An application of the energy–moment tensor relation to estimation of seismic energy radiated by point and line sources

Douglas W. McCowan and Adam M. Dziewonski

Applied Seismology Group, Lincoln Laboratory, Massachusetts Institute of Technology, 42 Carleton Street, Cambridge, Massachusetts 02142, USA

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Summary. We derive expressions for the seismic energy radiated from point and line sources in terms of the moment tensor of the source. Using moment tensors based on previously published fault plane solutions for the 1960 Chilean earthquake, the precursor to that event, and the 1964 Alaskan earthquake, we present power spectra for various fault length, rupture velocity, and source duration combinations. The results showed that, for point sources, a half-sine-wave increase in applied moment is necessary to insure a finite radiated energy asymptote at high frequencies. The results for a line source with finite propagation velocity were smoother than those for an equivalent point source. However, for reasonable combinations of fault plane parameters, the total radiated energy did not differ appreciably between the two source models. On the other hand, the energy radiated from a moving line source is quite sensitive to the rupture propagation velocity when that approaches the phase velocity of a mode. In our numerical experiments for these three events, we were always able to find combinations of source parameters which agreed with those from other investigators and that produced radiated energies consistent with either independent estimates or reported $M_\text{L}$ values.

1 Introduction

The advantages to expressing seismic radiation fields in terms of the moment tensor (Gilbert 1971) of the applied source are well known. The six independent quantities in this tensor provide information about the seismic source in the most general form; for example, it can describe a source representing a linear combination of three point source mechanisms: explosive, compensated linear vector dipole, and double-couple (Knopoff & Randall 1970). Furthermore, the time histories of the moment tensor elements can provide an insight into the nature of the processes in the source region (Dziewonski & Gilbert 1974). Here we show

*This work was sponsored by the Advanced Research Projects Agency of the Department of Defense.
†Present address: Department of Geological Sciences, Harvard University, Cambridge, Massachusetts 02138, USA.
how to solve the forward problem of directly calculating the radiated seismic energy if the
moment tensor (i.e. source mechanism) of an event and the earth structure are known.

The problem of evaluation of the energy associated with normal modes of the earth has
been treated by Backus & Gilbert (1967) and by Kovach & Anderson (1967). The latter
authors presented results of calculations of the kinetic energy associated with the funda-
mental spheroidal and toroidal modes normalized to 1 cm displacement at the earth's
surface; the effects of the source mechanism were not explicitly taken into account.

Abe (1970) considered the effects of the seismic source using Saito's (1967) formulæ for
excitation of normal modes. While Abe's theory is complete for the point sources and the
double couple source mechanism, he treats the case of a propagating fault using Ben-
Menahem's (1961) expression for surface waves, an approximation valid only for large
angular order numbers.

Our theory is developed in terms of the moment tensor approach of Gilbert (1971); the
moment tensor (or the moment rate tensor) may, in general, be a function of time or fre-
quency. Also, following the development of Dziewonski & Romanowicz (1977), we use the
exact expressions for a linear fault propagating with a constant rupture velocity. In our
numerical experiments we consider also the contribution of the overtones.

Presently, moment tensor source theories exist for two problems: the excitation of the
earth's free oscillations (Gilbert 1971) and the excitation of Love and Rayleigh surface
waves in a plane layered halfspace (McCowan 1976). Since the method is general it can be
applied to any dynamic elastic wave generation mechanism. Our development below is in
terms of the earth's free oscillations but the basic concept can be easily extended to other
cases.

2 Theory

Following Gilbert (1971), we can express the vector displacement of the earth's surface as
a sum over its normal modes

$$ u(r, t) = \sum_n s_n(r) \left[ M : S_n(r) \right] \frac{1 - \cos \omega_n t}{F_n \omega_n^2}. $$

In this expression $s_n$ is the displacement vector for the $n$th mode, $M$ is the moment tensor
of the source, $S_n(r)$ is the strain tensor of the $n$th mode evaluated at the source position,
$\omega_n$ is the angular frequency of the $n$th modes, and $F_n$ is the normalization integral

$$ F_n = \int \rho(r) |s_n(r)|^2 dv. $$

2.1 POINT SOURCE

The average kinetic energy over a cycle is

$$ \overline{\varepsilon_n(r)} = \frac{1}{2} \rho(r) \left| s_n(r) M : S_n(r) \right|^2 $$

This can then be integrated over the volume to give the kinetic energy in the mode, $K_n$.

