# A method of short-circuiting comparison with mixing indexes

David Dah-Wei Tsai, Rameshprabu Ramaraj and Paris Honglay Chen

## ABSTRACT

The improvement of reactor performance is being continuously studied and discussed. One of the most common phenomena is short-circuiting of reactor analysis. It is very difficult to compare the degree of short-circuiting from hydraulic indexes in reality. Mixing is one of the key mechanisms; accordingly it is possible to apply mixing indexes to compare the level of short-circuiting. Identification of reactor type was essential and necessary; we supplied several differentiation criteria to identify the reactor type to make the comparison achievable. Therefore, this study was to develop a simple and practical method for short-circuiting comparison with the 'distance method' through eight available mixing indexes. The results showed that all the indexes could correctly be compared. The 'distance method' was the method, considering the nature and type of studied reactor, using the distance between the study reactor and the identified type of ideal reactor on the axis of the index, and taking these values to differentiate the reactors for the degree of short-circuiting. This method could be applied to reactors from a lab scale to the field scale. We hope this process could further help us to understand the reactors better and to make the comparison possible.

**Key words** | high rate pond, mixing index, performance assessment, reactor analysis, short-circuiting

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## INTRODUCTION

There is an increasing need of reactor design to accomplish various physical, chemical and biological functions for biotech, biological, environmental and other purposes. Short-circuiting is a complicated phenomenon: it has large effects on reactor performance and it is one of the greatest hindrances to a successful design (Persson 2000). Not only resulting in dead zones (Metcalf & Eddy Inc. 2003) but also reducing proposed functions (Dierberg *et al.* 2005), it is the key factor to cause poor hydraulic efficiency (Singh *et al.* 2009).

According to Metcalf & Eddy Inc. it could be defined as 'non-ideal flow' caused by inadequate mixing, density currents, circulation, poor design, axial dispersion and dead space (Metcalf & Eddy Inc. 2003). Mixing is one of the key mechanisms of short-circuiting. Consequently, it's possible to do the evaluation on short-circuiting by mixing indexes. There are numerous papers focusing on mixing characteristics (Dusting & Balabani 2009; Gumery *et al.* 2009; Sardeshpande *et al.* 2009; Zhang *et al.* 2009a), index dynamics (Smith *et al.* 1993; Teixeira & Siqueira 2008; Siiriä & Yliruusi 2009), blender effect (Mehrotra & Muzzio 2009) and reactor performance (Bedoya *et al.* 2009; Zhang *et al.* 2009b).

At present, many mixing indexes have been developed and proposed. Theoretically each index could offer certain useful information to describe short-circuiting. Unfortunately, all the indexes show puzzling conclusions and difficult comparisons (Teixeira & Siqueira 2008) even by the same index. The difficulty comes mostly from the index itself; it is not suitable to compare indexes if they are of different 'reactor types'. Therefore, this study is to identify the reactor type with the differentiation criteria and to develop a method in order to promote the mathematical comparison of the indexes. Accordingly, the primary

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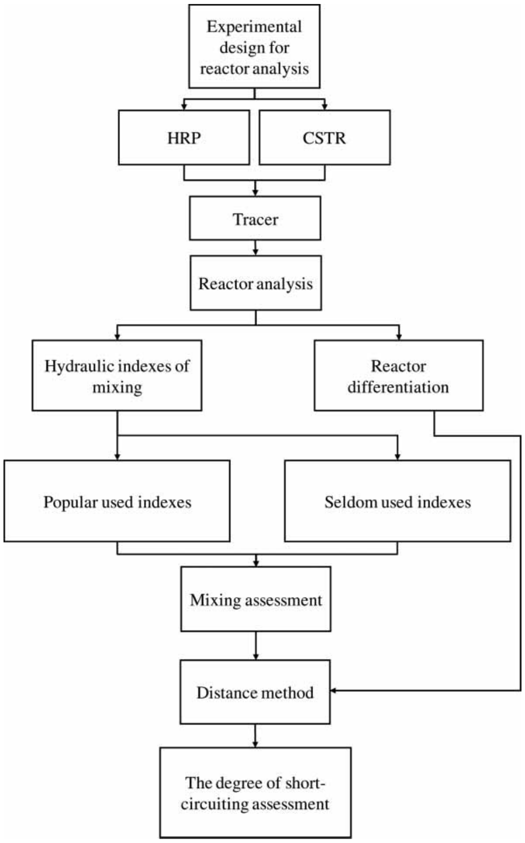

