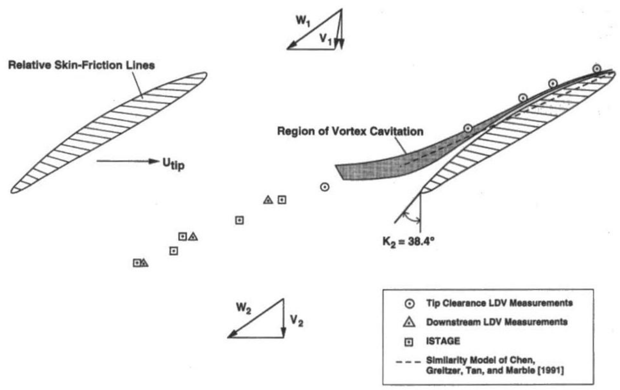


**Fig. 25 Trajectory of rotor tip clearance vortex core**

have been concerned for some time about a difference that appears in the authors' estimates for a parameter $A$ in a simple model for predicting the locus of the clearance vortex by Chen et al. (1991). Chen et al. suggested a value of 0.46, whereas Kang and Hirsch (1993b) suggested 0.19; the two values imply a difference of more than a factor of two in the circumferential displacement of the vortex from the suction side of the blade. In our discussion we emphasized the importance of endwall motion and a non-collateral inlet boundary layer, as would occur in a rotor.

In the authors' response (Kang and Hirsch, 1993b) they introduced speed of the blade tip (relative to the wall) as an independent variable. We have to admit to being unable to follow the process leading from Eq. (4) to Eq. (6) in the paper and by the subsequent treatment of Inoue's data. As regards the latter, reducing the flow coefficient from 0.5 to 0.33 does more than just alter the speed of the wall (assuming the axial velocity is held constant)—it alters the pressure distribution around the whole blade.

In the final part of section 5 of the present paper the authors return to our discussion of the earlier paper (Kang and Hirsch, 1993b), and the effect of a moving endwall. This time they are presenting numerical results with a moving endwall. As far as we can understand the calculations with a moving endwall refer to the condition of flow coefficient equal to 0.5. With an axial inlet flow to the rotor a flow coefficient of 0.5 means the axial velocity outside the boundary layer is equal to half the blade speed, so the inlet flow direction is $\cot^{-1} 0.5 = 63.4$ deg. The blade inlet direction is, however, 32.5 deg, so the specified flow coefficient would correspond to an incidence of almost 30 deg. It may be that we have misunderstood what was done, but it is clearly important that the conditions chosen be physically plausible if the effect of relative motion of the endwall is to be assessed.

At the conference at which this paper was presented there was a two-part paper by Zierke et al. [later consolidated and published as Zierke et al. (1985)], who examined clearance flows in a high-Reynolds-number axial pump. With Dr Zierke's permission, Fig. 4 is reproduced here as Fig. 25. This shows how well the model of Chen et al. fits the observations of the vortex position; it also shows how well it fits the predictions of Navier–Stokes solver, ISTAGE.

## References


Chen, G. T., Greitzer, E. M., Tan, C. S., and Marble, F. E., 1991, "Similarity Analysis of Compressor Tip Clearance Flow Structure," ASME JOURNAL OF TURBOMACHINERY, Vol. 113, pp. 260–271.

Zierke, W. C., Farrell, K. J., and Straka, W. A., 1995, "Measurements of the Tip Clearance Flow for a High-Reynolds-Number Axial-Flow Rotor," ASME JOURNAL OF TURBOMACHINERY, Vol. 117, pp. 522–531.


## Authors' Closure

The authors would like to thank N. Cumpsty and E. Greitzer for their discussion and for raising the question of the range of validity of the simplified model, developed by Chen et al. (1991), for the prediction of the tip vortex trajectory, which is represented as follows:

$$\frac{L_z/s}{x/c} = A \cdot \sqrt{\frac{c(\tan \beta_1 - \tan \beta_2)}{s \cos \beta_m}} \tag{1}$$

This discussion provides an opportunity for a more critical analysis of this model, based on inviscid considerations and on a number of assumptions, some of which deserve particular attention:

(*i*) It is fair to assume, following Chen et al., that the basic mechanism for the creation of the tip vortex is of inviscid nature, since it is mainly pressure driven;

(*ii*) It is also acceptable to consider that the influence of the tip clearance height is not a first-order effect, although a certain dependence of the tip vortex trajectory on clearance height, especially for small values, is clearly observed by Inoue et al. (1986), Fig. 9 in Chen et al. (1991);

(*iii*) The influence of the entrainment effect of the end wall is totally ignored, which is clearly the main weakness of the model of Chen et al.