For a non-attenuating system, the total energy is twice the kinetic energy

$$ E_n = 2K_n = \frac{1}{2} \left| \frac{M : S_n(r)}{F_n \omega_n} \right|^2. $$
The normalization integral (2) has been used in the numerator of this expression. Since a normal mode represents an oscillation between total kinetic and total potential energies, $E_n$ is also the total energy in the mode. The cumulative energy in the mechanical system is then the sum of these energy maxima because the normal modes are independent. This is

$$E = \frac{1}{2} \sum_n \left| \frac{M \cdot S_n(r_x)}{F_n \omega_n^2} \right|^2. \quad (5)$$

The relationship in (5) can be generalized for an event with an arbitrary time history if $M$ were to represent the spectrum of the moment rate tensor (Gilbert & Dziewonski 1975). In particular, the energy–moment tensor relations for the cases of a ramp function in applied moment and a half-sine-wave increase in applied moment both of duration $T$ and final value $M_0$ are easily shown to be

$$E = \frac{1}{2} \sum_n \left| \frac{M_0 \cdot S_n(r_x)}{F_n \omega_n^2} \right|^2 \left[ \left( \sin \frac{\omega_n T}{2} \right) \left( \frac{\omega_n T}{2} \right) \right]^2 \quad \text{(ramp)} \quad (6a)$$

$$E = \frac{1}{2} \sum_n \left| \frac{M_0 \cdot S_n(r_x)}{F_n \omega_n^2} \right|^2 \left[ \left( \pi^2 \cos \frac{\omega_n T}{2} \right) \left( \frac{\pi^2 - \omega_n^2 T}{2} \right) \right]^2 \quad \text{(half sine wave).} \quad (6b)$$

The three source time functions corresponding to equations (5), (6), and (7) are shown in Fig. 1. The third function, a half-sine-wave increase, is the same as used by Kanamori & Anderson (1975) for the precursor in their study of the 1960 Chilean earthquake.

### 2.2 Line Source

Dziewonski & Romanowicz (1977) derived expressions for excitation of normal modes by a line source that may be approximated by a fragment of a great circle, of angular extent $\Phi$,
connecting the beginning and the end points of the fault. They employed a specific rotation of the spherical coordinate system, first suggested in a different context by Backus (1964). Upon rotation, the fault represents a fraction of the equator where the origin point has a colatitude \( \pi/2 \), longitude 0, and the end point is at \( \pi/2, \Phi \). Assuming that the source mechanism does not change during the process of rupture, the expression for the spectral amplitude of a particular normal mode \( n \) of an angular order number \( l \) is

\[
\hat{u}_n(r, \omega) = \frac{C_n(\omega)}{F_n \omega_n^2} \sum_{m=-l}^{+l} s_n^m(r) \psi_n^m(\omega)
\]

where \( \hat{C}_n(\omega) \) is the resonance curve and

\[
\psi_n^m(\omega) = M_0(\omega) \cdot \frac{\sin \chi_m}{\chi_m} \exp(-i\chi_m) \approx M_0(\omega) \cdot \frac{\sin \chi_m}{\chi_m} \exp(-i\chi_m).
\]

If the moment release along the source line changes as \((\pi/2\Phi) \sin(\pi\phi/\Phi)\), then the excitation function is

\[
\psi_n^m(\omega) = M_0(\omega) \cdot \frac{\sin \chi_m}{\chi_m} \exp(-i\chi_m).
\]

\( M_0(\omega) \) is the spectrum of the moment rate tensor of the equivalent point source. Most frequently it is represented by a constant \( M_0(\omega) = M_0 \) (cf. Kanamori 1970), but it could be modelled to include the effect of the finite fault width, which would lead to an asymptotic \( \omega^{-2} \) decrease in amplitudes at high frequencies (Aki 1967).

