The Glauber Approximation and Evaluation of Ionization Cross Section

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In the theory of multiple scattering the eikonal approximation based on fixed scatterer and on-shell assumptions had, as is well known, remarkable success even in atomic system\(^1\) as well as in nuclear system.\(^2\) However the validity of this theory is not yet clear. Paying attention to this problem, we consider a break-up process in the most fundamental atomic system in which the interaction is completely well known; that is, ionization of a hydrogen atom by electron impact.

In this particular inelastic scattering the cross section has been computed by the Born, the Born-Oppenheimer and some modified approximations.\(^3\) But all these calculations give results which are in fairly large discrepancy with experimental values in the most interesting energy region, in which the total cross section has its maximum value, as seen in Fig. 1.

Now we examine the validity and applicability of the Glauber approximation to the above process. For this process the scattering amplitude of an incident electron with velocity \(v_i\) and momentum \(k_i\hbar\) is given by
Fig. 1. Total ionization cross section of the hydrogen atom for the electron impact (energy $E_i$) calculated by the Born approximation is compared with experimental values given by the full line, curve which is obtained from relative measurements normalized at high energy to absolute measurements ($x>10^3$).

$F_{ij}(q, \kappa) = \frac{i k_i}{2\pi} \times \int \psi_f^*(r, \kappa) \Gamma(b, r) e^{i q \cdot b} \psi_i(r) d^2 b$ drifts,

where the profile function is expressed by

$\Gamma(b, r) = 1 - \exp[i \chi(b, s)]$

with the phase shift function

$\chi(b, s) = \frac{1}{h \nu_t} \int_{-\infty}^{\infty} V(b, r, \zeta) d\zeta$

$= 2\beta \ln \left\{ \frac{|b - s|}{b} \right\}$. 

$\beta = \frac{\hbar^2}{\nu_t}$, $q$ is the momentum-transfer, and $b$ is the impact parameter, which is on the plane perpendicular to the incident particle. $s$ is the projection of $r$ onto the plane of impact parameter. $\psi_i(r)$ represents the initial atomic state, that is the ground state wave function for the hydrogen atom in our case. $\psi_f(r, \kappa)$ means the final state; the ionized continuous state with wave vector $\kappa$. $V(b, r, \zeta)$ is the sum of the coulomb interaction of the incident electron with the target electron and proton.

When the ejected electron velocity is smaller than $v_i$, the final state would be given by the appropriate coulomb wave function:

$\psi_f^*(r, \kappa) = \frac{\kappa}{(2\pi)^{1/2}} \frac{4\pi}{\kappa r} \times \sum_{i, m} Y_{im}^* (k) Y_{im} (r) i^{-1} e^{i \delta_i} f_i (\kappa, r)$,

where $\delta_i$ is the coulomb phase factor. We expand $F_i (\kappa, r)$ as follows:

$F_i (\kappa, r) = \sum_{l=0}^{\infty} A_l \psi_i (r)^{l+1+i}$.

In the case $l - m = odd$ the integral (1) vanishes by the selection rule, while for $l - m = even$, the spherical harmonics can be expanded as

$Y_{lm} (r) = \sum_{\mu=0}^{(l-m)/2} B_{lm}^{\mu} (s/r)^{2\mu+1} e^{i \mu \phi}$. 

Consequently the differential cross section for ionization process by the Glauber approximation is given by

$\frac{d^2 \sigma}{d\Omega d\kappa} = \sum_{l=0}^{\infty} \frac{k_i k_f}{2\pi^2} \times \left| \sum_{i, m} A_l B_{lm} I(l, |m|, \lambda, \mu) \right|^2$, (2)

where

$I(l, |m|, \lambda, \mu) = \int r^{l+1} e^{-\alpha r} \times \left[ 1 - \left( \frac{|b - s|}{b} \right)^{2l+1} \right] \times \left( \frac{r}{r} \right)^{2\mu+m} e^{i \mu \phi} d\phi d\phi_s d\kappa d\kappa_s dz$. (3)

The same kind of integral appears in cross section formulae for other scattering processes. For example the case of $\lambda = \mu = 0$ and $\alpha = 2$ corresponds to elastic scattering, and the case of $\lambda = 0, 1, \mu = 0$ and $\alpha = 3/2$ means the inelastic excitation (1s-2s)
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Fig. 2. Differential cross section by the Born (B) and Glauber (G) approximations (the incident electron energy \(E_i=100\) eV).

process, while \(\lambda\) must be varied up to infinity in the present case. The detailed reduction of the integral (3) will be given in the full paper.

A preliminary result of our evaluation for the differential cross section and comparison with values by the simple (coulomb) Born approximation are given in Figs. 2 and 3. It is to be noted that the forward \(p\)-wave contribution is dominant, and each magnitude of the \(d\), \(s\), \(f\) and \(g\)-wave contributions becomes successively (about one-tenth) smaller in this order, while the \(s\)-wave becomes superior in large angle. Moreover the Glauber differential cross section differs from the Born result especially for lower incident energy and wide angle.

The final evaluation of the total cross section is now in progress,\(^b\) while the validity of this approximation will be discussed in a separate paper.

5) Independent of our evaluation the preliminary report of the differential cross section was recently published: M. B. Hidalgo, J. H. McGuire and G. D. Doolen, J. Phys. 5B (1972), L70.