**Figure 1** | Flowchart of methodology.

objective of this study was to invent a simple and rational method to evaluate and to compare the short-circuiting by mixing indexes.

## METHODS

The methodology is illustrated in Figure 1. Since the high rate pond (HRP) could be used as a model of a plug flow reactor (PFR) in reality (Jupsin *et al.* 2003), it was taken as an example of a PFR in this study. The bench-scale HRP and lab-scale continuous stirred tank reactor (CSTR) were set up in the Sustainable Resources and Sustainable Engineering Research Lab (SRSE-LAB), Department of Soil and Water Conservation, National Chung-Hsing University, Taichung, Taiwan. The lab-scale HRP and CSTR designs are shown in Figures 2 and 3. Both reactors operated at 4 h retention time with spiking and other operational factors as listed in Table 1.

This study used the HRP and CSTR to discuss the degree of short-circuiting from the aspect of mixing and to establish the assessment method of index performance. Eight mixing indexes according to the literature were selected. The indexes were classified into two categories depending on

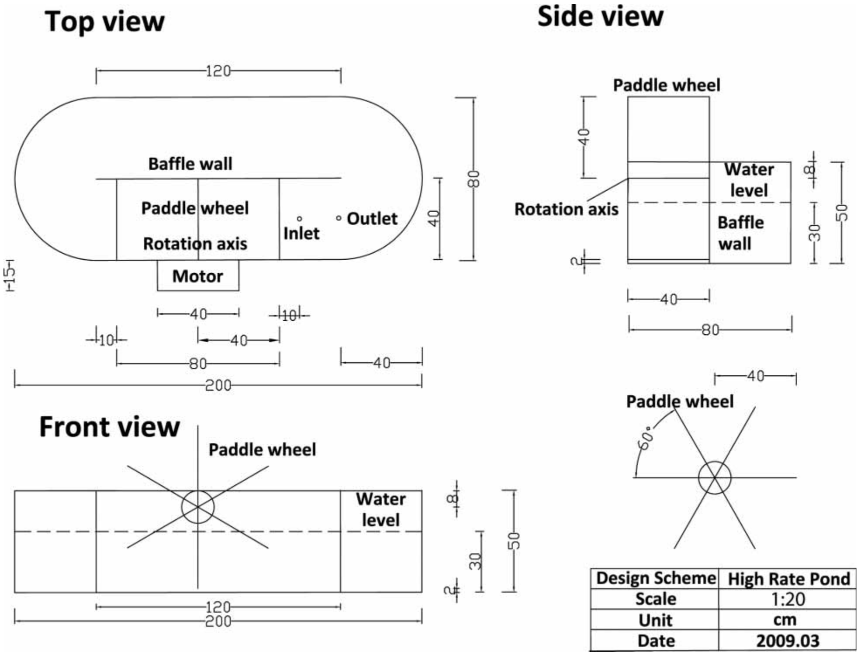

**Figure 2** | HRP design scheme.

**Figure 3** | CSTR design scheme.

**Table 1** | List of operational factors

| Operational factor | HRP | CSTR |
|---|---|---|
| Scale | Lab | Lab |
| Detention time | 4 h | 4 h |
| Reactor design | Paddle wheel with recirculation | 4 L up-flow flask |
| Water level | 15 cm | – |
| Water volume | 199 L | 4 L |
| Inflow speed | 820 $cm^3$/min | 16.8 $cm^3$/min |
| Tracer | NaCl | NaCl |
| Inflow type | Spike | Spike |
| Mixing speed (surface speed) | 10 cm/s | Magnetic mixer |
| Effluent measure interval | 15 min | 15 min |

their popularity as follows: (1) popularly used indexes and (2) seldom-used indexes.