\( S_n^m(r_0, \pi/2) \) is the strain tensor at coordinates \((r_0, \pi/2, 0)\), where \( r_0 \) is the constant source radius (depth). The expressions for \( S_n^m = E_n^m(r_0, \pi/2) \) can be found in Gilbert & Dziewonski (1975); equation (2.1.19) for the spheroidal modes and (2.1.20) for the toroidal modes.

Note that for \( \phi = 0 \)

\[
Y_l^m(\theta, \phi) = X_l^m(\theta) = (-1)^m \left[ \frac{2l + 1}{4\pi} \frac{(l - m)!}{(l + m)!} \right]^{1/2} P_l^m(\theta);
\]

computations of \( X_l^m \) for \( \theta = \pi/2 \) are particularly efficient.

The term

\[
\frac{\sin \chi_m}{\chi_m} \exp(-i\chi_m)
\]

where

\[
\chi_m = \frac{1}{2} \Phi \left( \frac{\omega_n r_0}{v} + m \right)
\]

is analogous to the directivity term of Ben-Menahem (1961, equation 2.19). Dziewonski & Romanowicz (1977) discuss this analogy in some detail.

Thus, the expression for the kinetic energy of the \( n \)th mode excited by a propagating fault is

\[
E_n = \frac{1}{2F_n \omega_n^2} \sum_{m=-l}^{+l} |M_0(\omega) \cdot S_n^m(r_0, \pi/2)|^2 \frac{\sin^2 \chi_m}{\chi_m^2}
\]

and for a modulated line source

\[
E_n = \frac{1}{2F_n \omega_n^2} \sum_{m=-l}^{l} |M_0(\omega) \cdot S_n^m(r_0, \pi/2)|^2 \left( \frac{\pi^2 \cos \chi_m}{\pi^2 - 4\chi_m^2} \right).
\]
It is obvious that when the fault length approaches zero, the expression above becomes identical to (4); also, when $\Phi \to 0$, $\Phi \omega_0 = T$, this expression is identical to (6a); similarly, application of equation (12b) will lead to (6b) under these conditions. Note that

$$\frac{1}{2\pi} \int_{-\infty}^{+\infty} |\tilde{C}_n(\omega)| \, d\omega = 1;$$

(13)

thus the kinetic energy of a mode is independent of attenuation.

To apply these expressions to actual events, it is necessary to have a complete catalogue of free oscillation frequencies and their associated strains. Then the formulae given by Gilbert & Dziewonski (1975) can be used to perform the sums and compute the actual energies. A suitable model of earth structure which satisfies a wide spectrum of standard earth data is, for example, Gilbert & Dziewonski's (1975) 1066A model for density, and compressional and shear velocities. For this model we had a set of all spheroidal modes $\nu S_j$ and toroidal modes $\nu T_j$ with periods greater than 45 s. This catalogue was computed by Buland & Gilbert (1975) and kindly made available to us.

3 Results

To illustrate the application of our method, we present results for three large events: the 1960 Chilean earthquake, the precursor to that event, and the 1964 Alaskan earthquake. These were large enough to excite the earth's free oscillations, observations of which were reported in numerous papers, and their measured eigenfrequencies were among the data used to derive model 1066A (Gilbert & Dziewonski 1975). The source mechanisms of these events were found, for example, by Plafker & Savage (1970), Kanamori & Anderson (1975), and Kanamori (1970) respectively. The pertinent data are listed in Table 1. Moment tensors were computed for these double-couple source models which, along with the catalogue of normal modes, were used to calculate the energies given below.

