Letters to the Editor

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Radiative Correction to Atomic Levels from Muon and from Hadron Vacuum Polarization Phenomena

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From the study of hadron productions in electron-positron colliding beam experiments, Greco and his collaborators1 have conjectured that (a) the \( I=1 \) vector meson spectrum is given by 
\[
m_n^2 = (2n+1)m_0^2, \quad n = 1, 2, \ldots
\]
as in the Veneziano model,2 (b) the hadron cross section behaves as \( 1/s \) at high energies on the average and (c) the isoscalar contribution is \( 1/3 \) times the isovector contribution. It was concluded that 
\[
R = \lim_{s \to \infty} \sigma_{\text{had}}(s)/\sigma_{\text{pair}}(s) \approx 2.5.
\]
Sakurai3 has looked at the above result from a somewhat more general point of view by assuming the superconvergence relation for the imaginary part of the difference between the hadronic part of the full vacuum polarization amplitude and a comparison amplitude (for simplicity the muon production amplitude multiplied by \( \sqrt{R} \) with various muon masses was chosen as the comparison amplitude in order to preserve the \( 1/s \) dependence of the hadron cross section), by making use of the unsubtracted dispersion relation for \( s(\sigma_{\text{had}}(s) - \sigma_{\text{comp}}(s)) \) at \( s=0 \). He concludes that the infinite series of vector-meson (a) contributions, when properly averaged, form an integral part of the smooth, asymptotic cross section from threshold up to infinite energies.

The purpose of this note is to estimate the radiative correction to the atomic levels from hadron vacuum polarizations. In what follows the Veneziano-like meson spectrum, if taken seriously, leads to the divergent correction. However, if we want to saturate the known discrepancy4 between the observed and the calculated Lamb shifts by muon vacuum polarization, i.e., the known hadron contributions together with \( I=1 \) vector mesons of mass spectrum (a) with a common coupling constant, we need \( n \) up to \( N=2.3 \cdot 10^{60} \). This corresponds to an extremely high energy so that the Greco et al.'s conjecture is consistent as far as the present knowledge of the Lamb shift of the hydrogen atom is concerned. In the following we describe our approach.

The scattering amplitude for the electron from hydrogen-like atom due to the vacuum polarization of hadrons may be given by a similar manner to the evaluation of the vacuum polarization effect due to the electron,5 viz.,
\[
A = i(4\pi\alpha)^2 \bar{u}_s \gamma_{\nu} u_1 a_{\nu}(q) \frac{C(q)}{q^2} V_{\mu\nu}(q), \tag{1}
\]
where the \( u_s \)'s are the electron spinors, \( a_{\nu}(q) \) is the external potential due to the hydrogen-like atom, \( C(q) \) is the Feynman cutoff function for the photon propagator and \( V_{\mu\nu}(q) \) represents the Fourier transform of the vacuum expectation value of the time-ordered product of the hadronic currents,6
\[
V_{\mu\nu}(q) = \frac{1}{F T} \langle 0 | T(J_{\mu}(x_1)J_{\nu}(x_2)) | 0 \rangle
\]
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Here $v(q^2)$ is related to the total hadron production cross section $\sigma_{\text{had}}(s)$ by

$$\sigma_{\text{had}}(s) = \frac{16\pi^2 s^2}{\bar{s}} \operatorname{Im}(iv(s))$$

where $m_\gamma^2/s$ is the photon and vector meson coupling constant.

The contribution to the level shift on the zero angular momentum state of the hydrogen-like atom may be obtained as the coefficient of $u \gamma^4 (C(q)/q^2) a_\nu(q) (q_\nu q_\nu - \delta_{\mu\nu} q^2)$ after performing a charge renormalization in the small $q^2$ limit:

$$\delta E_n^{(2)} = \sum_{\mu} \sum_{\nu} \frac{m_\nu^2}{g_\nu^2} (q^2 - m_\nu^2 + i m_\nu \Gamma_\nu),$$

where $m_\nu^2/g_\nu$ is the photon and vector meson coupling constant.

We consider that the term (8) is necessary as a threshold (or off-resonant) correction in addition to the relation (6) since we are considering the problem at small $q^2$.

If we choose $m_\gamma = 7754$ GeV, $m_\mu = 7839$ GeV, $m_\pi = 1.019$ GeV, $m_\rho = 7839$ GeV, $m_\pi = 1.019$ GeV, $g_\rho^2/4\pi = 2.56$, $g_\pi^2/4\pi = 19.2$ and $g_\rho^2/4\pi = 11.3$, we get $-0.0027$ MHz to the $2S_{1/2}$ state from Eq. (6) as the contribution of the three vector mesons, while Eqs. (7) and (8) lead $-0.0064$ and $-0.0001$ MHz, respectively. The sum of these contributes only to the last significant number of the presently known theoretical value. The difference between the theoretical and the average of two direct measurements of $n=2$ Lamb shift in hydrogen atom becomes $0.03 \pm 0.06$ MHz.

Let us investigate how many number of vector mesons of $n \geq 1$, type (a), can be allowed to contribute in order not to contradict with the discrepancy obtained in the above, neglecting the experimental error. We find the relation in a narrow-width approximation and with a common coupling constant $g_\pi^2/4\pi = 2.56$:

$$\sum_{n=1}^{138} \frac{1}{2n+1} = 0.00027 \text{ MHz from Eq. (6)}$$

The right-hand side is expressed by the Euler constant $\gamma$ and the digamma function of integer argument, or simply by $\ln(N+1) + \frac{\pi}{2} - 1$ in a large $N$ approximation. We find $N = 23.1069$ which is a very big number.

Our estimate is very crude and there is no reason to believe that the simplified vector meson model should saturate the whole gap between the theoretical and the experimental values of Lamb shift. However, the result obtained may be considered to give an additional support to Greco et al.'s conjecture and its importance stressed by Sakurai.

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   A. Bramón, E. Etim and M. Greco, Phys. Letters \textbf{41B} (1972), 609.