Due to the small values of tip clearance gap heights, the viscous effects from the end wall tangential motion, leading in addition to the end wall boundary layer skewness upstream of the blade, reinforces the tip vortex and drags it away from the suction surface. A rigorous analysis of the data of Fig. 15 of Chen et al. clearly indicates the nonnegligible effect of the flow coefficient. With higher flow coefficients, that is for smaller end wall entrainment at fixed axial velocity, the experimental position of the tip vortex is shifted toward the suction surface and the correspondence with the simplified model seen on this figure is, at best, qualitative. This appears, both from numerical simulations and from experimental data of Inoue et al. (1991) and Moyle et al. (1992) to be an important effect. In high-speed axial compressors, Adamczyk et al. (1993) and Jennions and Turner (1993) observed from their numerical results on NASA Rotor67 that with reducing mass flow the tip vortex inclined more toward the tangential direction.

On the other hand, the calibration of the constant in the model of Chen et al. to the value of 0.46 is based on data corresponding mostly to flow coefficients around 0.5, as seen from Table 1 in their paper. Only one set of data, from Smith, refers to a lower flow coefficient of 0.29. As seen from Fig. 5 in the paper of Chen et al., reproduced here as Fig. 26, the corresponding points (open diamonds) are off the main line of slope 0.46 and fit quite well to a slope of 0.55, indicated with a dashed line.

The assumptions made in our response to a previous discussion by the same authors on our paper (Kang and Hirsch, 1993b), are based on the following considerations:

(*i*) The quasi-linear variation of tip vortex position with the square root of main loading is well confirmed by experimental and numerical data;

(*ii*) The nearly linear variation of tip vortex position $L_z$ with axial distance is also well validated, at least over the range about 70 to 80 percent chord from the leading edge. The position $L_z$ is measured normal to the camber line as defined by Chen et al. (1991), even though a measurement from the chord is more convenient and may be even closer to a linear relation at certain conditions;

(*iii*) A linear variation of the entrainment effect due to the end wall relative motion, scaled by the axial velocity, is the simplest assumption. Based on our experimental and numerical data for a fixed end wall and the data of Inoue et al. (1991) and Moyle et al. (1992) for axial compressor rotors, measured at the trailing edge section, we tentatively calibrated the constant $A$ in the model of Chen et al. as