3.1 Point Source

Figs 2–4 show the effect of changing the point source time function on the spectral distribution of energy in the toroidal modes for the 1960 Chilean earthquake. The results are similar for the poloidal modes. These plots are cumulative energy in 0.004 rad/s intervals.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Chile, 1960 (1)</th>
<th>Chile Precursor, 1960 (2)</th>
<th>Alaska, 1964 (3)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Depth (km)</td>
<td>25</td>
<td>53</td>
<td>71</td>
</tr>
<tr>
<td>Strike (°)</td>
<td>10</td>
<td>10</td>
<td>246</td>
</tr>
<tr>
<td>Dip (°)</td>
<td>35.5</td>
<td>10</td>
<td>20</td>
</tr>
<tr>
<td>Slip (°)</td>
<td>270</td>
<td>90</td>
<td>270</td>
</tr>
<tr>
<td>Moment (dyne cm)</td>
<td>$8.6 \times 10^{29}$</td>
<td>$3.3 \times 10^{30}$</td>
<td>$7.5 \times 10^{29}$</td>
</tr>
<tr>
<td>$M_S$</td>
<td>8.3 (4)</td>
<td></td>
<td>8.5 (4)</td>
</tr>
<tr>
<td>Energy (erg)</td>
<td>$1.8 \times 10^{24}(5)$</td>
<td></td>
<td>$1.5 \times 10^{25}$</td>
</tr>
</tbody>
</table>

References
(1) Plafker & Savage 1970
(2) Kanamori & Anderson 1975
(3) Kanamori 1970
(4) USCGS
(5) Gutenberg–Richter relation
Figure 2. Spectral distribution of radiated energy for step source time function of applied moment. Results for the 1960 Chilean earthquake using Plafker & Savage's (1970) fault plane solution. Point source at 79.4 km depth.

Figure 3. Spectral distribution of radiated energy for a ramp source time function of applied moment. Results for the 1960 Chilean earthquake using Plafker & Savage's (1970) fault plane solution. 180° point source at 79.4 km depth.
Estimation of energy radiated by point and line sources

Figure 4. Spectral distribution of radiated energy for a half-sine-wave source time function of applied moment. Results for the 1960 Chilean earthquake using Plafker & Savage’s (1970) fault plane solution. 180 s point source at 79.4 km depth.

Figure 5. Energy as a function of source duration for the half-sine-wave source time function. Results for the 1970 Chilean earthquake using Plafker & Savage’s (1970) fault plane solution. Point source at 79.4 km depth. Magnitude from Gutenberg–Richter relation.
Figure 6. Energy as a function of source duration for the half-sine-wave source time function. Results for the 1970 Chilean earthquake precursor using Kanamori & Anderson's (1975) fault plane solution. Point source at 45.2 km depth. Magnitude from Gutenberg–Richter relation.

of angular frequency. The period range of the plots is from 18 000 s (to include the Slichter mode) at the left to 45 s at the right, a factor of 400 which is equivalent to a bandwidth of almost 9 octave; these limits encompass our normal mode catalogue.

The results in Fig. 2 show that the step function point source produces an energy spectrum which, in our bandwidth, increases as $\omega^{1.9}$ at short periods. If this asymptotic behaviour continues at shorter periods it implies an infinite radiated energy. Gilbert (1975) gives expressions for the high-frequency asymptote of the earth’s normal modes which, in fact, do confirm that equation (5) is divergent. The ramp function source results in Fig. 3 decrease as $\omega^{-0.32}$ which also, in our bandwidth, appear to be insufficient to guarantee convergence. Only the third source time function, a half-sine-wave increase, appears to produce an energy spectrum with a high-frequency asymptotic drop-off large enough to insure convergence. The short-period behaviour for this source (Fig. 4) varies as $\omega^{-2.6}$. We therefore reject the step source time function because our results supported by Gilbert’s (1975) asymptotic expressions indicate it leads to an infinite radiated energy. We also dismiss the ramp source time function because our results show that, at best, a significant amount of the radiated energy falls outside our bandwidth.

The dependence of the radiated energy on the source duration for all three events, assuming the half-sine-wave source time function, is shown in Figs 5–7. As can be seen in Figs 5 and 7, there is a substantial decrease in total radiated energy on the order of a factor of 400 between 0 and 200 s source duration. In Fig. 6 the decrease is more dramatic: a factor of $3 \times 10^5$ between 0 and 1000 s period. These energy ratios correspond to differences in $M_s$ of 1.8 and 3.6 units respectively. Furthermore, in all three cases this decrease is logarithmi-
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Figure 7. Energy as a function of source duration for the half-sine-wave source time function. Results for the 1964 Alaskan earthquake using Kanamori's (1970) fault plane solution. Point source at 45.2 km depth. Magnitude from Gutenberg–Richter relation.

cally non-linear, dropping off most rapidly at short source durations. The relative distribution of radiated energy into poloidal and toroidal components for the 1960 Chilean earthquake (Fig. 5) is also dependent on source duration. At short durations, the poloidal component dominates. At long durations, the radiated energy approaches equipartition between poloidal and toroidal components. This effect is also noticeable in the other two events, particularly so in the 1960 Chilean earthquake precursor (Fig. 6).