The so-called 'distance method' is the method we developed to compare each index on different reactor types using the distances between the study and ideal reactors after screening the reactor type by the criteria we set up (Tsai & Chen unpublished). The axis of comparison basis was formed by the index value itself. The comparison values were calculated by the definition of each index from the standard points of the ideal reactor type. There is no short-circuiting in the ideal reactor; therefore according to this concept, in reality the distance could offer comparable data on the degree of short-circuiting for the different types of reactors.

## RESULTS AND DISCUSSION

### Popularly used indexes

Since several mixing indexes are designed to describe the level of mixing, we could apply those results to compare the mixing simply and correctly. All the results in this category are shown in Table 2.

### Dispersion index (var $= \sigma^2/t_{mean}^2$)

The dispersion index (var) is one of the mixing indexes and was calculated as 0.53 in HRP and 0.59 in CSTR. Theoretically, var is based on the variance of the residence time distribution (RTD), which was defined as the standardized curve of the tracer response and the area under the normalized curve is equal to 1 to represent the mixing phenomenon (Teixeira & Siqueira 2008). By the 'distance method' we applied, the distances were estimated as $d_{HRP}$ 0.53 > $d_{CSTR}$ 0.41 in Figure 4. Since the 'distance' was defined as how far away the mixing in the study reactor is from the ideal reactor, which had no short-circuiting, it could be mathematically recognized as the degree of short-circuiting from the ideal reactor. Consequently, $d_{HRP} > d_{CSTR}$ demonstrated that more short-circuiting came from inadequate mixing in HRP than CSTR. From the lab observations, there were several dead spots and sedimentation zones in HRP shown in Figure 5 while there was no obvious sedimentation zone in CSTR.

### Peclet number ($P_e = 1/\bar{d}$)

The Peclet number ($P_e$) is calculated as the inverse of the dispersion number ($\bar{d}$) and its result describes the level of the mixing (Levenspiel & Smith 1957). $P_e$, which decreased with the increase of mixing, was computed as 2.33 in HRP and 1.85 in CSTR. The result showed that the mixing was stronger in CSTR than HRP. This index explained the

**Table 2** | Results in the category of popularly used indexes

| Index | Formula | Range | Ideal PFR | Ideal CSTR | Lab HRP (spike) | Lab CSTR (spike) | Literature differentiation criteria | Author suggested criteria[a] | Evaluation | Reference |
|---|---|---|---|---|---|---|---|---|---|---|
| Dispersion index | $\sigma^2/t_{\mathrm{mean}}{}^2$ | 0–1 | 0 | 1 | 0.53 | 0.59 | – | – | $d_{\mathrm{HRP}} > d_{\mathrm{CSTR}}$ (0.53 > 0.41) | Teixeira & Siqueira (2008) |
| Peclet number ($P_e$) | $1/\bar{d}$ | 0–8 | 8 | 0 | 2.33 | 1.85 | – | – | $d_{\mathrm{HRP}} > d_{\mathrm{CSTR}}$ (8 > 1.85) | Levenspiel & Smith (1957) |
| Tanks-in-series no. ($N$) | $t_{\mathrm{mean}}{}^2/\bar{d}^2$ | 1–8 | 8 | 1 | 1.90 | 1.71 | – | – | $d_{\mathrm{HRP}} > d_{\mathrm{CSTR}}$ (8 > 0.707) | Levenspiel (1989) |
| Morrill dispersion index (MDI) | $t_{90}/t_{10}$ | 1–8 | 1 | ~8 | 16.69 | 18.87 | 22 | 18 | $d_{\mathrm{CSTR}} > d_{\mathrm{HRP}}$ (8 > 15.69)[b] | Morrill (1932) |
| Volumetric efficiency ($V_e$) | 1/MDI | 0–100% | 100% | ~0 | 5.99% | 5.30% | – | – | $d_{\mathrm{HRP}} > d_{\mathrm{CSTR}}$ (94.01% > 5.3%) | Metcalf & Eddy Inc. (2003) |
| Dispersion number ($\bar{d}$) | $D/\mu L$ | 0–8 | 0 | 8 | 0.43 | 0.54 | – | 0.5 | $d_{\mathrm{CSTR}} > d_{\mathrm{HRP}}$ (9 > 0.43)[b] | Philipossian & Mitchell (2003) |

