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Enlightening Visualizations of Partially Filled Dielectric Capacitors

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This paper offers a technique for abstracting capacitors partially filled with a dielectric into parallel and series capacitor models with enlightening visualization approaches. Essential explanations, conditions, and limitations of these models lacking in textbooks are also discussed. It can serve as supplementary reading material for introductory physics learners and instructors for in-depth explanations or alternative perspectives on fundamental topics unavailable in textbooks.

Introduction

Introductory physics teaches ideal capacitors, infinitely large parallel conducting plates separated by a distance. These practically include parallel conducting plates that are separated by a distance far smaller than the size of the plates; thus, the electric field is uniform in between and vanishes outside. Since there has long been a recognition among introductory physics educators of students' difficulty understanding the inner workings of capacitor topics,[1–5] papers have been written on topics such as trapped charge,[6] charging and discharging in capacitor systems,[7] polarization and bound charge in a dielectric revisited by a field theory approach,[8] and a new pedagogical philosophy of resistor–capacitor circuits.[3] Related topics beyond the scope of introductory physics such as fringe fields,[9] the inverse sum rule of capacitance,[10] and electric field[11] due to nonideality have also been explored.

Common homework problems involving capacitors partially filled with a dielectric pose challenges for students; Jackson suggests that this is because conductors appearing in electrostatics and capacitors are typically taught as disjoint topics.[1] Most solutions to these problems lack analysis of how charges are distributed on various surfaces and how to distinguish bound charges from free, as well as essential intermediate steps toward abstracting them as capacitors in parallel or series. Students may readily accept or memorize the notion of capacitors in parallel and series on seeing corresponding patterns of partially filled capacitors, and commonly endow capacitors with "collecting" charges due to some unknown internal mechanism or think of charges as somehow "jumping" from one plate to the other, as has been demonstrated through surveys and interviews.[5] In this paper, the close connections between the capacitor and electrostatics will be reinforced, and capacitors partially filled with a dielectric in particular will be simplified into parallel or series models with enlightening visualizations. In addition, some frequently overlooked technicalities and limitations will be analyzed.

This paper will remain in the framework of ideal capacitors delivered with the language of introductory physics, with no extension to the full solution of nonideal capacitors and fringe fields.[10,11] All the figures in this paper assume infinitely large parallel conducting plates, and any nonideality is not to be considered. All the electric wires in this paper are assumed ideal, with no resistance. The spacing between any plate of a capacitor and a dielectric surface is exaggerated for illustration convenience in this paper, though assumed zero in theory.

Capacitors with dielectric in parallel

Although not directly taught in most textbooks, capacitors partially filled with a dielectric almost always appear as exercise problems in textbooks. There are as-yet unexplained concepts and technicalities that I deem missing but necessary in textbooks.[12–16] Without these explanations, students may lose sight of the essential physics.

The hardest part of understanding capacitors partially filled with a dielectric may be distinguishing free and induced charges (also known as bound charges) due to polarization on various surfaces and correctly drawing electric field lines in different regions. For our purposes, the figures show the free charges on the plates and the induced charges on the surfaces of the dielectric.

Consider a capacitor filled with a dielectric on the top half as shown in Fig. 1(a). This setup could be established by connecting the capacitor to a battery (hidden) first and then inserting the dielectric into the top half with the battery either staying connected or not. The distinction that whether or not the battery stays connected would bring is whether the potential difference across the plates or the charge on each