Our results for both the 1960 Chilean and 1964 Alaskan earthquakes show that, for shallow source depths, the radiated energy is relatively insensitive to changes in depth. In particular, increasing the source depth from 45 to 79 km increases the radiated energy by approximately half which is equivalent to only 0.1 surface-wave magnitude units.

The method also provides a means of directly calculating the energy in individual modes. One such mode, of interest in the dynamo theory of the earth's core, is $1S_1$, the Slichter mode (Slichter 1961). In all our calculations we never found an excitation energy for this mode larger than $10^{17}$ erg. This is in comparison with total radiated energies on the order of $10^{24}$–$10^{25}$ erg. Typical values are given in Table 2.

Table 2. Event energies for point source duration adjusted to give reported $M_s$ values.

<table>
<thead>
<tr>
<th>Event</th>
<th>$M_s$</th>
<th>Energy (erg)</th>
<th>Source duration (s)</th>
<th>Slichter energy (erg)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Chile 1960</td>
<td>8.3</td>
<td>$1.8 \times 10^{24}$</td>
<td>130</td>
<td>$6.3 \times 10^{16}$</td>
</tr>
<tr>
<td>Alaska 1964</td>
<td>8.5</td>
<td>$3.6 \times 10^{24}$</td>
<td>90</td>
<td>$2.0 \times 10^{16}$</td>
</tr>
</tbody>
</table>
Table 3. Total toroidal energy for the 1960 Chilean earthquake* as a function of fault length and rupture velocity.

<table>
<thead>
<tr>
<th>Toroidal energy (erg)</th>
<th>Fault length (km)</th>
<th>Rupture velocity (km/s)</th>
<th>Ratio (s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>8.96 x 10^{24}</td>
<td>200</td>
<td>3.5</td>
<td>57.1</td>
</tr>
<tr>
<td>3.98 x 10^{24}</td>
<td>400</td>
<td>3.5</td>
<td>114.3</td>
</tr>
<tr>
<td>1.85 x 10^{24}</td>
<td>600</td>
<td>3.5</td>
<td>171.4</td>
</tr>
<tr>
<td>9.10 x 10^{23}</td>
<td>800</td>
<td>3.5</td>
<td>228.6</td>
</tr>
<tr>
<td>4.84 x 10^{23}</td>
<td>1000</td>
<td>3.5</td>
<td>285.7</td>
</tr>
<tr>
<td>1.09 x 10^{23}</td>
<td>800</td>
<td>2.0</td>
<td>400.0</td>
</tr>
<tr>
<td>1.09 x 10^{23}</td>
<td>800</td>
<td>2.5</td>
<td>320.0</td>
</tr>
<tr>
<td>3.07 x 10^{23}</td>
<td>800</td>
<td>3.0</td>
<td>266.7</td>
</tr>
<tr>
<td>9.10 x 10^{23}</td>
<td>800</td>
<td>3.5</td>
<td>228.6</td>
</tr>
<tr>
<td>2.51 x 10^{23}</td>
<td>800</td>
<td>4.0</td>
<td>200.0</td>
</tr>
<tr>
<td>3.01 x 10^{23}</td>
<td>7.2</td>
<td>0.04</td>
<td>180.0</td>
</tr>
<tr>
<td>2.70 x 10^{23}</td>
<td>36.0</td>
<td>0.2</td>
<td>180.0</td>
</tr>
<tr>
<td>2.74 x 10^{23}</td>
<td>72.0</td>
<td>0.4</td>
<td>180.0</td>
</tr>
<tr>
<td>4.48 x 10^{23}</td>
<td>360.0</td>
<td>2.0</td>
<td>180.0</td>
</tr>
<tr>
<td>2.96 x 10^{24}</td>
<td>720.0</td>
<td>4.0</td>
<td>180.0</td>
</tr>
</tbody>
</table>

*Source at 79.4 km depth, Plafker & Savage (1970) fault plane parameters. Double-couple moment 8.6 x 10^{29} dyne cm in all calculations.