[a] The suggested criteria were calculated by the authors.

[b] The conclusion was inconsistent with other indexes.

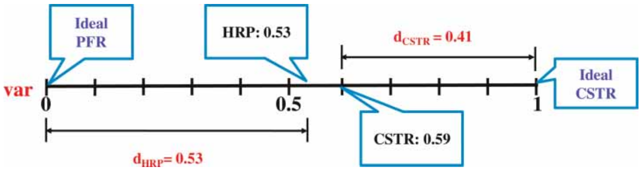

**Figure 4** | The distances of var from ideal reactors in HRP and CSTR.

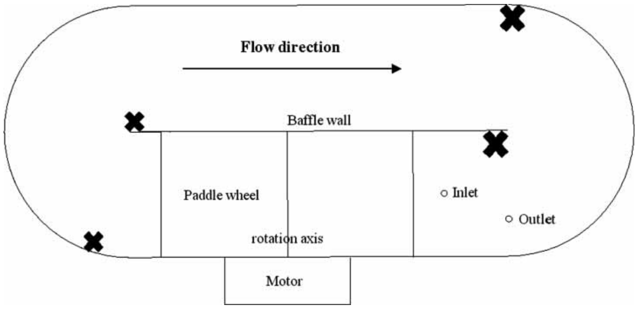

**Figure 5** | Location of dead spots observed in HRP. *Remark:* x: location of dead spot and the size of x shows the degree of dead spot.

mixing well and it is consistent with the lab observations. For short-circuiting, the distance method concluded that $d_{\mathrm{HRP}}$ 8 > $d_{\mathrm{CSTR}}$ 1.85 in Figure 6 and that the degree of short-circuiting caused by inadequate mixing in HRP was more than in CSTR.

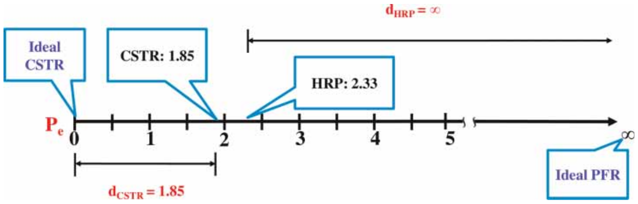

**Figure 6** | The distances of $P_e$ from ideal reactors in HRP and CSTR.

## Discussion

In this study, the definition of mixing was the flow homogeneity or the turbulent strength in the reactor, which the mixing index could describe well. On the other hand, the short-circuiting, which we defined in the introduction, was the degree of non-ideality from the ideal reactor and the short-circuiting was difficult to compare among different types of reactor by all the mixing indexes. Consequently, we would like to examine the mixing level by indexes themselves and the degree of short-circuiting by the 'distance method' to accomplish the study's purpose of short-circuiting comparison.