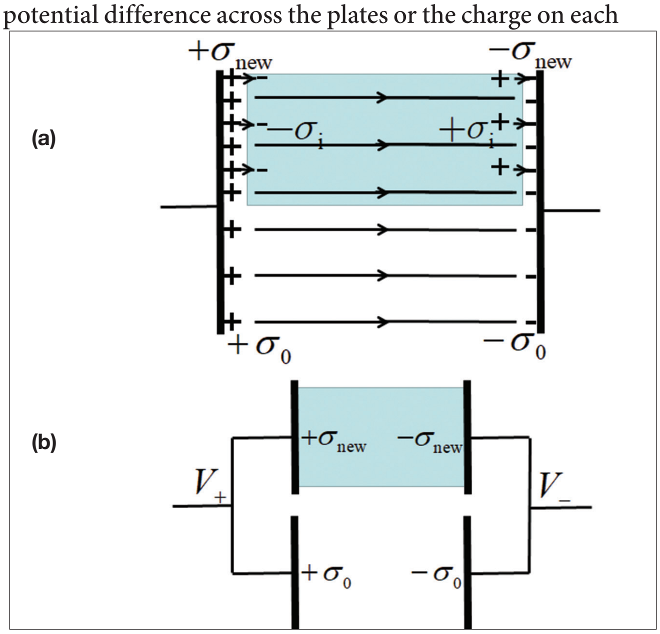

Fig. 1. Ideal capacitor filled with a dielectric in the top half. The charge densities $\sigma_{new} = 2\sigma_i = 2\sigma_0$ and permittivity in the dielectric $\varepsilon = 2\varepsilon_0$ are taken as such for illustrative purposes. (a) Physical capacitor with free $\pm\sigma_{new}$ and surface induced $\pm\sigma_i$ charge densities in the top half and free charge densities $\pm\sigma_0$ in the bottom half. The electric field is uniform throughout, though each plate is charged with different charge densities in the vacuum and dielectric regions. (b) Ideal capacitors in parallel, an equivalent abstraction of (a). The voltage difference is $V_+ - V_- = \sigma_0 d/\varepsilon_0 = \sigma_{new} d/\varepsilon$, with d the distance between the plates.

plate stays the same as it was before the dielectric was inserted. Therefore, this condition of the battery will not affect the effective capacitance, which lets us directly assume a potential difference across the plates $V_+ - V_-$ *after* the dielectric is inserted as the starting configuration of Fig. 1(a).

Each of the conducting plates is equipotential on itself, so the electric field in between should be uniform in *both* the dielectric and vacuum [Fig. 1(a)]. Maintaining the same amount of electric field, the charges on both plates are redistributed (from uniform when there was no dielectric) such that the top dielectric half should now carry higher densities $\pm\sigma_{new}$ of newly distributed free charges than in the bottom vacuum half $\pm\sigma_0$, the original free charge densities ($\sigma_0 < \sigma_{new}$), since some field lines starting from the positive free charges need to terminate on some induced surface charges with densities $\pm\sigma_i$. If one calculates the induced surface charge density σ_i, applying Gauss's law in the dielectric and vacuum yields $\sigma_{new}/\varepsilon = (\sigma_{new} - \sigma_i)/\varepsilon_0$, where ε_0 and ε are the permittivity in vacuum and the dielectric, respectively. The charge densities $\sigma_{new} = 2\sigma_i = 2\sigma_0$ and permittivity in the dielectric $\varepsilon = 2\varepsilon_0$ are taken as such for illustrative purposes in Fig. 1(a). Figure 1(b) turns the original physical capacitor into an equivalent model of two capacitors in parallel, one filled with a dielectric and the other unfilled. The left conducting plate in Fig. 1(a) is split into two with the same voltage in Fig. 1(b), and the same treatment applies to the right plate in Fig. 1(a). The voltage difference across the two capacitors is related to the free charges through $V_+ - V_- = \sigma_0 d/\varepsilon_0 = \sigma_{new} d/\varepsilon$, with d the distance between the two plates in Fig. 1(a). Most solutions provided in textbooks or exercise books stop here.

