \( \varepsilon^{-\lambda z} \), where \( 1/\lambda \sim 15 \text{cm Pb} \). This figure of collision mean path is equal to that of nucleons obtained by Cocconi et al.

Although this interpretation, of course, can not be free from ambiguity caused by the estimation of the knock-on level in Fig. 2, this absorption coefficient will not be far from reality, since both \( A_0 \) and \( A_\Pi \) are converging to the estimated level.

(2) The initial increase of NPS is far steeper in paraffin than in lead, and
its material dependence is about $A^{-1/3}$. This means that NPS is caused by nucleons.

These evidences seem to support strongly the nucleonic hypothesis, but the relation between the second maximum and NPS is not clear at once. To make this relation clear, we classify NS (only for the case $\Sigma_i \geq 10$ cm Pb) into two types, as seen in Fig. 3.

We first separate the shower producing material into two parts, i.e. $\tau (=10$ cm) and $\sigma$ cm, and denote the shower generated in each part by $\tau'$ and $\sigma'$ types respectively, where $\sigma'$ type, of course, does not involve the shower which can not penetrate through $\tau$.

According to these definitions, NS ($\Sigma_i \geq 10$ cm Pb) consists of $\sigma' (\sigma = \Sigma_i - \tau)$ and $\tau' (\tau = 10$ cm Pb) types, while NPS ($\Sigma_i = \sigma$, $\Sigma_2 = \tau = 10$ cm Pb) contains only $\sigma'$ type and $A_{10} (\Sigma_1 = \sigma$, $\Sigma_2 = \tau = 10$ cm Pb) contains only $\tau'$ type.

These relation can be shown by the following schemes:

\[
\begin{align*}
\text{NS} &= \sigma' + \tau', \\
\text{NPS} &= \sigma', \\
A_{10} &= \tau'.
\end{align*}
\]
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and then

$$\text{NPS}(\Sigma_1=\sigma) = \text{NS}(\Sigma_1-\tau=\sigma)-A_{20}(\Sigma_0=\sigma).$$

This equality must be held exactly, provided we ignore the detection probability of counter trays. NS—A and NPS can be obtained from the data in Fig. 2 and they are compared in Fig. 4.

The agreement can be said as fairly good, considering the difference of geometrical conditions. The relation between NPS and NS is now evident, the second maximum of the latter corresponding to the first maximum of the former. Then if we want to know the character of the second maximum, we have only to study the character of the first maximum of NPS.

§ 4. Frequencies of showers

The frequency of the radiation producing the second maximum is about 1 $h^{-1}$ in our case, and this can also be explained consistently by nucleonic primaries. As already pointed out, Clay's interpretation is not appropriate in this case.

Putting the directional intensity of primaries as $j_\perp \cos^\delta \theta$, the intensity of incoming rays in the circular cone with vertical angle $\delta$ is represented by

$$j(\delta) = 2\pi \int_0^\delta d\theta j_\perp \cos \theta \sin \theta = \frac{2\pi j_\perp}{n+1} (1 - \cos^{n+1} \delta). \quad (1)$$

As $\delta$ is sufficiently small in our case, (1) can be reduced to

$$j(\delta) = \pi j_\perp \delta, \quad (2)$$
and it follows
\[ \frac{j_N(\theta)}{j_E(\theta)} = \frac{j_{1N}}{j_{1E}}, \quad \frac{j_N(\theta)}{j_H(\theta)} = \frac{j_{1N}}{j_{1H}}. \]
The values of right hand sides are given by Rossi as
\[ j_{1N}/j_{1E} \sim 1/50, \quad j_{1N}/j_{1H} \sim 1/200. \] (4)

We put here \( j_{1L} \cos^{\theta} \) for nucleonic, hard, or electronic component as \( j_{1N} \cos^{\theta}, \)
\( j_{1H} \cos^{\theta}, \) or \( j_{1E} \cos^{\theta} \) and, their momentum ranges of these components are taken following Rossi:

- **nucleonic component**: \( \geq 10^8 \text{eV/c}, \)
- **hard**, 
  \( \geq 3 \times 10^8 \text{eV/c}, \)
- **electronic**, 
  \( \geq 10^7 \text{eV/c}. \)

From (3), (4) and the total intensity* (coincidence of counters 1, 2) obtained by Kameda and Miura, we can estimate the intensity of each component as follows.

\[ j_N \sim 6.10^2 j_{1N}/(j_{1N}+j_{1H}) \sim 3h^{-1}, \] (5)
\[ j_E \sim 6.10^2 j_{1E}/(j_{1N}+j_{1H}) \sim 120h^{-1}, \] (6)
\[ j_H \sim 6.10^2 j_{1H}/(j_{1N}+j_{1H}) \sim 480h^{-1}. \] (7)

From (5), it is plausible to assume that only nucleons with momenta larger than \( 2 \text{BeV} \) contribute to the second maximum, because the actual frequency of the maximum is about \( 1h^{-1}. \) For the first maximum it appears at the thickness of about \( 2.5 \text{cm Pb}. \) The electrons capable to contribute to it, therefore, must have energy larger than \( 20E_i, \) where \( E_i = 7 \text{MeV} \) is the critical energy for lead.