Beside var and $P_e$, there are four other mixing indexes in Table 2: $N$, $V_e$, MDI and $\bar{d}$ (Tsai & Chen unpublished). All showed an accurate comparison of mixing: stronger mixing in CSTR than HRP. But in term of short-circuiting, after applying the 'distance method', the mixing indexes of $N$ and $V_e$ concluded the truthful result as var and $P_e$ in Table 2; MDI and $\bar{d}$ appeared to be an erroneous comparison because the assumption of ideal CSTR value equals 8, which makes $d_{\mathrm{CSTR}} \gg d_{\mathrm{HRP}}$. Accordingly, we focus further discussion on the method performance of MDI and $\bar{d}$, as follows.

## Ideal values related to infinity

In the only exceptional indexes, MDI and $\bar{d}$, the assumption of 8 in ideal CSTR would cause a difficulty in calculating the degrees of short-circuiting and might produce confusing results, as mentioned above. The definitions of MDI make the denominator close to zero for ideal CSTR, causing the index values to approach infinity and to extend the erroneous distance. However, if the index criteria of ideal reactors in reality are such that $\mathrm{MDI} = 22$ (Metcalf & Eddy Inc. 2003) or $\mathrm{MDI} = 18$ (which was calculated by the authors), the distance results would be applicable and the calculation results would be correct and consistent as shown in Figure 7. Furthermore, ideal CSTR with huge axial dispersion makes the numerator of $\bar{d}$ close to infinity and $\bar{d}$ equal to 8, which also expands the distance incorrectly. We selected the criteria of $\bar{d} = 0.5$ (which was calculated by the authors); the 'distance method' would be accurate and consistent too as shown in Figure 8. Consequently, after a slight modification of the suggested criteria, the 'distance method' could perform an accurate evaluation for all the mixing indexes universally.

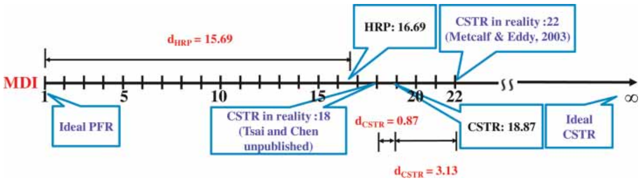

**Figure 7** | The distances of MDI from ideal reactors in HRP and CSTR.

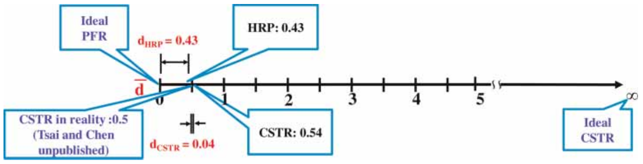

**Figure 8** | The distances of $\bar{d}$ from ideal reactors in HRP and CSTR.

## Seldom-used indexes

### Mixing dispersion indicators I and II ($M_{\mathrm{I}} = t_{75} - t_{25}$, $M_{\mathrm{II}} = t_{90} - t_{10}$)

Searching through the literature, two seldom-used indexes were evaluated in Table 3.

(I) $M_{\mathrm{I}}$ could show the dispersion in cumulative RTD by the difference between the time at 25 and 75% tracer flowing out ($t_{25}$ and $t_{75}$) (Stamou 2008). Table 3 shows the results of $M_{\mathrm{I}}$ as 1.42 in HRP and 1.26 in CSTR. The index adapted the interquartile to express the variation of the curve to describe conditions of mixing. But this result drew the incorrect conclusion: there is more mixing in HRP than CSTR. Because the interquartile concept only showed the amount of dispersion rather than the distribution of variation change, if applying it to different distribution curves (different reactor types), erroneous results were possibly concluded.

By the 'distance method' with $M_{\mathrm{I}}$ as 0 in an ideal PFR and 1.099 in ideal CSTR which were estimated by Stamou, the distance results of the HRP and CSTR, $d_{\mathrm{HRP}}$ 1.42 > $d_{\mathrm{CSTR}}$ 0.16, were calculated in Figure 9 and the results confirmed the correct conclusion: more short-circuiting in the HRP was caused by inadequate mixing than in CSTR.