The equivalent model of Fig. 1(b) is indeed more intelligible than Fig. 1(a) and facilitates the evaluation of the effective capacitance involved in capacitor circuits. However, some discussion of the conditions and limitations of this homework problem remains. A necessary condition for this Fig. 1(a) setup is some external constraint needed to maintain the dielectric on the top half by force, for otherwise it would be pulled in toward the center due to the interaction between the dielectric and the plates. This can be understood by either a force or energy analysis. This homework problem also comes with a geometric limitation. Introductory physics learners are generally not used to seeing nonuniform charges on one conducting surface and rather are more used to, say, spherical conducting shells charged uniformly. The free charges with densities $+\sigma_{new}$ and $+\sigma_0$ are on the same left plate in Fig. 1(a), and it is tempting to wonder if this nonuniform configuration can be held at all. The answer is yes when invoking the geometric limitation that our capacitors in question are ideal, ignoring any deviations from ideality close to the dielectric–vacuum interface. The electric fields of both ideal capacitors in Fig. 1(b) are *closed* without fringe fields and electrically shield each other, so the bottom capacitor exerts no force on the surface charges on the $\pm\sigma_{new}$ plates, which are thus only acted upon by forces from the top capacitor itself along the horizontal direction. Therefore, those $\pm\sigma_{new}$ (and similarly $\pm\sigma_0$) free charges cannot move along their conducting surfaces and are able to stay in equilibrium. When nonideality is considered, the above argument is only true at places far away from the dielectric–vacuum interface.

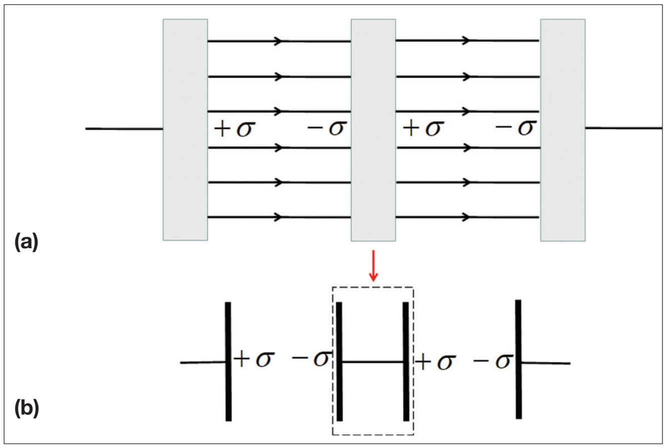

Fig. 2. Three infinitely large parallel conducting plates with surface charge densities shown. (a) Physical conducting plates. (b) Ideal capacitors in series, an equivalent model to (a). The middle plate in (a) is equivalent to the H-shaped two conducting plates connected with an ideal wire enclosed by the dashed box in (b).

Capacitors with dielectric in series

Before discussing capacitors with dielectric in series, it is necessary to consider a triple conductor-plate model, shown in Fig. 2(a), that will be associated with an equivalent model of two capacitors in series. With the charged configuration in Fig. 2(a) assumed, it can be modeled as two capacitors in series, as shown in Fig. 2(b), in which the middle plate in Fig. 2(a) is equivalent to the H-shaped isolated island. Both plates of the H-island in Fig. 2(b) represent both surfaces of the middle plate in Fig. 2(a), and the virtual ideal wire between the plates of the H-island causes the electric field to vanish in electrostatics. It is not that students cannot associate the configuration of Fig. 2(a) with the simplified model of Fig. 2(b), but they might lack the above convincing argument to support the equivalence between the two.

Now consider a capacitor filled with a dielectric on the right half in between plates charged with free charge densities $\pm\sigma$ [Fig. 3(a)], where $\sigma = 2\sigma_i$ and permittivity in the dielectric $\varepsilon = 2\varepsilon_0$ are taken as such for illustrative purposes. As the electric field in a dielectric is reduced by the dielectric constant, the electric field is denser in a vacuum (left). The electric field lines starting from the positive conducting plate terminate on either the surface induced charges on the left dielectric surface or the right conducting plate. The gaps between the surface charges and plates are negligible but have been exaggerated in Fig. 3. It is a usual practice to associate this model with two capacitors in series, as shown in Fig. 3(c).