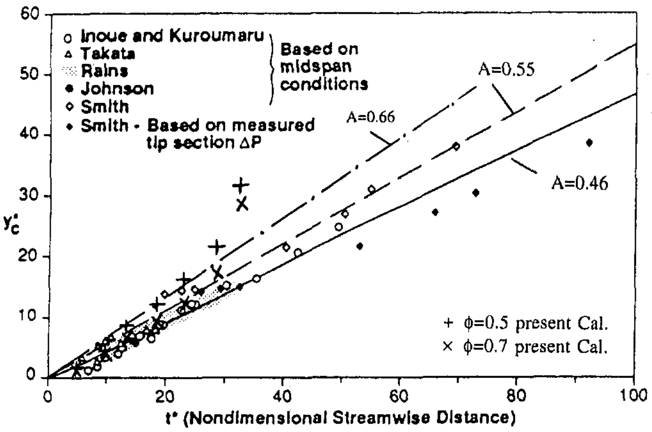


**Fig. 26 Representation of Fig. 5 in the paper of Chen et al. (1991), with the data of present computations**

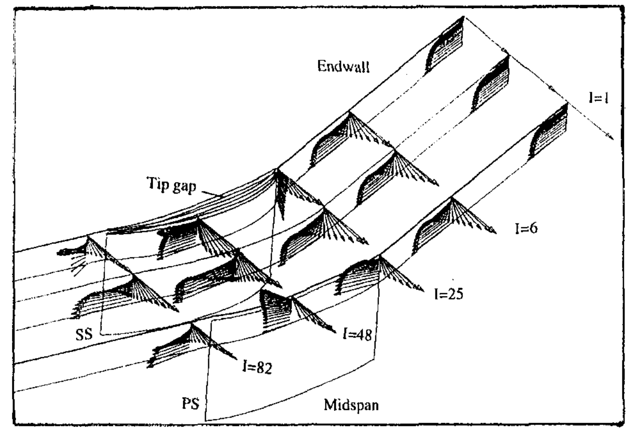


**Fig. 27 Three-dimensional velocity vectors near the moving endwall at some axial sections from the inlet of computation domain, $I = 1$ (150%c upstream), to $I = 28$ (12%c behind trailing edge), for $\phi = 0.7$**

$$A = A_0 + Au/\phi = 0.19 + 0.27/\phi \quad (2)$$

where $\phi$ is the flow coefficient. For $\phi = 0.5$ and 0.29, the factors $A$ calculated with this formula are respectively 0.73 and 1.12. These data are apparently larger than what one reads from Fig. 26 since the calibration was based only on the data at the exit section. Behind the exit of a compressor blade row, the slope of tip leakage vortex trajectory is generally greater than inside the blade passage. This variation may be even stronger at higher tip clearance and lower flow coefficient, as one can observe from Fig. 9 in the paper of Chen et al. (1991). Due to blade stagger, the entrainment effects on the vortex trajectory should be related to the projection of the wall speed along the normal to the blade. Hence, with a factor cos ($\gamma$), the coefficient $A$ may be assumed as

$$A = A_0 + Au \cos(\gamma)/\phi \quad (3)$$

Table 1 shows the calculated $A$ for some of the experiments in Fig. 26, for which both the flow coefficient and stagger angle are known, with a suggestion of $Au = 0.24$ and $A_0 = 0.19$.

With regard to the conditions of our numerical simulations, it has to be specified that they are performed at fixed *relative* flow inlet angles and hence correspond to a nonaxial absolute flow field. It is true, as indicated by the discussors, that in an actual axial compressor changing the flow coefficient will modify the blade loading. Our simulations, however, allow variations of the flow coefficient independently of blade loading, enabling us to separate the effects. In addition, as seen from Fig. 27, which shows the three-dimensional velocity vectors near the endwall at $\phi = 0.7$, our simulation is performed with a relative motion of the end wall and represents the inlet end wall flow skewness, typical of compressor flows.

Figure 28 presents the computed three-dimensional streamlines, issuing from the tip gap near the leading edge, and the

**Table 1 Factor $A$ for different cases ($A_u = 0.24$)**

| Author | Stagger Angle | ϕ | A |
|---|---|---|---|
| Inoue | 56.2 | 0.33 | 0.59 |
| Inoue | 56.2 | 0.50 | 0.46 |
| Rains | 57.4 | 0.45 | 0.48 |
| Katana | 48.0 * | 0.50 | 0.51 |
| Katana | 48.0 * | 0.53 | 0.49 |
| Katana | 48.0 * | 0.62 | 0.45 |
| Zierke | 58.1 | 0.74 | 0.36 |
| Present Cal. | 10.0 | 0.50 | 0.66 |
| Present Cal. | 10.0 | 0.70 | 0.53 |

* This stagger angle is measured from Fig.15 in Chen et al's paper.