(II) $M_{\mathrm{II}}$ is another index of mixing and it was calculated by the difference of $t_{90}$ and $t_{10}$. Adapting the same concept of interquartile as $M_{\mathrm{I}}$, the index was also falsely concluded in mixing. But by the distance method with the comparison of 0 in an ideal PFR and 2.197 in ideal CSTR (Stamou 2008), the results, $d_{\mathrm{HRP}}$ 2.41 > $d_{\mathrm{CSTR}}$ 0.03 in Figure 10, provided concrete evidence to prove more short-circuiting in an HRP than in CSTR.

Therefore, in this category $M_{\mathrm{I}}$ and $M_{\mathrm{II}}$ themselves couldn't provide precise information about the mixing level in different types of reactors. But by the 'distance method' the results of short-circuiting for inadequate mixing were demonstrated consistently and correctly.

## Mathematical basis of the distance method

Although the term 'short-circuiting' has no precise technical definition, it could be defined as 'non-ideal flow' (Metcalf & Eddy Inc. 2003). The degree of short-circuiting could be

**Table 3** | Results in the category of seldom-used indexes

| Index | Formula | Range | Ideal PFR | Ideal CSTR | Lab HRP (spike) | Lab CSTR (spike) | Evaluation | Reference |
|---|---|---|---|---|---|---|---|---|
| Mixing dispersion indicator I ($M_I$) | $t_{75} - t_{25}$ | 0–1.099 | 0 | 1.099 | 1.42 | 1.26 | $d_{HRP} > d_{CSTR}$ ($1.42 > 0.16$) | Stamou (2008) |
| Mixing dispersion indicator II ($M_{II}$) | $t_{90} - t_{10}$ | 0–2.197 | 0 | 2.197 | 2.41 | 2.23 | $d_{HRP} > d_{CSTR}$ ($2.41 > 0.03$) | Stamou (2008) |

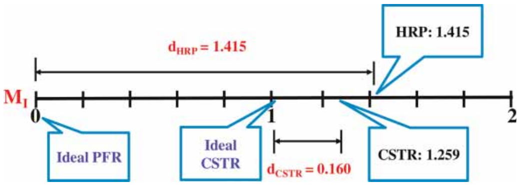

**Figure 9** | The distances of $M_I$ from ideal reactors in HRP and CSTR.

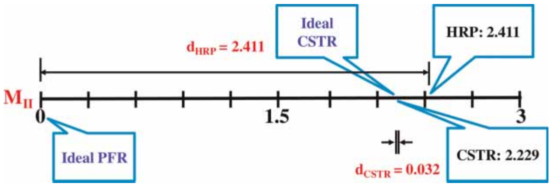

**Figure 10** | The distances of $M_{II}$ from ideal reactors in HRP and CSTR.

recognized as the level of similarity of RTD distribution to the ideal reactor's (PFR and CSTR). Since each reactor was identified and compared to the proper distribution type of the identified ideal reactor, both the central tendency and variance could mathematically describe their difference. From the definition of 'short-circuiting' we could use the central tendency indicator, such as $t_{mean}$, $t_{centroid}$, $t_{50}$, etc., in many short-circuiting indexes (Tsai & Chen unpublished). On the other hand, most mixing indexes in this study adapted the variance (var, $N$, $P_e$, $\bar{d}$) or distribution variation (MDI, $V_e$, $M_I$, $M_{II}$) to describe the changes of the curves. Accordingly, the mathematical concept made it possible to explain the basis for the comparison by mixing indexes for short-circuiting.

Furthermore, as regards the mathematical basis of the distance method, we based it on both the concept of RTD distribution in mathematics and no short-circuiting in ideal reactors to develop this method to differentiate and compare the different type of reactors or to evaluate the degree difference of the same type of reactors. Therefore, the 'distance method' is to standardize the comparison process and then to compare diverse reactors in the same category of distribution, according to the reactor type. In other words, we have to screen the reactor type first as PFR or CSTR and then do the distance comparison. This method standardizes all reactors to the correct distribution for comparison. Those standardizations of the process could offer a more accurate estimation of short-circuiting. As in Table 4, $M_I$ and $M_{II}$ had problems to describe the mixing level and all the other indexes could not express the accurate results of short-circuiting by index values themselves. But by the 'distance method', the correct results were concluded and make the indexes better. The adequate distribution comparison was the crucial key to compare the degree of short-circuiting successfully here.