In order to estimate the intensity of the electrons with energy larger than \( 140 \text{MeV}, \) we tentatively take the integral spectrum of electron \( F(E) \) as
\[ F(E) \propto E^{-1} \text{ for } E < 50 \text{MeV}, \]
\[ \propto E^{-2} \text{ for } E > 50 \text{MeV}, \] (8)
then
\[ j_\text{cascade} = F(140 \text{MeV}) \sim 5h^{-1}. \] (9)

This roughly agrees with the frequency of the first maximum.

Next, we discuss a difficulty of Clay's interpretation. From his experimental results, the intensity of the rays producing the second maximum is about \( 10^{-4} \) times of that of the incoming hard component. If the maximum were caused by knock-on showers, this ratio would have to be constant for both narrow and wide angle primaries, but it is about \( 10^{-2} \) in our case, so that his presumption can not be accepted.

* This amounts about \( 6 \times 10^2 h^{-1}. \)
Our interpretation, however, is also valid in his experiment, as shown in what follows. As his apparatus selects wide angle primaries, the ratio of the hard to the nucleonic component can be estimated by referring to the relation (1) and (4) as

\[
\frac{\text{Hard component}}{\text{Nucleonic component}} = \frac{j_{N'}}{j_{N}}/3 = 6 \times 10^2, \tag{10}
\]

and if we take only nucleons with momenta larger than 2BeV/c, this ratio becomes 2 \cdot 10^3. Considering the contribution of knock-on showers, the value 10^4 can be reduced to the same order as the above estimated value.

Next we consider the reasons why the second maximum distinctly appears in our experiment. In our apparatus, the solid angle of the counter train is very small, which gives much larger values of \(j_{N_1}/j_e\) than for wide angle primaries. This seems to be the reason why this maximum appears distinctly in our case. If we took wide angle primaries, the tail of the cascade shower and the background of knock-on electrons would mask the second maximum.

In order to make sure of such considerations, it is desirable to attempt the following experiments:

1. The experiment with different solid angles, because \(j_{N_1}/j_e\) varies with the solid angle.
2. The experiment at different altitudes, because \(j_{N_1}/j_e\) varies with altitude.

§ 5. Features of secondary rays

The composition of secondary rays is supposed to largely affect the characteristic shape of the transition curve. As already pointed out, it is sure that the second maximum of NS is shifted to the first maximum of NPS, and then it is plausible to assume that such a maximum appears only at the position of \(\Sigma_1 + \Sigma_2 = \text{constant} \sim 15\text{cm Pb}\). Although it is not clear whether or not the second maximum of NPS is the same type as above mentioned, the experimental evidence for the maximum of penetrating showers (this corresponds to NPS for \(\Sigma_2 \sim 50\text{cm Pb}\)) at the position \(\Sigma_1 \sim 15\text{cm Pb}\) makes us infer these two kinds of the maxima as the same ones, which appear at the thickness of \(\Sigma_1 \sim 15\text{cm Pb}\).

Then the nature of secondary particles which produce such a maximum can easily be explained as follows. Putting the thickness of \(\Sigma_1\) and \(\Sigma_2\) as \(x_1\) and \(x_2\) (see Fig. 5) respectively, we represent the absorption curves of primary and secondary rays as \(e^{-\lambda x}\) and \(g(x)E(R-x)\), where \(E(x)\) has values 1, 1/2, or 0 for \(x\) positive, zero or negative and \(R\) means the maximum range of secondaries. The frequency of NPS can be expressed as

* Hereafter, we denote this case as case 1.
* We denote this case as case 2.
Transforming the integral variable, (11) is reduced to
\[
N(x_1) \sim e^{-\lambda x_1 + \tau_1} \{F(x_1 + x_2) - F(x_2)\},
\]
where
\[
F(x) = \int_0^x e^{y \gamma} g(y) E(R-y) dy
\]
and
\[
H(x) = \int_0^x e^{y \gamma} g(y) dy.
\]

We then get the expressions for \(N(x_1)\) corresponding to three different ranges of \(R\):

\[
\begin{align*}
N(x_1) & \sim 0 \quad \text{for } R \leq x_2, \quad (15a) \\
N(x_1) & \sim e^{-\lambda x_1 + \tau_1} \{H(R) - H(x_2)\} \quad \text{for } x_2 \leq R \leq x_1 + x_2, \quad (15b) \\
N(x_1) & \sim e^{-\lambda x_1 + \tau_2} \{H(x_1 + x_2) - H(x_2)\} \quad \text{for } x_1 + x_2 \leq R. \quad (15c)
\end{align*}
\]

For (15b), \(N(x_1)\) is a decreasing function, so that the maximum appears at the minimum value of \(x_1\), i.e.
\[
x_{1\text{max}} = R - x_2. \quad (16b)
\]

For (15c), the maximum position depends on the function \(H\), but in any case
\[
x_{1\text{max}} \leq R - x_2. \quad (16c)
\]

Referring to these relations, we study the character of function \(g(x) \times E(R-x)\) for respective cases.