Yet most students and instructors miss a necessary step transitioning from Fig. 3(a) to Fig. 3(c). Capacitors in series should be equally charged, but how the vacuum–dielectric interface in Fig. 3(a) is charged is still ambiguous. How could this interface be transformed into two plates with equal and opposite charges to fit into this model of capacitors in series? I propose a simple approach of adding a virtual conductor, infinitesimally thin and initially neutral, *just* to the left of this vacuum–dielectric interface shown in Fig. 3(b). The spacing

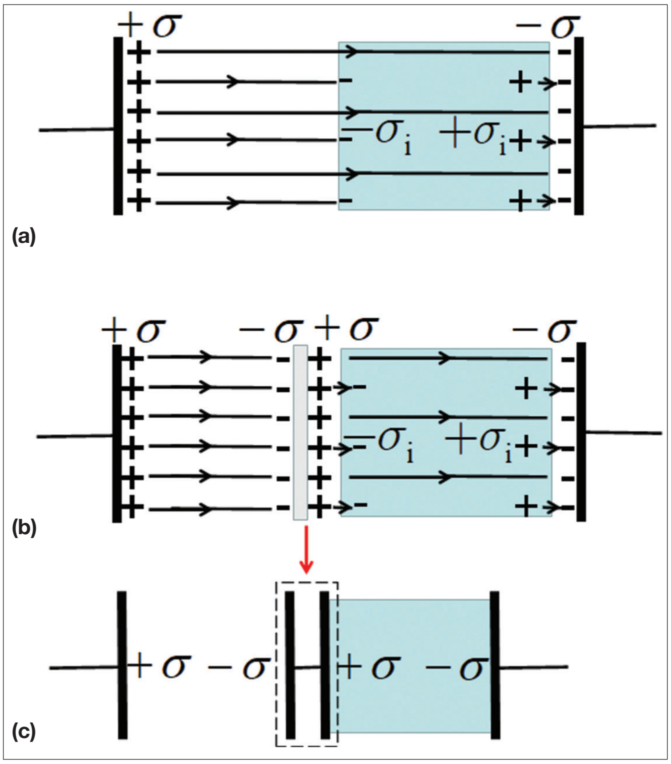

Fig. 3. Ideal capacitors filled with a dielectric on the right half. The free charge densities $\sigma = 2\sigma_i$ and permittivity in the dielectric $\varepsilon = 2\varepsilon_0$ are taken as such for illustrative purposes. (a) Physical capacitor with free $\pm\sigma$ and surface induced $\pm\sigma_i$ charge densities shown. The electric field lines terminate either at the left surface of the dielectric or the right plate of the capacitor. (b) Virtual infinitely thin conductor inserted *just* to the left of the dielectric in (a). All electric field lines starting from the left plate of the capacitor terminate on the virtual conductor. (c) Capacitors in series, an equivalent abstraction of (a) and (b). The virtual conductor in (b) is equivalent to the isolated H-shaped plates–wire combination enclosed in the dashed box.

between this virtual conductor and the interface is assumed negligible. Now the left surface of this virtual conductor will be induced with a density $-\sigma$ by the left $+\sigma$ plate, and the right surface of the virtual conductor will be induced with a density $+\sigma$ to conserve charge. The field lines starting from the positives on the "imaginary conductor" either terminate on the dielectric or on the right plate of the capacitor. Following the above approach in Fig. 2, the left/right surface of the virtual conductor in Fig. 3(b) corresponds to the left/right plate in the dashed enclosure in Fig. 3(c), and the interior of the virtual conductor is equivalent to the middle connecting wire in Fig. 3(c). This approach does an excellent job in fitting the single capacitor in Fig. 3(a) into the series model in Fig. 3(c) without affecting the essential physics. The strengths of the electric fields in the vacuum and dielectric in Fig. 3(a) are unaffected by this approach, and so is any voltage drop throughout.

Conclusions

Discussing these problems in class should help provide students with a better understanding of the relevant physics. One could imagine asking them how to analyze a situation like in Fig. 3 but where there are *two* different dielectrics between the plates, each of which takes up some fraction of the space between them.

A thorough analysis has been done on converting capacitors partially filled with a dielectric into models of capacitors in parallel and series, with emphasis on such matters frequently overlooked as how to best explain and visualize equivalent models with special treatments and how surface charges are mechanically balanced. Conditions and limitations of these models are also discussed. This work serves as supplementary reading material for introductory physics learners and instructors who seek solutions or alternative perspectives on ideal capacitor topics unavailable in textbooks.

Acknowledgments

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