## General discussion

This study selected six popular mixing indexes to make comparisons, including (1) dispersion index (var), (2) Peclet

**Table 4** | The final results of explaining short-circuiting in each hydraulic index

| Type | Index | Mixing level Index value | Short-circuiting Index value | Short-circuiting By distance method |
|---|---|---|---|---|
| Popularly used | Var | ✓ | × | ✓ |
| | $P_e$ | ✓ | × | ✓ |
| | $N$ | ✓ | × | ✓ |
| | MDI | ✓ | × | ✓[a] |
| | $V_e$ | ✓ | × | ✓ |
| | $\bar{d}$ | ✓ | × | ✓[a] |
| Seldom used | $M_I$ | × | ✓ | ✓ |
| | $M_{II}$ | × | ✓ | ✓ |

[a] Comparing to ideal CSTR in reality, the correct result of short-circuiting would be concluded.

number ($P_e$), (3) tanks-in-series number ($N$), (4) Morrill dispersion index (MDI), (5) volumetric efficiency ($V_e$) and (6) dispersion number ($\bar{d}$). Each index provided practical ways to portray the mixing phenomena in reactors and we discussed the detailed content further, as follows:

1. The var index is computed by variance divided by the mean square from a normalized tracer response curve and this standardized variance could be a dispersion index. The standardized variance concept was similar to the 'coefficient of variation' in statistics. Both indexes were used to compare data in different sample distributions and to measure the dispersion level by the unit variance. In an ideal PFR the standardized variance would be equal to zero and in ideal CSTR it was 1. From those characteristics of ideal reactors, var could describe and evaluate the mixing degree of the reactor well.
2. MDI suggested by Morrill (1932) was defined by the ratio of the 90th percentile over the 10th percentile of the cumulative tracer curve as an index of dispersion. In an ideal PFR, the distribution could be equivalent to a normal distribution with infinite kurtosis, the 10th and 90th percentile would be equal, and therefore MDI equal to 1.