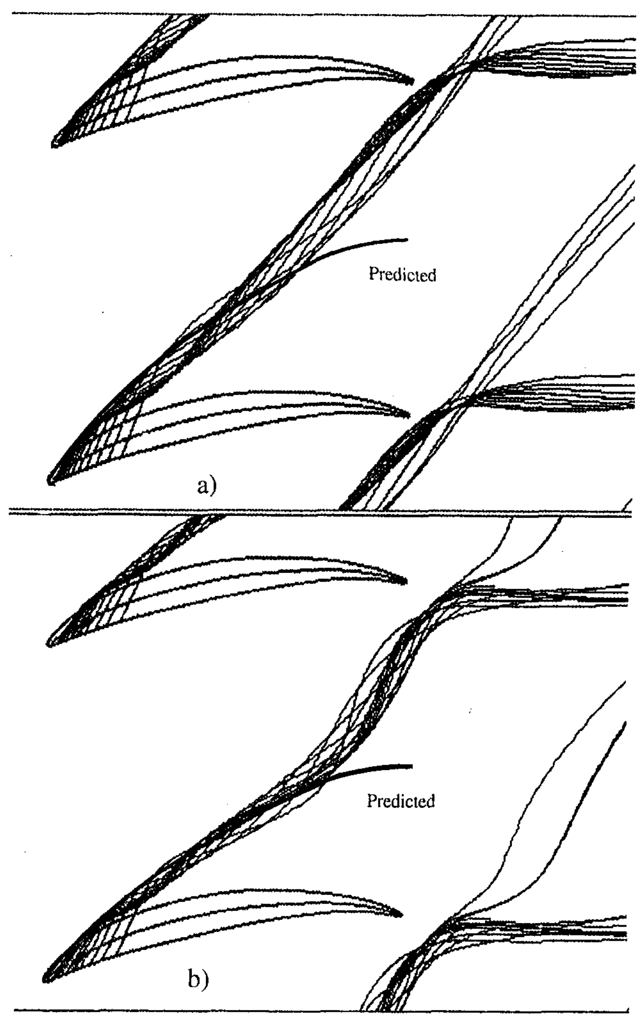


**Fig. 28 Computed leakage vortex trajectories, compared with the predictions of the modified model: (a) $\phi = 0.5$; (b) $\phi = 0.7$**

predicted leakage vortex trajectories with the modified model Eq. (3), for $\phi = 0.5$ and 0.7. The computed vortex trajectories are also presented in Fig. 26. The computations clearly show the good correspondence between the Navier–Stokes results and the corrected model of Chen et al., except near the trailing edge section. At $\phi = 0.7$, a strong variation of the trajectory slope is observed behind 70 percent chord. Behind the trailing edge the vortex follows the mainstream direction. Due to the entrainment effect, the streamlines closer to the wall move tangentially away from the vortex. It is seen that near the trailing edge (with large $t^*$ in Fig. 26) the computed vortex trajectories are off the linear model, even though they are on a linear line in Fig. 28($a$), as the vortex trajectories are measured perpendicularly to the camber line.

The discussors also refer to the difference between cascade and turbomachinery blade row data, raising doubts as to the validity of cascade data and computations for assessing real life phenomena. It appears that under this general statement, the discussors refer essentially to the effects of the presence of the inlet boundary layer skewness. As pointed out by Storer in his discussion of the paper of Chen et al., the main effect on the tip vortex is to shift its origin toward the leading edge of the blade, as a consequence of the high leading edge loading created by the relative motion of the end wall. This is indeed seen also from Fig. 28. It should be noted that the simulated cascade is characterized by a low stagger angle (10 deg), high blade camber, and thick blade profile. The computed flow is then close to a stator blade row with a hub clearance, rather than the tip flow in an unshrouded rotor.

Finally, we would like to emphasize, if needed, that the development of reliable CFD tools and of powerful computer hardware opens the way to a more systematic use of numerical simulations of complex phenomena, such as tip clearance flows, in order to gain a better understanding, particularly of flow behavior not accessible to experiments. This should allow also, when coupled to reliable experimental data, to establish limits and range of validity of simplified models, which are still necessary and useful for design purposes.

## References


Adamczyk, J. J., Celestina, M. L., and Greitzer, E. M., 1993, "The Role of Tip Clearance in High-Speed Fan Stall," ASME JOURNAL OF TURBOMACHINERY, Vol. 115, pp. 28–39.

Inoue, M., Kuroumaru, M., and Fukuhara, M., 1986, "Behavior of Tip Leakage Flow Behind an Axial Compressor Rotor," ASME *Journal of Engineering for Gas Turbines and Power,* Vol. 108, pp. 7–14.

Hirsch, Ch., and Kang, S., 1996, "The 3D Flow in Krain's Centrifugal Impeller," ERCOFTAC Workshop on 3D Turbomachinery Flow Prediction, Jan. (unpublished).

Jennions, I. K., and Turner, M. G., 1993, "Three-Dimensional Navier–Stokes Computations of Transonic Fan Flow Using an Explicit Flow Solver and an Implicit $k$–$e$ Solver," ASME JOURNAL OF TURBOMACHINERY, Vol. 115, pp. 261–272.