**Case 1.** In this case the maximum appears only at the thickness of \(x_1 + x_2\) = constant \(\sim 15\text{cm Pb}\), and accounting for (16b) and (16c) we can conclude \(x_{1\text{max}} + x_2\) must be equal to \(R\).

This condition is, of course, restricted by the functional form of \(H\).

As a simple example, we put
\[
g(x) = e^{-\mu x},
\]
then
\[
H(x) = \frac{1}{\lambda - \mu} (e^{(\lambda - \mu)x} - 1), \quad (14')
\]
and for \(x_1 + x_2 \leq R\)
Interpretation of the Second Maximum of Rossi Curve

\[ N(x_1) \sim \frac{1}{\lambda - \mu} e^{-\mu x_2} (e^{-\mu x_2} - e^{-\lambda x_1}). \]  

(15c')

This gives

\[ x_{1\text{max}} = \frac{1}{\lambda - \mu} \ln \left( \frac{\lambda}{\mu} \right) \text{ provided } \frac{1}{\lambda - \mu} \ln \left( \frac{\lambda}{\mu} \right) \leq R - x_2, \]

\[ x_{1\text{max}} = R - x_2 \text{ provided } \frac{1}{\lambda - \mu} \ln \left( \frac{\lambda}{\mu} \right) \geq R - x_2. \]

Since \( x_{1\text{max}} = R - x_2 \), \( \ln(\lambda/\mu)/(\lambda - \mu) \) must be larger than \( R \). This means that the absorption of secondary particle is mainly due to the ionization, and that almost of them have the ranges about 15cm Pb.

Case 2. In this case, the maximum appears only at the position \( x_1 = \text{constant} \approx 15\text{cm Pb} \). In the limiting case of \( x_2 = \infty \), we must take \( R = \infty \). Hence, \( N(x_1) \) is always given by \( (\infty, \infty) \).

Differentiating it by \( x_1 \), \( x_{1\text{max}} \) is given by the relation

\[ \lambda - \lambda \frac{H'(x_1 + x_2)}{H(x_2)} + \frac{H'(x_1 + x_2)}{H(x_2)} = 0. \]  

(17)

As \( x_{1\text{max}} \) does not depend on \( x_2 \), \( H(x) \) must be an exponential function as easily seen from (17). This leads to \( g(x) \) of exponential type, which means that the secondary particles responsible to the maximum have high energy.

Summarizing these two results, we may conclude that the second maximum of NS consists of the overlap of two kinds of maxima, mainly due to the particles with definite range. This character of secondaries seems to be favourable to explain some features of the second maximum.

(1) From the general consideration of the shower curve, it can be concluded that the decrease after maximum must be slower than that of the primaries. Only in our case, that the secondaries have a definite range, it is equal to that of the primaries. The experimental decrease is too steep to explain its behaviour by any other absorption law of the secondaries.

(2) The second maximum appears at about the same thickness (in g/cm²) for various materials².

This seems to suggest that the absorption of the secondaries is mainly due to ionization, which results in a definite range.

§ 5. Difficulties of our interpretation

The above interpretation can explain various features of transition curve, but seems to contain some difficulties.

1)* The frequency of showers produced by a single act is generally repre-

* This defect was first pointed out by Dr. Y. Sekido.
stented by $|e^{-\lambda x} - e^{-\mu x}|$, where $\lambda$ and $\mu$ means the absorption coefficients of primaries and secondaries respectively.

This function is convex for the small values of $x$, whereas the shape of the initial increase of NS is concave, as represented in Fig. 2 by dotted curve. If this experimental result be correct, NS must mainly consist of the showers produced by two or more times of collisions. Here one should note that the statistical error may not be small enough to discuss such a detailed point.

2) The decrease after the maximum of NPS is much steeper than that expected from our presumption.** The experimental data, however, are not so accurate that this defect may also be due to the statistical error of the data.

3) The assumption that the penetrating showers have two kinds of secondaries has already pointed out by Walker, but the range of shorter secondaries obtained by him is much shorter than that of ours:

- in our case $R \sim 15\text{cm Pb}$,
- in his case $R \sim 1\text{cm Pb}$.

Although the first maximum of NPS can also be explained by considering the detection probability of counters (3 and 4) making use of his result, the relation between this and the second maximum of NS becomes obscure. This also seems to mean that the detection probability does not play an essential role in our case. If we adopt the momenta of primaries as larger than $2\text{BeV/c}$, which is plausible as the lower limit from the absolute intensity, a shower contains two or more fast particles capable to penetrate through the $10\text{cm}$ lead. Among these particles about a half may be protons which mainly undergo ionization loss. Our interpretation is, therefore, supposed to be not far from reality, and these considerations will be testified by the experiment varying the thickness of $\Sigma$.

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**According to our presumption, it must be slower than that of primaries.

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