   In reality, if the reactor was closer to PFR, MDI was approaching 1. In contrast, CSTR had a tendency of a smaller $t$ of the 10th percentile from the fast flowing-out speed, and the tendency to approach zero as does the ideal CSTR. Accordingly, MDI would be too exaggerated and sensitive to overstate the mixing phenomenon. If we applied the 'distance method', the infinity which came from ideal CSTR would cause a mathematical operational problem. Consequently, we needed a minor modification of the method as mentioned previously for CSTR-type reactors in this index. Since the percentile ratio could sense the major tracer flowing-out characteristic, a PFR-type reactor was nearer the 10th and 90th percentiles and MDI came close to 1. In other words, the higher kurtosis of the distribution, the closer the reactor to the ideal PFR would be.
3. The $\bar{d}$ index was designed to estimate the axial dispersion of a reactor and was calculated from the coefficient of axial dispersion per unit of fluid velocity and reactor length. $\bar{d}$ could discern a range from 0 for the ideal PFR to infinity for the ideal CSTR. Since $\bar{d}$ design set the ideal PFR as 0, it tended to apply and to evaluate a PFR's axial dispersion in practice (Metcalf & Eddy Inc. 2003). Moreover, as caused by the infinity abstraction operational problem, CSTR required a minor modification in the 'distance method' as in the MDI adjustment with the index criteria of an ideal reactor in reality ($\bar{d} = 0.5$). The exact coefficient of axial dispersion ($D$) was difficult to measure; therefore the similar concept of curve distribution as mentioned in the var index was applied to evaluate dispersion and the spread of distribution (variance of curve distribution) of the tracer response curve could exhibit the dispersion level well. Consequently, since we could recognize the parameters of $t_{\text{mean}}$, fluid velocity and reactor length as constants in one reactor, the variance would have a correlation relationship of $D$ with those constant correlation coefficients. In this case, $D$ and var would be similar indexes mathematically; in fact, var $= 2\bar{d}$ in small dispersion and var $= 2\bar{d} + 8\bar{d}^2$ in large dispersion ($\bar{d} = D/\mu L$).
4. $N$, the tank-in-series number, was used to describe an ideal PFR with the concept of the composite of infinite CSTR units in series. In other words, the flow characteristics of an ideal PFR was recognized as a series of infinite CSTRs and $N$ was equal to infinity. An ideal CSTR is the unit here when we used the exponential equation to describe CSTR's RTD curves and the curve would gradually skew to the left and approach the normal distribution when the number of CSTRs increased. Finally, the RTD would be performed as a pulse curve in an ideal PFR when $N$ approached to infinity. Thus the characteristics of the RTD curve could be connected to the $N$ estimation. Consequently, $N$ had a certain meaningful sense to express the mixing phenomenon between the PFR and CSTR. The lower the $N$ number, the more mixing was in the reactor. $N$ could explain the PFR characteristic well, conceptually, but since the small number $N$ explained CSTR mixing well, this index had a tendency to describe CSTR better, especially in a series CSTR design.
5. $V_e$ was the inverse of MDI and was applied to calculate the volumetric efficiency of the reactor type of PFR conventionally. Because in an ideal PFR $V_e$ was equal to 1, it was

recognized that 100% volume was fully used. $V_e$ had a tendency to describe how soon the tracer would flow out of the reactor. In a PFR, if there was a dead zone or axial dispersion, $V_e$ could detect and present the index value as less than 100%. It is a very simple and efficient index to detect dispersion in a PFR. On the other hand, in a CSTR-type reactor, not only an inefficient volume was used but also insufficient mixing would cause a higher $V_e$ value compared to ideal CSTR. Consequently, this index could sense and express precisely the mixing level in a CSTR reactor, especially in the comparisons of CSTR-type reactors.

6. $P_e$ could present the advection per unit of axial dispersion, which was the inverse of $\bar{d}$. In the PFR, there was major mass transport by advection and little axial dispersion to cause an enormous index value. Since there is no axial dispersion in an ideal PFR, the $P_e$ value will be infinity. On the other hand, the strong axial dispersion in CSTR would get a very small $P_e$ value and ideal CSTR would be zero.

Moreover, according to the dispersion value, if there was a small dispersion in the reactor $P_e$ was approximately equivalent to $2N$. If there was large dispersion in the reactor with closed boundary, the relationship between $P_e$ and $N$ could be modified as $N = P_e^2/[2(P_e + e^{-Pe} - 1)]$. According to those two relationships, $P_e$ could apply the sense of $N$ to evaluate the mixing degree in all types of reactor conceptually. In short, $P_e$ could be transferred to a similar sense of $N$.

## CONCLUSIONS

Although the degree of short-circuiting among different types of reactor is very difficult to compare, this study demonstrates the unique and the innovative 'distance method' can successfully accomplish this mission. All the mixing indexes could practically describe the level of mixing in all types of reactors except the seldom-used indexes ($M_I$, $M_{II}$). With the identification of the reactor nature and type to ideal reactors, the 'distance method' was the key to solve the comparison difficulty in which all the mixing indexes met. According to Metcalf & Eddy's definition of short-circuiting in this study, we further demonstrated the 'distance method' with reactor differentiation could provide more information beyond mixing itself on short-circuiting to give a practical guide to improve reactor design. We hoped this innovative method would help us understand all the reactors better and make reactor short-circuiting comparison possible.

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