3.2 Line Source

The results for a modulated line source are presented in Table 3 and in Figs 8-10. Our choice of the model has been dictated by its asymptote \( \omega^{-2} \) amplitude behaviour at high frequencies, cf. equation (8b); in agreement with Aki (1967). These are all for the 1960 Chilean earthquake using Plafker & Savage's (1970) fault plane solution with the line source at 79.4 km depth. Table 3 lists the cumulative energy in the toroidal modes for a variety of fault length and rupture velocity combinations. Thus, one can see here the effects of three experiments: (1) changing the fault length with a constant rupture velocity (3.5 km/s), (2) changing the rupture velocity with a constant fault length (800 km), and (3) changing the fault length and rupture velocity but keeping the ratio constant (180 s). The most rapid variation in the energy values given in Table 3 occurs when the rupture velocity is comparatively high: 3.5–4.0 km/s.

Figs 8–10 show typical toroidal energy spectral distributions from the above-mentioned experiments. In particular, Fig. 8 results from a low rupture velocity (2.0 km/s). There the spectral dropoff rate is similar to the half-sine-wave point source but the scallops in the power envelope are not as prominent. Fig. 9 shows the results from a high rupture velocity (4.0 km/s). These are similar to those for a step function point source (Fig. 2). This is explained by the fact that a moving source radiates energy most efficiently in its direction of travel and is spectrally enhanced at high frequencies by the Doppler effect. Seen from the front, where the radiated energy is concentrated, it would appear as a step function point source. Finally, Fig. 10 shows the results from intermediate values of fault length and rupture velocity. Here the energy distribution is smooth with a peak near 160-s period. The distribution also drops off cleanly above this peak.

4 Discussion and conclusions

Energies for half-sine-wave point source durations adjusted to give the reported \( M_s \) values for the 1960 Chilean and 1964 Alaskan earthquakes are given in Table 2. The corresponding
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Figure 8. Spectral distribution of radiated energy for the 1960 Chilean earthquake for an 800 km fault with 2.0 km/s rupture velocity. Line source at 79.4 km depth using Plafker & Savage's (1970) fault plane solution.

Figure 9. Spectral distribution of radiated energy for the 1960 Chilean earthquake for a 720 km fault with 4.0 km/s rupture velocity. Line source at 79.4 km depth using Plafker & Savage's (1970) fault plane solution.
source duration times, 130 and 90 s respectively, can be interpreted in terms of the finiteness of the sources. Thus, although they only strictly apply to our point source model, they can be interpreted as point source representations of larger sources which, because of their dimensions, require finite time to act. In the first case Kanamori & Cipar (1974) have estimated the fault dimensions and rupture velocity for the Chilean earthquake to be $800 \times 200 \text{ km}^2$ and $3.5 \text{ km/s}$ respectively. Kanamori (1970) estimated the same for the Alaskan earthquake to be $500 \times 300 \text{ km}^2$ and $3.5 \text{ km/s}$ respectively. In both cases our point source times are close to the times that it would take the rupture to propagate half the estimated fault length.

In the excitation of the earth’s free oscillations there is a trade-off between double couple moment and dip angle. For shallow dip-slip events, the strains $S_{r\theta}$ and $S_{r\phi}$ are very small, as they vanish at the free surface, and the quantity which determines the amplitude of the modes is the product of the moment and the sine of twice the dip angle. The effect of this trade-off is to reconcile the larger moment reported for the 1960 Chilean earthquake by Kanamori & Cipar (1974) with that reported by Plafker & Savage (1970). To obtain equal amplitudes one needs approximately three times greater moment for a shallow dip angle of $10^\circ$ (Kanamori & Cipar 1974) than for an angle of $35^\circ$ (Plafker & Savage 1970).

Kanamori (1970) estimated the released strain energy from the 1964 Alaskan earthquake to be $1.5 \times 10^{25} \text{ erg}$ using a method due to Aki (1966). Our result, $3.6 \times 10^{26} \text{ erg}$, for a half-sine-wave point source of 90-s duration is smaller by almost a factor of four. This, we feel, may be due to our rather simple point source model and, also, the limited bandwidth ($T > 44 \text{ s}$) of our calculations. On the other hand, Kanamori’s energy estimate is larger than the Gutenberg–Richter value by almost half a surface-wave magnitude unit.

Our point source results for the precursor to the 1960 Chilean earthquake using Kanamori & Anderson’s (1975) moment and fault plane parameters show that a precursor having a

Figure 10. Spectral distribution of radiated energy for the 1960 Chilean earthquake for an 800 km fault with 3.5 km/s rupture velocity. Line source at 79.4 km depth using Plafker & Savage’s (1970) fault plane solution.
half-sine-wave source duration of 300 s will release energy equivalent to a 7.8 mag event. This is clearly large enough to have been observed. In fact it may be the precursor reported to have occurred 15 min and 23 s before the main shock (USCGS 1960). However, as Kennett & Simons (1976) noted concerning a precursor to the 1970 Columbian earthquake, events with long source durations are difficult to observe directly at large distances from the source.

The Slichter mode \( (1, S_1) \) calculations presented in Table 2 show point source excitation energies much less than those estimated by Won & Kuo (1973). Their work indicated that, in a \( 10^{25} \) erg earthquake, \( 2 - 4 \times 10^{21} \) erg would be found in the \( 1, S_1 \) inner core oscillation. Our results show that this figure is high by at least four orders of magnitude which supports the previous conclusion by Smith (1976). Thus, since Won & Kuo concluded that the Slichter mode was almost within the possibility of observation, our results and Smith’s place it well below that level and probably beyond direct observation.

Results for the line source model with finite rupture velocity do not substantially alter these conclusions. In particular, the cumulative toroidal energy which we computed for the 1960 Chilean earthquake using the fault length and rupture velocity values estimated by Kanamori & Cipar (1974) (800 km and 3.5 km/s) is \( 9.1 \times 10^{23} \) erg. This corresponds to a half-sine-wave point source duration of 140 s (see Fig. 5) which is very close to that value obtained when the source duration was adjusted to give the reported \( M_L \) value. The total energy radiated from our smooth model of the Chilean earthquake is approximately equal to that estimated from the Gutenberg–Richter relation. Since the actual faulting is probably more complicated than that represented by our smooth model, the total energy actually radiated is probably larger than estimated here. Probably the Gutenberg–Richter relation underestimates the energy radiated by these large earthquakes.

The principal difference between the point source and line source results is in the shape of the power spectrum envelope. Going to a more realistic finite source with a propagating rupture gives a smoother power spectrum. However, increasing the rupture velocity to 4.0 km/s from 3.5 km/s produces an unacceptable envelope shape which, in our frequency band, resembles that due to a step function point source. Since the shear velocity of the 1066A model at 80 km depth is approximately \( 4.5 \) km/s, we conclude that, for this event, the radiated energy is quite sensitive to rupture propagation velocity approaching the source region shear velocity.

The following two general conclusions can also be drawn from these results. First, from the shape of our spectral energy distribution curves (Figs 2–4 and 8–10), it can be concluded that the long-period band, 18000 to 45 s period, contains a substantial amount of the strain energy released in earthquakes. In fact, a spread of plausible source duration times can usually be found which brackets the energy determined from \( M_L \) values. This leads to the second general conclusion. The total radiated energy released by an earthquake (and calculated by using our method) depends significantly on the source duration. For both the 1960 Chilean and 1964 Alaskan earthquake mechanisms, the \( M_L \) difference between source time functions having durations a few tens of seconds apart should be observable, assuming an observation error in \( M_L \) of 0.2 units. Clearly the rate at which strain energy is released in an earthquake source region is a prime consideration in gauging the size and destructive power of earthquakes.

Finally, we would like to note that the events we studied were relatively shallow thrust events. Since it is apparent from the energy–moment tensor relation that the radiated energy depends on the fault plane solution and source depth, our results cannot be extended to deep events of different source mechanisms without modification. In particular the trade-off between moment and depth will become much more complicated for deep events.
References


